Jiří Adámek; Jan Reiterman

Cartesian closed hull of uniform spaces

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Jiří Adámek and Jan Reiterman

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A concrete category is called cartesian closed topological CCT if it is initially complete, fibre small and has canonical hom-objects. The CCT-hull of a concrete category, introduced by H. Herrlich and L.D. Nel, is the least CCT category in which the original category is a concrete, full, finitely productive subcategory.

Definition. A bornology on a set X is a collection $\mathcal{B}$ of its subsets (called bounded subsets) such that (i) each finite subset is in $\mathcal{B}$, (ii) if B_1, $B_2 \in \mathcal{B}$ then $B_1 \cup B_2 \in \mathcal{B}$, (iii) $B \in \mathcal{B}$ implies $B' \in \mathcal{B}$ for all $B' \subset B$. A bornological uniform space is a triple $(X, \mathcal{U}, \mathcal{B})$ where $(X, \mathcal{U})$ is a uniform space, $\mathcal{B}$ is a bornology on X such that each set $A \subset X$ with the property

"for every cover $\alpha \in \mathcal{U}$ there is $B \in \mathcal{B}$ with $A \subset \mathrm{st}_\alpha B$" is in $\mathcal{B}$. Morphisms $f : (X, \mathcal{U}, \mathcal{B}) \longrightarrow (Y, \mathcal{V}, \mathcal{C})$ of bornological spaces are those maps $f : X \rightarrow Y$ which preserve bonded sets and are uniformly continuous on bounded sets. Each uniform space is regarded as a bornological uniform space with bornology consisting of all subsets.

Theorem. The CCT-hull of the category Unif of uniform spaces is the category Bunif of bornological uniform spaces.