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Dual Ramsey type theorems

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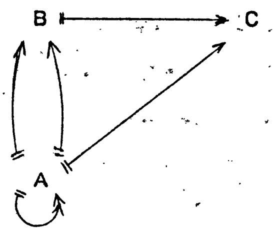
Dual Ramsey type theorems

J. Nešetřil and V. Rödl

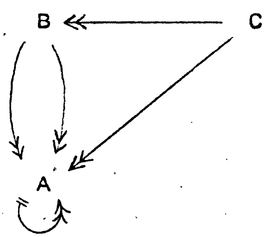
The finite Ramsey theorem deals with the following schema:

(*)



in the category Set of all finite sets and all 1-1 mappings. This schema may be considered also in other categories as done by Deuber, Leeb, Graham, Rothschild, Voigt and authors.

Here we consider the schema (*) in the category Set^{op} :



<u>Theorem</u> (dual Ramsey theorem):

Set^{op} is the Ramsey category.

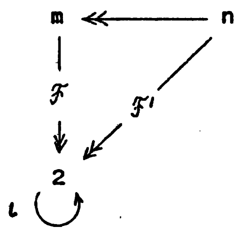
Explicitely:

For every k,p,m there exists n with the following property: for every coloring of the set $\Pi_p(X)$ of all p-partitions of a set X with at least n elements there exists an m-partition π of X (i.e. $\pi \in \Pi_m(X)$) such that all the p-partitions of X which are coarser than π (i.e. all those p--partitions of which π is a refinement) are colored by one color only. (Of course partitions are just onto-mappings modulo a permutation of the range.)

This theorem was proved in this form in 1979 by the authors. However, implicitely, it was proved in about the same time by B. Voigt (in the context of Ramsey properties of distributive lattices) and earlier by Graham-Rothschild (1969) as a special case of parameter sets.

The fact that Set^{op} has 2-Ramsey property was known earlier (Rado-Sanders-Folkman). In this special case several strenghtenings of the above theorem can be proved. Their generalizations to k-Ramsey properties for $k > 2$ are presently not known.

<u>Theorem</u> (induced Rado theorem):

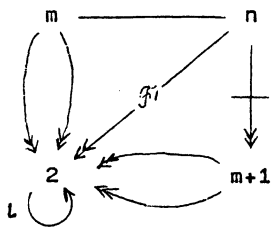


For every family $\mathfrak{F}$ of onto-mappings $m \twoheadrightarrow 2$ and for every k there exists a family $\mathfrak{F}'$ of onto mappings $n \twoheadrightarrow 2$ such that for every k-coloring c of $\mathfrak{F}'$ which satisfies $c(f) = c(\iota f)$ for every $f \in \mathfrak{F}'$ there exists an onto-mapping $g : n \twoheadrightarrow m$ such that

i. $fg \in \mathfrak{F}'$ iff $f \in \mathfrak{F}$;

ii. c restricted to the set $\mathfrak{F}g = \{fg;\ f \in \mathfrak{F}\}$ is a constant.

<u>Theorem</u> (forbidden Rado theorem):



For every m and k there exists a family $\mathcal{F}'$ of onto-mappings $n \twoheadrightarrow 2$ with the following properties:

i. There does not exist an onto mapping $g : n \twoheadrightarrow m+1$ such that $fg \in \mathcal{F}'$ for every $f : m+1 \twoheadrightarrow 2$;

ii. for every k-coloring c of $\mathcal{F}'$, $c(f) = c(\iota f)$ for every $f \in \mathcal{F}'$, there exists $g : n \twoheadrightarrow m$ such that $fg \in \mathcal{F}'$ for every $f : m \twoheadrightarrow 2$ and c restricted to this set is a constant.

The notions not defined in this paper may be found in:

J. Nešetřil, V. Rödl: Partition theory and its applications. In: Surveys in Combinatorics, Cambridge University Press, Cambridge (1979), 96-156.

Proofs are going to appear elsewhere.