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LIE DERIVATIVES AND NATURAL OPERATORS

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Given two manifolds M, N, a smooth map $f: M \to N$, a vector field ξ on M and a vector field η on N, one defines the Lie derivative of f with respect to ξ and η by

$$L_{(\xi,\eta)}f := Tf \circ \xi - \eta \circ f : M \to TN.$$

Let F, G be two prolongation functors of the category of smooth manifolds into the category of smooth fibered manifolds. Every vector field ξ on M is prolonged into a vector field $F\xi$ on FM and into another vector field $G\xi$ on GM, [1]. Having a base-preserving morphism $\varphi : FM \to GM$, we define its Lie derivative with respect to ξ by

$$L_\xi \varphi := L_{(F\xi, G\xi)}\varphi : FM \to VGM,$$

where V denotes the vertical tangent bundle. If the values of G lie in the subcategory of smooth vector bundles, then $L_\xi \varphi$ can be considered as a map $L_\xi \varphi : FM \to GM$.

Let $\bar{F}$, $\bar{G}$ be two further prolongation functors. An operator A transforming any base-preserving morphism $\varphi : FM \to GM$ into a base-preserving morphism $A\varphi : \bar{F}M \to \bar{G}M$ is called natural, if it holds

$$A((Gf)^{-1} \circ \varphi \circ Ff) = (\bar{G}f)^{-1} \circ A\varphi \circ \bar{F}f$$

for every diffeomorphism f. Operator A is said to be regular, if it transforms any smoothly parametrized family of morphisms into a smoothly parametrized family.

Theorem. Let F, G, $\bar{F}$, $\bar{G}$, ξ, φ and A be as above and let the values of G and $\bar{G}$ lie in the category of smooth vector bundles.

If A is natural, $\mathbb{R}$-linear and regular, then A commutes with the Lie derivative, i.e.

$$L_{\xi}(A\varphi) = A(L_{\xi}\varphi)$$

for all φ and ξ.

REFERENCES

[1] S. A. SALVIOLI, On the theory of geometric objects, J. Differential Geometry, 7(1972), 257-278