Boris A. Pasynkov

Projection spectra and dimension

In: (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the symposium held in Prague in September 1961. Academia Publishing House of the Czechoslovak Academy of Sciences, Prague, 1962. pp. 298--300.

Persistent URL: http://dml.cz/dmlcz/700988

PROJECTION SPECTRA AND DIMENSION

B. PASYNKOV

Moscow

1.

1. We shall consider for a bicompactum X three types of inverse spectra $S = (X_\alpha, \pi_\alpha^{\alpha'})$:

a) *Combinatorial spectra* – the X_α are finite complexes (= finite T_0-spaces),[1]) and the projections are continuous mappings of T_0-spaces.

b) *Polyhedral spectra*: the X_α are polyhedra, the projections $\pi_\alpha^{\alpha'}$ are continuous ("into").

c) *Simplicial spectra*: the X_α are polyhedra, each projection $\pi_\alpha^{\alpha'}$ is a simplicial continuous mapping of the polyhedron $X_{\alpha'}$ (with a certain triangulation) into the polyhedron X_α (also with a certain triangulation).

2. Let us define the dimension of each spectrum $S = (X_\alpha, \pi_\alpha^{\alpha'})$ as

$$\text{ind } S = \sup_{\alpha \in S} \text{ind } X_\alpha ;$$

thus for a bicompactum X there result the combinatorial dimension $\dim_c X$, the polyhedral dimension $\dim_p X$ and the simplicial dimension $\dim_s X$, each defined as the minimum of dimensions ind S of all spectra of the given kind (combinatorial, polyhedral, simplicial) having the bicompactum X as limit space.

It is known that every bicompactum is the limit space of a simplicial spectrum (with projections which are in general not onto) — this is proved in the monograph [1] of S. Eilenberg and N. Steenrod; there are still older results of P. Alexandroff and A. Kurosch stating that every bicompactum is the limit space of a combinatorial spectrum (whose elements are finite simplicial complexes in the classical sense with projections onto); the Alexandroff-Kurosch theorem has been generalized to paracompact spaces by V. Ponomarev (see his communication).

3. The following results seem to be new (for the proofs see [2] to appear in the Matematičeskij Sbornik).

I. There exist bicompacta which cannot be represented as limit spaces of polyhedral (a fortiori of simplicial) spectra with projections "onto".

---

[1]) Every finite T_0-space can be realized as a finite simplicial complex in the general sense: a face of a simplex of the given complex may not belong to this complex.

II. The following relations hold for every bicompactum:

$$\dim X \leqq \dim_p X \leqq \dim_s X ,$$
$$\operatorname{Ind} X \leqq \dim_c X \leqq \dim_s X .$$

If $\dim_p X \leqq 1$ then moreover

$$\operatorname{Ind} X \leqq \dim_p X .$$

III. There exists a bicompactum X with

$$\dim X = \operatorname{ind} X = \operatorname{Ind} X = \dim_c X = 1$$

and

$$\dim_p X > 1 .$$

IV. For $n = 1, 2, 3, \ldots$ there exist bicompacta X_n with

$$\dim X_n = \operatorname{ind} X_n = \operatorname{Ind} X_n = \dim_c X_n = 1$$

and

$$\dim_s X = n .$$

These results shows a certain analogy with the beautiful results of P. VOPĚNKA (concerning $\dim X$, $\operatorname{ind} X$, $\operatorname{Ind} X$).

V. The "dimensional sum theorem" for a countable number of summands holds neither for $\dim_p X$ nor for $\dim_s X$; it does not hold for $\dim_c X$ even for two summands.

The following questions remain open, as far as I know:

a) Does there exist a bicompactum X with

$$\operatorname{Ind} X < \dim_c X .$$

b) Is the sum theorem true for $\dim_p X$ and $\dim_s X$ in the case of a finite numbers of summands.

2.

By means of inverse spectra of the form $S = (X_\alpha, \pi_\alpha^\beta)$, where the X_α are Hausdorff spaces (and the projections are continuous) the following theorem can be proved (see [3], [4]).

Theorem[2]). *Let G be a local bicompact group and H a closed subgroup of G. Then for the quotient space $X = G/H$ the following identity holds*:

$$\operatorname{ind} X = \operatorname{Ind} X = \dim X = \operatorname{ind} G - \operatorname{ind} H .$$

(As a corollary we obtain that

$$\operatorname{ind} G = \operatorname{Ind} G = \dim G , \quad \operatorname{ind} H = \operatorname{Ind} H = \dim H) .$$

For the case $\operatorname{ind} X < \infty$ (which includes the case $\operatorname{ind} G < \infty$), as well as for the case $\operatorname{ind} H < \infty$ I gave a direct proof of this theorem; in the infinite dimensional case the following theorem of E. SKLYARENKO [5] has been used: If $\dim X = \infty$, then X contains a topological image of the infinite dimensional Hilbert cube.

[2]) This theorem answers a problem raised by E. MICHAEL.

References

[1] *S. Eilenberg* and *N. Steenrod*: Foundations of Algebraic Topology. Princeton, 1952.

[2] *Б. Пасынков:* О спектрах и размерности топологических пространств. Матем. сб. (в печати).

[3] *Б. Пасынков:* Об обратных спектрах и размерности. Докл. АН СССР *138* (1961), 1013—1015.

[4] *Б. Пасынков:* О совпадении различных определений размерности для локально бикомпактных групп. Докл. АН СССР *132* (1960), 1035—1037.

[5] *Е. Скляренко:* О бесконечномерных однородных пространствах, Докл. АН СССР *141* (1961). 811—813.