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ALGEBRAIC PROPERTIES OF FUNCTION SPACES

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Bucureşti

Let X be an H-space with multiplication $\mu : X \times X \to X$ and inversion $\nu : X \to \to X$. The basic commutator map is the composition

$$\varphi : X^2 \xrightarrow{\Delta} X^2 \times X^2 \xrightarrow{1^2 \times \nu^2} X^2 \times X^2 \xrightarrow{\mu \times \mu} X \times X \xrightarrow{\mu} X$$

in which $X^2 = X \times X$, Δ is the diagonal map and 1 is the identity map; the commutator map of weight $n + 1$ is the composition

$$\varphi_{n+1} : X^{n+1} = X^n \times X \xrightarrow{\varphi_n \times 1} X \times X \xrightarrow{\varphi} X$$

in which φ_n is the commutator map of weight $n \geqq 1$ with $\varphi_1 = 1$. The nilpotency class nil X is defined as the least integer $n \geqq 0$ for which φ_{n+1} is nullhomotopic; if no such integer exists, we put nil $X = \infty$. Next, for any space X define $\cup$-long X as the least integer $n \geqq 0$ such that for any commutative coefficient field the cup product of any $n + 1$ singular cohomology classes of positive dimension vanishes; also, let wcat X denote the least integer $n \geqq 1$ for which the composition

$$X \xrightarrow{\Delta} X^n \xrightarrow{p} X^{(n)}$$

is nullhomotopic (the symbol $X^{(n)}$ stands for the smashed n-fold product).

Let X be a Hausdorff space with base-point $a \in X$ and let G be an arbitrary H-space with unit $e \in G$. The compact-open topologized space $(G, e)^{(X,a)}$ of all continuous maps $(X, a) \to (G, e)$ is an H-space, and the main result of this paper consists of the inequalities

$$\cup\text{-long } X \leqq \sup \text{nil} (G, e)^{(X,a)} \leqq \text{wcat } X - 1$$

in which G ranges over all H-spaces. The second inequality improves a result due to G. W. Whitehead (Comment. Math. Helv. *28* (1954), 320—328). Proofs and further details may be found in a joint paper by I. Berstein and the present author (Illinois J. Math. *5* (1961), 99—130).