

Toposym 1

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ALGEBRAIC PROPERTIES OF FUNCTION SPACES

T. GANEA

București

Let X be an H -space with multiplication $\mu : X \times X \rightarrow X$ and inversion $\nu : X \rightarrow X$. The basic commutator map is the composition

$$\varphi : X^2 \xrightarrow{\Delta} X^2 \times X^2 \xrightarrow{1^2 \times \nu^2} X^2 \times X^2 \xrightarrow{\mu \times \mu} X \times X \xrightarrow{\mu} X$$

in which $X^2 = X \times X$, Δ is the diagonal map and 1 is the identity map; the commutator map of weight $n + 1$ is the composition

$$\varphi_{n+1} : X^{n+1} = X^n \times X \xrightarrow{\varphi_n \times 1} X \times X \xrightarrow{\varphi} X$$

in which φ_n is the commutator map of weight $n \geq 1$ with $\varphi_1 = 1$. The nilpotency class $\text{nil } X$ is defined as the least integer $n \geq 0$ for which φ_{n+1} is nullhomotopic; if no such integer exists, we put $\text{nil } X = \infty$. Next, for any space X define $\cup\text{-long } X$ as the least integer $n \geq 0$ such that for any commutative coefficient field the cup product of any $n + 1$ singular cohomology classes of positive dimension vanishes; also, let $\text{wcat } X$ denote the least integer $n \geq 1$ for which the composition

$$X \xrightarrow{\Delta} X^n \xrightarrow{p} X^{(n)}$$

is nullhomotopic (the symbol $X^{(n)}$ stands for the smashed n -fold product).

Let X be a Hausdorff space with base-point $a \in X$ and let G be an arbitrary H -space with unit $e \in G$. The compact-open topologized space $(G, e)^{(X, a)}$ of all continuous maps $(X, a) \rightarrow (G, e)$ is an H -space, and the main result of this paper consists of the inequalities

$$\cup\text{-long } X \leq \sup \text{nil } (G, e)^{(X, a)} \leq \text{wcat } X - 1$$

in which G ranges over all H -spaces. The second inequality improves a result due to G. W. WHITEHEAD (Comment. Math. Helv. 28 (1954), 320—328). Proofs and further details may be found in a joint paper by I. BERSTEIN and the present author (Illinois J. Math. 5 (1961), 99—130).