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Proximity and construction of compactifications with given properties

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PROXIMITY AND CONSTRUCTION OF COMPACTIFICATIONS WITH GIVEN PROPERTIES[1])

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A. A topological non-compact space X may have many compactifications. For example, if X is an infinite countable discrete space then it can be completed into a compactification $cX = X \cup N$ by an arbitrary separable compact space N. On the other hand, not every space N can be the remainder $cX \setminus X$ for a fixed space X. In fact, a disconnected space N cannot be the remainder of a half-line or of an Euclidean space E^n with $n \geqq 2$. And therefore it is natural to ask this question: *how the properties of a space X determine which spaces N can complete the space X into a compactification and which not?* The following question, interesting in itself, is connected with it: *how the properties of a space X_c, as a subspace of a fixed compactification cX, determine whether or not the remainder $N_c = cX \setminus X_c$ has some considered properties?*

We shall deal with the first question in the following form:

Problem I. *Find necessary and sufficient conditions $\mathcal{R}_{\mathcal{P}}$, at least for the class of spaces with a countable base, for each space X of this class to have at least one remainder $N = cX \setminus X$ with some given topological property $\mathcal{P}$.*

The second question is essentially a question on a duality of some topological properties of N_c with some properties of its complement $X_c = C \setminus N_c$ (in a given compact space C). However, not quite: first, C is an arbitrary compact space and need not be a manifold, secondly, the set X_c is not arbitrary (it is dense in C), thirdly, the sought for properties of X_c are not simply topological, but "proximal". For example, in a closed ball, a boundary point N_0, a closed arc N_1 of a meridian and a closed disc N_2 of the boundary sphere have homeomorphic complements. What does the word "proximal" mean?[2])

B. *It is well-known that every compactification cX of a space X defines, in a quite natural way, a binary relation A c B (A is proximal to B) between subsets*

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[1]) A short communication on this theme was presented in August 1966 at the International Congress of Mathematicians in Moscow. A detailed publication will appear in Matematičeskiĭ Sbornik.

[2]) The reader familiar with the theory of proximity spaces may pass immediately to Section C.

of X, by the formula:

(c) $A \, c \, B \Leftrightarrow \bar{A}^c \cap \bar{B}^c \neq \emptyset$.[3])

It is easy to verify that the following conditions are satisfied:

(s) $A \, c \, B \Rightarrow B \, c \, A$,
(m) $A' \supset A,\ A \, c \, B \Rightarrow A' \, c \, B$,
(a) $(A \cup A') \, c \, B \Rightarrow A \, c \, B$ *or* $A' \, c \, B$,
(i) $x \in X \Rightarrow \{x\} \, c \, \{x\}$,
($\emptyset$) $\emptyset \, \bar{c} \, X$,[4])
(t) $A = \bar{A} \Leftrightarrow \{x\} \, c \, A$ *implies* $x \in A$.

If cX is a T_1-space, respectively, a T_2-space, then

(o_p) $\{x\} \, c \, \{x'\} \Rightarrow x = x'$,

respectively,

(o_s) *If* $A \, \bar{c} \, B$ *then there exist sets* A' *and* B' *such that* $A \, \bar{c} \, B'$, $A' \, \bar{c} \, B$ *and* $A' \cup B' = X$.

A binary relation c, satisfying conditions (s), (m), (a), (i) *and* ($\emptyset$) *on an abstract set X, is said to be a proximity relation, or shortly a proximity, and the set X together with the proximity relation c is said to be a proximity space* (*or a general proximity space*).[5])

It is easy to see that any proximity *c induces*, in accordance with formula (t), a topology on a set X. Conditions (o_p) and (o_s) are of character of separating axioms (for points or, respectively, for sets).

The main theorem of the theory of proximity spaces asserts that *the correspondence* $\varphi : \{cX\} \to \{c\}$, *defined by formula* (c), *is an isotonic*[6]) *one-to-one mapping of the set of all compactifications cX of a given space X onto the set of all proximities c inducing the topology of the space X* (see [23], theorems 10 and 11).

Here, and in the sequel, it is natural to consider *Hausdorff* compactifications cX and hence *completely regular* spaces X only and proximity spaces (proximities) are considered only those satisfying the separating axioms (o_p) and (o_s).

The inverse correspondence φ^{-1} can be obtained in various ways. For example, using *maximal centred systems* of sets (the "*ends*", see [23] p. 551–552), having appeared in the papers of P. S. Alexandroff [2], Freudenthal [10] and Carathéodory [7], using Gel'fand-Kolmogoroff-Šilov's *theory of rings of functions* (see [9] and [27]) or, finally, in the following way:

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[3]) $\emptyset$ is the void set, $\bar{A}^c$ is the closure of A in cX.

[4]) $A \, \bar{c} \, B$ denotes that A and B are *distant* (i.e. non-proximal), $\bar{A}$ is the closure of A in X.

[5]) The origins of the theory of proximity spaces can be found in the papers of Riesz [6], [18], Wallace [33] and, of Efremovič [8], who introduced the important axiom (o_s) on separation. A sufficiently elaborated theory of proximity spaces has appeared in the papers [23], [24], [25], [26].

[6]) I.e. order-preserving in both directions.

Let X_c and Y_d be proximity spaces with proximities c and d. A mapping $f : X \to Y$ is called *proximally continuous* ("δ-mapping", see [23], p. 550) if the images of proximal sets are proximal, i.e. if $A\, c\, B \Rightarrow fA\, d\, fB$.

Given a proximity space X_c, consider the set $C(X_c)$ of all real-valued bounded proximally continuous functions $f_\alpha : X \to I_\alpha$ (I_α is a minimal segment of reals containing the image $f_\alpha X$). Then *the canonical mapping* $f : X \to \prod I_\alpha$ (*where* $f(x) = \{f_\alpha(x)\}$) *of* X_c *into the cartesian product of all* I_α*'s, endowed with the Tychonoff topology, is one-to-one and proximally continuous in both directions, i.e. it is a proximal embedding of the space* X_c *into the Tychonoff cube* $\prod I_\alpha$, *and the closure of* fX *in this cube is a compactification of the space* X *inducing* (by formula (c)) *the given proximity* c (see [26]).

C. It follows that *a property of the space* X_c, *corresponding to a compactification* cX, *to have a remainder* $N_c = cX \setminus X$ *with a given topological property* $\mathscr{P}$ *is a proximal property* (i.e. it is invariant under proximal homeomorphisms). Now the first question may be put in the proximal form: *Find necessary and sufficient conditions* $\mathscr{R}_{\mathscr{Q}}$ *for every space* X *of a considered class to possess at least one proximity* c, *such that the space* X_c *has a given proximal property* $\mathscr{Q}$. Therefore the second question is nothing else but a question of a "translation" of the language of topological properties of the remainder N_c into the language of proximal properties of the space X_c:

Problem II. *Find, at least for the class of the spaces with a countable base, necessary and sufficient conditions* $\mathscr{Q}_{\mathscr{P}}$ *for the remainder* $N_c = cX \setminus X$ *to have a given topological property* $\mathscr{P}$, *for each proximity space* X *of this class.*

D. The present report is devoted to solving these two problems for some topological properties $\mathscr{P}$.

First, notice that Problems I and II have relatively simple solutions for the following properties, in the class of all completely regular spaces (respectively, all proximity spaces: $\mathscr{P}_0$ – *compactness*, $\mathscr{P}_{1(n)}$ – *consisting of* n *points* ($n = 0, 1, 2, \ldots$), $\mathscr{P}_2$ – *connectedness*. Problems I and II have non-simple solutions for the following properties: $\mathscr{P}_3$ – *Lindelöf property*,[7]) $\mathscr{P}_{4(n)}$ – *dimension at most* n.[8])

It is interesting that the properties $\mathscr{Q}_{\mathscr{P}_3}$ and $\mathscr{R}_{\mathscr{P}_3}$ coincide and they are of purely topological character:

$\mathscr{R}_{\mathscr{P}_3}$ – *for each compact subset* K' *of the space* X (*respectively*, X_c) *there exist a compact subset* K *and a countable system of neighbourhoods* U_n *of* K *such that* $K' \subset K$ *and if* O *is a neighbourhood of* K *then* $U_n \subset O$ *for some* n.

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[7]) See Isbell [13], Smirnov ([21], p. 446–447).

[8]) For $n = 0$ see Freudenthal [11], [12], Morita [16], Skljarenko [20]. For arbitrary n see Aarts [1] and Smirnov [28], [30] (They got independently different solutions of different degree of generality).

This property $\mathscr{R}_{\mathscr{P}_3}$ will be called a *compact axiom of countability*. The class $\mathfrak{S}$ of all spaces satisfying the compact axiom of countability is rather wide: *it contains all locally metrizable and all locally complete (in the sense of Čech) spaces.*[9])

Remark 0. *The solutions of Problems I and II presented here for the properties* $\mathscr{P}_{fH(n)}$, $\mathscr{P}_{f\Pi}$, $\mathscr{P}_{H(n)}$ *and* $\mathscr{P}_{\Pi}$ (see below) *has been obtained for this class* $\mathfrak{S}$ *only. This fact, for the sake of brevity, will not be further referred to and, in the sequel, all given spaces are assumed to belong to the class* $\mathfrak{S}$.

E. Now, let us formulate properties $\mathscr{P}_{H(n)}$ and $\mathscr{P}_{\Pi}$:

$\mathscr{P}_{H(n)}$ — *the n-dimensional cohomology group of the space N is a given group H.*[10])

$\mathscr{P}_{\Pi}$ — *the space N is "Π-like", where Π is a polyhedron or a system of polyhedra* (see [15]).

Let us explain it in detail. Let ω be an open cover of a space X. A mapping $f : X \to Y$ is called an *ω-mapping* if for each point y of Y there exists a neighbourhood Oy such that the inverse image $f^{-1}Oy$ is contained in some element of ω, Further, let Π be a family of polyhedra.[11])

Definition Π. *A space X is called Π-like if for each open cover ω of X there exists a continuous ω-mapping of X onto one of the polyhedra of the family Π.*

Definitions $f\Pi$ **and** fH. *The properties* $\mathscr{P}_{f\Pi}$ *and* $\mathscr{P}_{fH(n)}$ *are defined in the same way as the corresponding properties* $\mathscr{P}_{\Pi}$ *and* $\mathscr{P}_{H(n)}$ *with the only difference that finite covers, or, respectively, compact polyhedra are considered only. The property* $\mathscr{P}_{f\Pi}$ *will be called, accordingly, fΠ-likeness.*[12])

Notice that all the above mentioned properties $\mathscr{P}_0, \ldots, \mathscr{P}_{4(n)}$ are special cases of Π-likeness; they all except $\mathscr{P}_0$ and $\mathscr{P}_3$ are special cases of $f\Pi$-likeness. In fact, to obtain $\mathscr{P}_0$, we take as Π the family of all finite polyhedra, for $\mathscr{P}_{1(n)}$ — the polyhedron consisting of n points, for $\mathscr{P}_2$ — the family of all connected polyhedra, for $\mathscr{P}_3$ — the family of all countable polyhedra, for $\mathscr{P}_{4(n)}$ — the family of all at most n-dimensional polyhedra.

F. The following definition appeared to be useful for a solution of Problems I and II in the case of properties $\mathscr{P}_{fH(n)}$ and $\mathscr{P}_{f\Pi}$.

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[9]) The class $\mathfrak{S}$ was proposed by me to E. G. Skljarenko, my aspirant at that time, for a final general solution of Problem I with respect to the property $\mathscr{P}_{4(0)}$. For properties of this class see [31], § 6 and [4].

[10]) We consider the cohomology groups based on locally finite covers (over some group of coefficients).

[11]) Not necessarily compact (finite).

[12]) This definition was preceded, historically, by "treelikeness" (Π is the family of all "trees", i.e. one-dimensional acyclic polyhedra) and "snakelikeness" (Π is a segment) for the case of compact metric spaces (see [34], [5]).

Definition 1*f*. *A system* $\{\Gamma_1, \ldots, \Gamma_s\}$ *of open sets of a proximity space* X_c *is called an extensionable fringe if the set* $K = X \setminus \Gamma_1 \setminus \ldots \setminus \Gamma_s$ *is compact and for every neighbourhood* O *of* K *the system* $\{O, \Gamma_1, \ldots, \Gamma_s\}$ *is a proximal cover*[13]) *of the space* X_c (see [28]).

Definition 1'. *A system of sets* $\Gamma_1, \ldots, \Gamma_s$ *will be called non-compact if the closure of each non-void intersection of these sets is not compact.*

It can be shown that the system of all extensionable fringes is a directed set with respect to the relation "α refines β". Therefore, analogously to the construction of spectral (Čech) cohomology groups $H^n(X)$ which uses covers, *we can define, over a given group of coefficients, spectral groups* $F^n(X_c)$ *of a proximity space* X_c, *using extensionable fringes.*

Theorem 1. *The group* $H^n(N_c)$ *of the remainder* N_c *is canonically isomorphic to the group* $F^n(X_c)$ *of the space* X_c.[14])

Corollary. *The remainder* N_c *has the property* $\mathcal{P}_{fH(n)}$ *if and only if the group* $F^n(X_c)$ *(over some group of coefficients) is isomorphic to the given group* H.

Now, we shall say that a family Π is *hereditary* if it contains, with each polyhedron, any of its subpolyhedra. We shall say that *the number of components of a family* Π *is finite* if the number of components of each polyhedron of the family Π is not greater than some number $k(\Pi)$, $k(\Pi) < \infty$.

Let Π be a system of (compact) polyhedra P_i. Choose, for each i, any triangulation K_i of the polyhedron P_i and denote by K_{ij} the complex which is the j-th barycentric subdivision of K_i.

Theorem 2. *The remainder* N_c *possesses the property* $\mathcal{P}_{f\Pi}$ *(where* Π *is a hereditary family or the number of components of* Π *is finite) if and only if every extensionable fringe of the space* X_c *may be refined by a non-compact*[15]) *extensionable fringe the nerve of which is one of the complexes* K_{ij}.

Thus Problem II is solved for the properties we are interested in.

G. To solve Problem I, we need the following, according to our opinion natural, definitions:

Definition 2. *A fringe of a (completely regular) space* X *is every extensionable fringe of the proximity space* X_β, *where* β *is the maximal proximity.*[16])

[13]) A system of sets $G_1, \ldots, G_s$ of a proximity space X_c is called a proximal cover ("δ-cover") if there exist sets $H_1, \ldots, H_s$ such that $X = H_1 \cup \ldots \cup H_s$ and $H_i \bar{c}\, X \setminus G_i$ for each i (see [23], p. 559).

[14]) This also holds for groups based on infinite covers and, respectively, infinite fringes (see below).

[15]) This need not be requested if the family Π is hereditary.

[16]) It is induced by the maximal (Čech) compactification βX.

If X is normal then the definition may be simplified:

A fringe of a normal space X is a system $\{\Gamma_1, \ldots, \Gamma_s\}$ of open sets such that the complement $X \setminus \Gamma_1 \setminus \ldots \setminus \Gamma_s$ is compact.

Definition 3. *A system of fringes of a space X is called a structure of fringes if, for any two fringes of this system, there exists a fringe of this system which is a star-refinement of each of them.*

Definition 4*t*. *A system Σ of fringes of a space X is called topological if for each point x of X and each neighbourhood Ox of x there exist a neighbourhood U of x and a fringe γ from the given system Σ such that* $\mathrm{St}_\gamma U \subset Ox$ ("base property", see [28]).

Theorem 3. *A space X has at least one at most n-dimensional remainder N if and only if there exists a topological structure of fringes of X with order $\leqq n+1$; moreover, we can achieve the weight of the compactification $cX = X \cup N$ to be equal to the weight of the space X.*[17])

It is to be noted that (in spite of my assertion in [28], Theorem 5), *for $n \geqq 1$, a maximal compactification among all compactifications with at most n-dimensional remainders need not exist.* B. Levšenko proved that *it does not exist for any Euclidean space E^N with $N \geqq 2$ in any dimension n, where $1 \leqq n \leqq N-1$.*

Let us remark that Theorem 3 does not hold, in general, with property $\mathscr{P}_{4(n)}$ replaced by any $f\Pi$-likeness.[18]) In the general case, it is necessary to consider structures Σ of more special type.

For this purpose, observe that *every structure Σ of fringes of a space X generates, in the following natural way, a (general) proximity c_Σ:*

$$(\Sigma)\ A\, \bar{c}_\Sigma\, B \Leftrightarrow \bar{A} \cap \bar{B} = \emptyset \text{ and } A \cap \mathrm{St}_\gamma B = \emptyset \text{ for some } \gamma \text{ from } \Sigma.$$

If the structure Σ is topological then the proximity c_Σ also satisfies the separating axioms and induces the topology of the given space X. Hence, every topological structure Σ of fringes of a space X defines uniquely some compactification $c_\Sigma X$ of the space X and also the remainder N_{c_Σ} (with a given property). That is what the main idea of the proof of here obtained results consists in.

Definition 4*p*. *A structure Σ of fringes of a space X is called proximal if any binary extensionable fringe $\{\Gamma_1, \Gamma_2\}$ of the proximity space X_{c_Σ}, generated by the structure Σ, has some refinement from Σ.*

[17]) For details see [31]; for a short exposition see [28], [29], [30].

[18]) Unless we require the property $\mathscr{P}$ to be, in a certain sense, "countably monotonic" (If the sum of countably many compact sets is a subset of a compact space with the property $\mathscr{P}$ then it also possesses the property $\mathscr{P}$). However, there are, apparently, very few such properties, different from $\mathscr{P}_{4(n)}$.

Theorem 3'. *A space X has at least one $f\Pi$-like remainder N (where the family Π is hereditary or the number of components of Π is finite) if and only if there exists a proximal structure of non-compact*[15]) *fringes in X, the nerves of which are the complexes K_{ij}* (see Section F).

As mentioned above, it cannot be required here, in general, the sought for structure of fringes to be topological only. But *it is always possible if the space X is locally compact and then the weight of the compactification $cX = X \cup N$ can be achieved to be equal to the weight of the space X.*

Theorem 4. *A space X has at least one remainder N with property $\mathscr{P}_{fH(n)}$ if and only if there exists a proximal structure Σ of fringes such that the spectral group $F^n(\Sigma)$, constructed (over some group of coefficients) by means of fringes of this structure,*[19]) *is isomorphic to the group H.*

H. To obtain analogous results for properties $\mathscr{P}_{\Pi}$ and $\mathscr{P}_{H(n)}$ it is necessary, first to use arbitrary (not only finite) fringes and secondly to replace proximal structures of fringes by uniform ones. The following procedure is used:

Definition 1. *A system γ of open sets Γ_α of a proximity space X_c is called an extensionable fringe if the set $K = X \setminus \bigcup_\alpha \Gamma_\alpha$ is compact and for every neighbourhood O of K there exist sets $\Gamma_{\alpha_1}, \ldots, \Gamma_{\alpha_s}$ of the system γ such that the system $\{O, \Gamma_{\alpha_1}, \ldots, \Gamma_{\alpha_s}\}$ is a proximal cover* (compare with Definition 1*f*).

Definition 4*u*. *A structure Σ of fringes of a space X is called uniform if every extensionable fringe of the proximity space X_{c_Σ}, generated by the structure Σ, has some refinement from Σ.*

By fringes of a space X, extensionable fringes of the proximity space X_β are understood. Definition 3 for infinite fringes is the same. Theorems 1 and 3 also hold for arbitrary fringes in the same formulations. Let us turn our attention to one more theorem.

Theorem 5. *The limit space of the projection spectrum*[20]) *consisting of nerves of all star-finite extensionable fringes of a proximity space X_c, with canonical projections arising by refining, is canonically homeomorphic to the remainder N_c. The limit space of the projection spectrum consisting of nerves of all finite extensionable fringes of the space X_c is canonically homeomorphic to the Čech compactification βN_c of the remainder N_c.*

[19]) It is constructed by means of fringes of the structure Σ in the same way as the group $F^n(X_c)$ by means of extensionable fringes.

[20]) In the sense of Alexandroff-Švedov (see [3] and [19]). The main difference from the projection spectra considered in the report of Alexandroff-Ponomarev (see page 25) consists in the fact that the projections are multivalued.

I. We win some more generality if we are not interested, in Problems I and II, in the properties of remainders but in the properties of compactifications themselves. However, in applying to properties $\mathscr{P}_{f\Pi(n)}$ and $\mathscr{P}_{f\Pi}$, such a formulation does not yield much new information: *we obtain analogous answers with the only difference that it is necessary to take proximal covers instead of extensionable fringes and open covers instead of fringes* (*Remark* 0 *will be, of course, unnecessary*). For example, we have the following.

Theorem 6. *A space X has a compactification cX with dimension at most n if and only if there exists a topological structure,*[21]) *in X, of normal covers*[22]) *of order* $\leqq n+1$, *moreover, we can achieve the weight of cX to be equal to the weight of X.*

As corollaries, we obtain well-known theorems of Hurewicz [14] and Skljarenko [22] on existence of compactifications with the same dimension and weight as the original space. Finally, let us mention the following modification of an interesting Orevkov's theorem [17]:

Theorem 7. *Let a countable collection of families* Π_k *consisting of finite polyhedra be given; let each family* Π_k *be either finite or have a finite number of components. If a normal space X is* $f\Pi_k$*-like for each k then it has a compactification of the same weight as X and it is also* $f\Pi_k$*-like for each k.*

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[21]) See Definition 3 (in other words, it is a base of a uniform precompact structure in the sense of Tukey [32]). The condition of topologicality is here equivalent to this common condition: the stars of points form a base of the space *X*.

[22]) A normal cover of a space *X* is a proximal cover of the space X_β. Normality of covers is necessary for non-normal spaces only.

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