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ORDER STRUCTURES AND TOPOLOGICAL STRUCTURES

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The primary interest here is to determine the extent to which the concept of order can be used in studying topological concepts. In particular, we focus our attention on the following situation. When can a topological space X be embedded in a complete lattice Ω so that topological convergence and order convergence (o-convergence) are equivalent? (See [1, p. 59], [2], [3], [4].) The answer to this question is contained in the following theorem.

Theorem. *A topological space X can be embedded in a complete lattice Ω so that topological convergence and o-convergence are equivalent if and only if X is a regular Hausdorff space.* (See [3].)

A simple example shows that it is not always possible to fully embed X in Ω (i.e., so that the image of X is all of Ω). It is conjectured that this can be done if X is a compact Hausdorff space, but this is not necessary [4]. For example, any discrete space can be fully embedded in a complete lattice.

Suppose then that a topological space X can be fully embedded in a complete lattice Ω. What order properties are related to what topological properties? Very interesting results in this connection have been obtained by Dyer and Shields [5]. As a sample, we ask the following questions.

1. When is the function $f(x, y) = x \vee y$ continuous in one or both variables?
2. When is every maximal chain (= linearly ordered subset) connected?
3. When is the lattice distributive?
4. When are there enough isotone continuous real-valued functions to distinguish points?

References

[1] *G. Birkhoff:* Lattice theory. Rev. ed., Amer. Math. Soc. Colloq. Publ. Vol. *25*, Amer. Math. Soc., Providence, R. I., 1948.

[2] *R. E. DeMarr:* Partially ordered spaces and metric spaces. Amer. Math. Monthly *72* (1965), 628–631.

[3] *R. E. DeMarr:* Order convergence and topological convergence. Proc. Amer. Math. Soc. *16* (1965), 588–590.

[4] *R. E. DeMarr:* Complete lattices and compact Hausdorff spaces (research problem). Bull. Amer. Math. Soc. *72* (1966), 223.

[5] *E. Dyer and A. Shields:* Connectivity of topological lattices. Pac. Jour. of Math. *9* (1959), 443–448.