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RAMIFICATION SYSTEMS AND SPACES OF ULTRAFILTERS

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Let α be an infinite cardinal. $U(\alpha)$ denotes the space of uniform ultrafilters on α . $\Omega(\alpha^+)$ denotes the space of subuniform ultrafilters on α^+ , i.e., those ultrafilters on α^+ which are not uniform, but which have only elements of cardinality at least α . The Stone-Čech compactification of a space S is denoted by βS .

Theorem. *If $\alpha = \alpha^{\aleph}$, then $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ (= the growth of $\Omega(\alpha^+)$) has the following property: for every non-empty closed subset F of $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ which is equal to the intersection of at most α open-and-closed sets and every family $\{V_\eta, \eta < \alpha\}$ of non-empty pairwise disjoint open-and-closed subsets of $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ there are p in F and an open-and-closed set N , with p in N and such that $|\{\eta < \alpha : V_\eta \cap N \neq \emptyset\}| < \alpha$.*

The property of this Theorem was studied by the author in The Existence of certain uniform ultrafilters, *Ann. of Math.* 90 (1969), 23–32, where it was proved that the space $U(\alpha)$ has the property (for regular α).

Corollary. *If $\alpha = \alpha^{\aleph}$ then $U(\alpha^+)$ is not homeomorphic to $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$. In particular $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ is not C^* -embedded in $\beta(\alpha^+)$.*

For $\alpha = \omega$ this last statement has been proved by Mrs. N. M. Warren (Doctoral Dissertation, University of Wisconsin, 1970) using the fact that ω^+ has property Q (in the sense of Erdős and Tarski, *On some problems involving inaccessible cardinals*, *Essays on the Foundations of Mathematics*, Magnes Press, 1966, pp. 50–82). By contrast we can derive the more general result of E. Specker (slightly improved by D. Monk):

Corollary. *If $\alpha = \alpha^{\aleph}$ then α^+ has property Q in the sense of Erdős and Tarski.*

Finally the following result can be proved directly.

Theorem. *If α is weakly compact (i.e., α is strongly inaccessible and does not have property Q), then the space $N(\alpha)$ of non-uniform, non-principal ultrafilters on α is C^* -embedded in $\beta(\alpha)$.*

The details will appear in *Trans. Amer. Math. Soc.*