RAMIFICATION SYSTEMS AND SPACES OF ULTRAFILTERS

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Let α be an infinite cardinal. $U(\alpha)$ denotes the space of uniform ultrafilters on α. $\Omega(\alpha^+)$ denotes the space of subuniform ultrafilters on α^+, i.e., those ultrafilters on α^+ which are not uniform, but which have only elements of cardinality at least α. The Stone-Čech compactification of a space S is denoted by βS.

Theorem. *If* $\alpha = \alpha^{\underset{\smile}{\alpha}}$, *then* $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ (= *the growth of* $\Omega(\alpha^+)$) *has the following property: for every non-empty closed subset* F *of* $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ *which is equal to the intersection of at most* α *open-and-closed sets and every family* $\{V_\eta, \eta < \alpha\}$ *of non-empty pairwise disjoint open-and-closed subsets of* $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ *there are* p *in* F *and an open-and-closed set* N, *with* p *in* N *and such that* $|\{\eta < \alpha : V_\eta \cap N \neq \emptyset\}| < \alpha$.

The property of this Theorem was studied by the author in The Existence of certain uniform ultrafilters, Ann. of Math. *90* (1969), 23–32, where it was proved that the space $U(\alpha)$ has the property (for regular α).

Corollary. *If* $\alpha = \alpha^{\underset{\smile}{\alpha}}$ *then* $U(\alpha^+)$ *is not homeomorphic to* $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$. *In particular* $\beta(\Omega(\alpha^+)) \setminus \Omega(\alpha^+)$ *is not* C^**-embedded in* $\beta(\alpha^+)$.

For $\alpha = \omega$ this last statement has been proved by Mrs. N. M. Warren (Doctoral Dissertation, University of Wisconsin, 1970) using the fact that ω^+ has property Q (in the sense of Erdös and Tarski, On some problems involving inaccessible cardinals, Essays on the Foundations of Mathematics, Magnes Press, 1966, pp. 50–82). By contrast we can derive the more general result of E. Specker (slightly improved by D. Monk):

Corollary. *If* $\alpha = \alpha^{\underset{\smile}{\alpha}}$ *then* α^+ *has property* Q *in the sense of Erdös and Tarski.*

Finally the following result can be proved directly.

Theorem. *If* α *is weakly compact (i.e.,* α *is strongly inaccessible and does not have property* Q*), then the space* $N(\alpha)$ *of non-uniform, non-principal ultrafilters on* α *is* C^**-embedded in* $\beta(\alpha)$.

The details will appear in Trans. Amer. Math. Soc.