S. Mardešić

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A SURVEY OF THE SHAPE THEORY OF COMPACTA

S. MARDEŠIĆ

Zagreb

1. The notion of shape

The notion of shape has been introduced by K. Borsuk [2], [3] as a modification of the notion of homotopy type. The idea was to take into account the global properties of compacta and neglect the local ones. This allows an effective application of the theory to compacta as opposed to homotopy theory which only works well on spaces with nice local properties like the ANR's.

In shape theory of compacta one introduces and studies a new category whose objects are compact spaces and whose morphisms, called shape maps, are modifications of classes of homotopic maps. Two compacta are said to be of the same shape if they are isomorphic objects in this category. Moreover, there is a covariant functor S from the category of compact spaces and maps into the shape category. It keeps objects fixed and assigns to every map f a shape map said to be generated by f. In contrast to the case of homotopy theory, not every shape map is generated by a map.

The shape category and the functor S have the following essential features (W. Holsztyński [34]).

(i) S factors through the homotopy category, i.e., homotopic maps generate the same shape map and consequently spaces of the same homotopy type are of the same shape.

(ii) If Y is an ANR, then every shape map into Y is generated by a map unique up to homotopy and therefore on ANR's shape coincides with homotopy type.

(iii) The functor S is continuous with respect to inverse limits.

As an example one can consider the "Polish circle" (the closure of the graph of $\sin 1/x$, $x \in (0, 1]$, closed by an arc) which is easily obtained as an inverse limit of circles with bonding maps of degree 1. After applying the functor S and condition (iii) it is clear that the "Polish circle" is of the shape of a circle although their homotopy types differ.

Shape can be viewed as a "Čech homotopy type" and its relationship to homotopy type is analogous to the relationship of Čech homology to singular homology.

In order to define shapes Borsuk [2] considers compacta X embedded in the Hilbert cube $I^\infty = \Pi I_i$, $I_i = [-1, 1]$, $i \in N$. Instead of mappings $f: X \to Y$ he considers fundamental sequences $\boldsymbol{f}: X \to Y$ defined as sequences of maps $f_n: I^\infty \to I^\infty$ with the property that for every neighborhood V of Y in I^∞ there exist a neighborhood U of X in I^∞ and an integer $n_0 \in N$ such that for $n, n' \geqq n_0$ the restrictions $f_n \mid U$ and $f_{n'} \mid U$ are homotopic in V. Consequently, X is not mapped into Y itself, but into its neighborhoods in I^∞, which can always be chosen with nice local properties.

The composition $\boldsymbol{h} = \boldsymbol{g}\boldsymbol{f}: X \to Z$ of fundamental sequences $\boldsymbol{f}: X \to Y$ and $\boldsymbol{g}: Y \to Z$ is by definition the fundamental sequence consisting of the maps $h_n = g_n f_n: I^\infty \to I^\infty$. The identity fundamental sequence $\boldsymbol{1}_X: X \to X$ consists of a sequence of identity maps $1_{I^\infty}: I^\infty \to I^\infty$.

Two fundamental sequences $\boldsymbol{f}, \boldsymbol{f}': X \to Y$ are considered to be homotopic, $\boldsymbol{f} \simeq \boldsymbol{f}'$, provided for every neighborhood V of Y in I^∞ there exist a neighborhood U of X in I^∞ and an integer $n_0 \in N$ such that for $n \geqq n_0$, $f_n \mid U$ and $f'_n \mid U$ are homotopic in V. It readily follows that $\boldsymbol{f} \simeq \boldsymbol{f}': X \to Y$ and $\boldsymbol{g} \simeq \boldsymbol{g}': Y \to Z$ implies $\boldsymbol{g}\boldsymbol{f} \simeq \boldsymbol{g}'\boldsymbol{f}': X \to Z$.

According to Borsuk, Y is said to fundamentally dominate X provided there exist fundamental sequences $\boldsymbol{f}: X \to Y$ and $\boldsymbol{g}: Y \to X$ such that $\boldsymbol{g}\boldsymbol{f} \simeq \boldsymbol{1}_X$; if also $\boldsymbol{f}\boldsymbol{g} \simeq \boldsymbol{1}_Y$, then X and Y are said to be of the same shape, $\operatorname{Sh} X = \operatorname{Sh} Y$. In the case of fundamental domination one writes $\operatorname{Sh} X \leqq \operatorname{Sh} Y$. Equality of shape is an equivalence relation.

To every map $f: X \to Y$ Borsuk assigns the fundamental sequence $\boldsymbol{f}: X \to Y$ consisting of any sequence $f_1, f_1, \ldots$ of extensions $f_1: I^\infty \to I^\infty$ of f. Homotopy of maps implies homotopy of the assigned fundamental sequences. Therefore, the equality of homotopy types implies the equality of shapes.

The notion of shape was also defined for pairs of compacta (X, A) and in particular for pointed compacta [2]. Recently Borsuk has shown in [11] (also see [17]) that the role of the Hilbert cube I^∞ in founding shape theory can also be taken by any compact AR M and the resulting notion of shape is independent of M. The notion of shape was also defined for metric non-compact spaces by Borsuk [11] and by R. H. Fox [26]. For an account of this aspect of shape theory see [18].

Čech cohomology and homology groups are important shape invariants. Borsuk has proved that two continua in the plane E^2 are of the same shape if and only if their first Betti numbers coincide. Thus, there are $\aleph_0$ different shapes of planar continua, as their representatives one can take a single point (trivial shape), a bouquet of n circles, $n \in N$, and the infinite bouquet of circles. A. Trybulec has proved that every 1-dimensional Peano continuum is of the shape of a planar continuum. S. Godlewski [27] has shown that there are $2^{\aleph_0}$ different shapes among continua in E^3; in fact that many different shapes can be found among solenoids. A complete shape classification of n-sphere-like continua was given in [44] and of projective plane-like continua in [53]. Borsuk has shown that there are $2^{\aleph_0}$ different shapes among compacta in E^2. In fact, in [44] it is shown that in E^1 there are at least $\aleph_1$ different shapes of compacta.

2. ANR-system approach to shapes

In [44] the author and J. Segal have developed shape theory on the basis of the notion of an ANR-system. In this approach shapes are defined for arbitrary Hausdorff compact spaces. We shall describe here only the case of metric compacta, where ANR-systems can be replaced by ANR-sequences.

An ANR-sequence $\boldsymbol{X} = \{X_i, p_{ii'}\}$ is an inverse sequence of compact metric ANR's X_i and maps $p_{ii'} : X_{i'} \to X_i$, $i \leqq i'$, $i, i' \in N$. A map of ANR-sequences $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y} = \{Y_j, q_{jj'}\}$ consists of an increasing function $f : N \to N$ and a sequence of maps $f_j : X_{f(j)} \to Y_j$ such that the following diagram commutes up to homotopy

$$\begin{tikzcd} X_{f(j)} \arrow[d, "f_j"'] & X_{f(j')} \arrow[l, "p_{f(j)f(j')}"'] \arrow[d, "f_{j'}"] \\ Y_j & Y_{j'} \arrow[l, "q_{jj'}"] \end{tikzcd}$$

The composition $\boldsymbol{h} = \boldsymbol{gf}$ of maps of sequences $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$, $\boldsymbol{g} : \boldsymbol{Y} \to \boldsymbol{Z} = \{Z_k, r_{kk'}\}$ consists of the map $h = fg : N \to N$ and of the sequence of maps $h_k = g_k f_{g(k)} : : X_{fg(k)} \to Z_k$. The identity map of sequences $\boldsymbol{1} : \boldsymbol{X} \to \boldsymbol{X}$ consists of the identity $1 : N \to N$ and of the sequence of identity maps $1_{X_i} : X_i \to X_i$. Two maps of sequences $\boldsymbol{f}, \boldsymbol{f}' : \boldsymbol{X} \to \boldsymbol{Y}$ are considered to be homotopic, $\boldsymbol{f} \simeq \boldsymbol{f}'$, provided for every $j \in N$ there exists an integer $i \geqq f(j), f'(j)$, such that

$$f_j p_{f(j)i} \simeq f'_j p_{f'(j)i} \, .$$

It is well-known that every metric compactum X admits an inverse sequence $\boldsymbol{X}$ of ANR's (even of polyhedra) such that $X = \text{Inv lim } \boldsymbol{X}$; every such ANR-sequence $\boldsymbol{X}$ is said to be associated with X. If $\boldsymbol{X}$ and $\boldsymbol{Y}$ are associated with X and Y respectively, then a map of sequences $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$ is said to be associated with a map $f : X \to Y$ provided for every $j \in N$ the following diagram commutes up to homotopy:

$$\begin{tikzcd} X_{f(j)} \arrow[d, "f_j"'] & X \arrow[l, "p_{f(j)}"'] \arrow[d, "f"] \\ Y_j & Y \arrow[l, "q_j"] \end{tikzcd}$$

Here $p_i : X \to X_i$ and $q_j : Y \to Y_j$ denote the respective projections.

An essential step in this approach to shapes consists in the result [44] that every map $f : X \to Y$ admits an associated map of sequences $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$ for any choice of $\boldsymbol{X}$ and $\boldsymbol{Y}$; $\boldsymbol{f}$ is determined uniquely up to homotopy. Moreover, if $f \simeq f'$, then

$\boldsymbol{f} \simeq \boldsymbol{f}'$. In particular, if $\boldsymbol{X}$ and $\boldsymbol{X}'$ are two ANR-sequences associated with X, then there are maps of sequences $\boldsymbol{i} : \boldsymbol{X} \to \boldsymbol{X}'$ associated with the identity map $1_X : X \to X$, determined uniquely up to homotopy. This enables one to compare maps of sequences $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$ and $\boldsymbol{f}' : \boldsymbol{X}' \to \boldsymbol{Y}'$ between different ANR-sequences associated with X and Y respectively. $\boldsymbol{f}$ and $\boldsymbol{f}'$ are now said to be homotopic, $\boldsymbol{f} \simeq \boldsymbol{f}'$, provided the following diagram commutes up to homotopy

$$\begin{tikzcd} \boldsymbol{X} \arrow[r, "\boldsymbol{f}"] \arrow[d, "\boldsymbol{i}"'] & \boldsymbol{Y} \arrow[d, "\boldsymbol{j}"] \\ \boldsymbol{X}' \arrow[r, "\boldsymbol{f}'"'] & \boldsymbol{Y}' \end{tikzcd};$$

here $\boldsymbol{j}$ is any map of sequences associated with $1_Y : Y \to Y$. The homotopy relation classifies all the maps of ANR-sequences $\boldsymbol{X}$ associated with X to ANR-sequences $\boldsymbol{Y}$ associated with Y. The corresponding classes are called shape maps from X to Y and are denoted by $f: X \to Y$ (using the notation of a representative $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$). Shape maps correspond to homotopy classes of fundamental sequences in the Borsuk approach. Compact metric spaces and shape maps form the shape category. The functor S from the category of metric compacta to the shape category keeps the objects fixed and sends every $f : X \to Y$ into the shape map whose representative $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$ is any map of sequences associated with f [42].

The value of this approach stems mainly from the fact that in studying the shape of X any ANR-sequence expansion $\boldsymbol{X}$ of X can be used. In many cases the space X itself is defined by means of an ANR-sequence, e.g., in the case of solenoids which are defined by an inverse sequence of circles. If X is already an ANR, one can use the sequence where all $X_i = X$ and all $p_{ii'} = 1_X$. Then one concludes immediately that maps into such sequences are associated with maps of spaces and so (ii) is satisfied. This approach is also more categorical and therefore allows generalizations to some abstract categories [34].

The equivalence of the ANR-sequence approach to shapes and the Borsuk approach was established in [45]. It is easy to construct a decreasing sequence of closed neighborhoods X_i of a compactum $X \subset I^\infty$ in such a way that the intersection yields X and each X_i is an ANR. Therefore, such a sequence can be considered as a special ANR-sequence associated with X. By suitably restricting the members of a fundamental sequence, one can easily produce a map of such ANR-sequences $\boldsymbol{f}: \boldsymbol{X} \to \boldsymbol{Y}$. To assign to every map of such ANR-sequences a fundamental sequence is more delicate and requires a repeated application of the Borsuk extension theorem (see [45]).

3. The method of infinite-dimensional manifolds

Recently an important and somewhat unexpected connection between shape and infinite-dimensional manifolds has been established. As far as we know the first application of these methods to homotopy is due to K. Borsuk [12], who has used a homeomorphism extension theorem of V. Klee. This approach has been further expanded by D. W. Henderson [33]. The final result in this direction is the following characterization theorem due to T. A. Chapman [21]: Two compacta X, Y contained in the pseudointerior s of the Hilbert cube I^∞, $s = \prod_{i \in N} I_i^\circ$, $I_i^\circ = (-1, 1)$, are of the same shape if and only if their complements $I^\infty \setminus X$ and $I^\infty \setminus Y$ are homeomorphic. It is of interest to point out that this theorem converts the problem of shape, which is essentially a homotopy problem, to a homeomorphism problem. Thus, shape theory can also be considered as a part of general topology. Chapman's result looks less surprising if one recalls that in ∞-dimensional manifolds homotopy and homeomorphism problems often are equivalent. Chapman's work is based on recent deep results on Hilbert cube manifolds due to several authors. It uses specially results by R. D. Anderson, T. A. Chapman, D. W. Henderson, R. M. Schori, J. E. West and R. Y. T. Wong.

Chapman has also succeeded in obtaining an analogous characterization theorem for finite-dimensional compacta [24]. A compactum $X \subset E^n$ is said to be stably embedded in E^n provided X lies in a Euclidean subspace of E^n of codimension $\geqq 2(\dim X) + 1$. A theorem of V. Klee insures that any two stable embeddings of X in E^n are equivalent by a space homeomorphism. Now Chapman's result can be stated as follows: For every integer $m > 0$ there exists an integer $n_1 > 0$ such that whenever X, $Y \subset E^n$ are stably embedded compacta of dimension $\leqq m$ and $n \geqq n_1$, then $\mathrm{Sh}\, X = \mathrm{Sh}\, Y$ if and only if the complements $E^n \setminus X$ and $E^n \setminus Y$ are homeomorphic. $n_1 = 10m + 17$ suffices. The same results holds with E^n replaced by the sphere S^n. The general structure of the proof of this result follows the one for the infinite-dimensional case, but the techniques are ones from piecewise linear topology.

4. Shapes of some quotient spaces

In this section we discuss some specific results concerning the shape of some quotient spaces. Borsuk has proved in [12] that for homeomorphic compacta A, B contained in the n-cell I^n the quotients I^n/A and I^n/B are of the same homotopy type and therefore of the same shape. D. W. Henderson [33] has obtained the same conclusion under the weaker hypothesis that A and B are of the same homotopy type. Using ANR-sequences the author has proved that for A, $B \subset I^n$ and $\mathrm{Sh}\, A = \mathrm{Sh}\, B$, the pairs (I^n, A) and (I^n, B) are of the same shape, which implies that $\mathrm{Sh}\,(I^n/A) = \mathrm{Sh}\,(I^n/B)$. Improving this result Borsuk has shown in [16] that the same theorem holds with I^n replaced by any AR. Finally, using his characterization of shape, Chapman has proved the following: if X is any compactum, A and B closed subsets of X of the same shape and there exists a closed subset $Y \subset X$ of trivial shape such that $A \cup B \subset Y$, then $\mathrm{Sh}(X/A) = \mathrm{Sh}(X/B)$ [23].

Using Chapman's characterization of shape R. D. Anderson [1] has recently proved a "Vietoris theorem" for shapes: Let X and Y be metric compacta and $f: X \to Y$ a continuous surjection with the property that $\mathrm{Sh}(f^{-1}(y))$ is trivial for every $y \in Y$. If in addition the set $Y' \subset Y$ of all points with nondegenerate $f^{-1}(y)$ is of finite dimension, then f is a shape equivalence and $\mathrm{Sh}\,X = \mathrm{Sh}\,Y$.

5. Movable compacta

Borsuk has introduced in [5] an interesting shape invariant property of compacta called movability. $X \subset I^\infty$ is said to be movable provided for every neighborhood U of X in I^∞ there exists a neighborhood $V \subset U$ of X in I^∞ such that for any neighborhood W of X in I^∞, V can be deformed into W within U. If X fundamentally dominates Y and is movable, then so is Y [5]. Every planar compactum is movable. A product of a finite or a countable collection of movable compacta is again movable [5]. The suspension of a movable compactum is movable [10].

In [46] the author and J. Segal have redefined movability in terms of ANR-sequences. X is movable provided every ANR-sequence $\boldsymbol{X}$ associated with X has the following property: for every $i \in N$ there exists an $i' \in N$, $i \leqq i'$, such that for every $i'' \in N$, $i \leqq i''$, there exists a map $r^{i'i''}: X_{i'} \to X_{i''}$ satisfying the homotopy relation $p_{ii''} r^{i'i''} \simeq p_{ii'}$. If one of the associated sequences $\boldsymbol{X}$ has this property then so does everyone. Using this property the author and J. Segal [46] have exhibited a 1-dimensional continuum X which is acyclic (all Čech homology and cohomology groups vanish) but fails to be movable. Nevertheless, its suspension is movable and even of trivial shape [41].

The author has proved [39] that every n-dimensional LC^{n-1} continuum is movable. The proof was simplified by R. Overton and J. Segal in [51]. Borsuk [6] has exhibited a 2-dimensional locally connected (LC^0) continuum which fails to be movable, which shows that the dimension in the above theorem cannot be improved.

K. Kuperberg [37] has shown that for movable compacta a form of the Hurewicz theorem holds. One uses the Čech homology groups and the shape groups $\pi_n(X, x_0)$, which are defined as the inverse limit of the inverse sequence $\{\pi_n(X_i, x_{i0}), p_{ii'*}\}$, where $(\boldsymbol{X}, x_0) = \{(X_i, x_{i0}), p_{ii'}\}$ is an ANR-sequence associated with (X, x_0).

The definition of movability given in [46] and applied to pairs yields the notion of a movable pair of compacta. R. Overton has shown that Čech homology with integer coefficients is exact on movable pairs of metric compacta [50].

Recently M. Moszyńska has introduced uniformly movable compacta, a subclass of movable compacta, as metric compacta X such that every ANR-sequence $\boldsymbol{X}$

associated with X has the properties as in the definition of movability and in addition $p_{i''i'''}r^{i'i'''} \simeq r^{i'i''}$ whenever $i \leqq i'' \leqq i'''$, i.e., the maps $r^{i'i''} : X_{i'} \to X_{i''}$, $i \leqq i''$, form a map of sequences $\boldsymbol{r}^{i'} : X_{i'} \to \boldsymbol{X}^{i} = \{X_{i''}, p_{i''i'''}, i'' \geqq i\}$. Uniform movability is also preserved under fundamental domination and is therefore a shape invariant too. Using the notion of uniform movability of pointed compacta Moszyńska has proved the following form of the Whitehead theorem: If (X, x_0) and (Y, y_0) are finite-dimensional uniformly movable pointed compacta and $\boldsymbol{f}: (X, x_0) \to (Y, y_0)$ is a shape map which induces isomorphisms of the shape groups $\pi_n(X, x_0) \to \pi_n(Y, y_0)$ for all $n \in N$, then $\boldsymbol{f}$ is a shape equivalence.

6. Retraction in shape theory

In [4] Borsuk has defined and studied fundamental or shape retraction. In terms of ANR-sequences the definition assumes this form: Let X and Y be metric compacta, $X \subset Y$. A shape retraction of Y to X is a shape map (in the sense of § 2) $\boldsymbol{r} : Y \to X$ such that $\boldsymbol{ri} = \boldsymbol{1}_X$, where $\boldsymbol{i} : X \to Y$ is the shape map generated by the inclusion $i : X \to Y$; if such an $\boldsymbol{r}$ exists, then X is said to be a shape retract of Y. Notice that this definition means that for any two ANR-sequences $\boldsymbol{X}$ and $\boldsymbol{Y}$ associated with X and Y respectively, there exists a map of sequences $\boldsymbol{r} : \boldsymbol{Y} \to \boldsymbol{X}$ such that $\boldsymbol{ri} \simeq \boldsymbol{1}_{\boldsymbol{X}}$. Thus, shape retraction generalizes the notion of homotopy retraction. The techniques of [45] and a theorem of H. Patkowska [52] enable one to show that this definition of a shape retract is equivalent to the original Borsuk definition. Let us also mention that M. Moszyńska has defined the notion of retraction at the level of ANR-sequences [47], which is closer to Borsuk's definition. However, after passing to space, it yields the same notion as described above.

Once shape retracts are defined, it is clear how to define absolute shape retracts ASR (called by Borsuk fundamental absolute retracts FAR) and absolute neighborhood shape retracts ANSR (called by Borsuk FANR) [4], [42].

Borsuk has characterized ASR's as compacta having the shape of a point (trivial shape) (see [9] or [42]). The author has characterized ASR's also as compacta obtainable as the intersection of a decreasing sequence of Hilbert cubes [43]. The same result has been obtained by T. A. Chapman [21] using ∞-dimensional manifolds techniques; the author's proof is based on elementary techniques from *PL*-topology. R. C. Lacher [38] and the author [43] have independently characterized finite-dimensional ASR's as cellular sets in some Euclidean space.

Many results known for AR's carry over to ASR's. For example, if $X_1, X_2, X_0 = X_1 \cap X_2$ are ASR's, then so is $X = X_1 \cup X_2$ [4]. If X and X_0 are ASR's, then so are X_1 and X_2 [22].

ANSR's form an interesting shape invariant class of compacta. Every ANR is an ANSR and ANSR's are movable and uniformly movable. Borsuk has characterized metric ANSR's by means of a property called strong movability [9].

The property of being an ANSR is preserved under fundamental domination ([7] or [42]). Therefore, shape retracts of ANR's are ANSR's. The converse also holds, i.e., every ANSR is a shape retract of some ANR ([4] or [42]).

M. Moszyńska has considered the notion of a shape deformation retract X of Y and has characterized it by the property that the shape map $\boldsymbol{i}: X \to Y$ generated by the inclusion is a shape equivalence. She has also introduced the notion of the mapping cylinder $C_{\boldsymbol{f}}$ of a shape map $\boldsymbol{f}$ and has proved a "Fox theorem" for shapes: A shape map $\boldsymbol{f}: X \to Y$ is a shape equivalence if and only if X is a shape deformation retract of the mapping cylinder $C_{\boldsymbol{f}}$. This result is used in the proof of her Whitehead theorem. She has also proved that $\mathrm{Sh}\, X = \mathrm{Sh}\, Y$ if and only if there exists a compactum Z such that both X and Y are shape deformation retracts of Z [42].

7. The shape dimension

K. Borsuk has also defined and studied the shape or fundamental dimension $\mathrm{Sd}\, X$ as the minimum of $\dim Y$, where Y runs through all compacta which fundamentally dominate X [3]. For example, every cell has fundamental dimension 0. Clearly, $\mathrm{Sd}\, X \leqq \dim X$. Holsztyński has shown that one obtains the same value for $\mathrm{Sd}\, X$ by letting Y run through all compacta having the same shape as X.

It is well-known that the nth cohomotopy group $\pi^n(X)$ is defined if $\dim X < 2n - 1$. Godlewski shows in [30] that $\pi^n(X)$ is defined even if $\mathrm{Sd}\, X < 2n - 1$ and that it is a shape invariant [27].

Recently Borsuk and Holsztyński [20] have proved a "Hopf classification theorem" for shapes: Let X be a compactum with $\mathrm{Sd}\, X \leqq n$ and let Y be any compactum fundamentally dominated by the n-sphere S^n. Two shape maps $\boldsymbol{f}, \boldsymbol{g}: X \to Y$ coincide (i.e., the corresponding maps of sequences are homotopic) if and only if they induce the same homomorphism of homology groups, $\boldsymbol{f}_* = \boldsymbol{g}_*$. From here one can easily deduce that S^n fundamentally dominates only its own shape and the trivial shape.

References

[1] *R. D. Anderson:* Point-like decompositions of the Hilbert cube (in preparation).

[2] *K. Borsuk:* Concerning homotopy properties of compacta. Fund. Math. *62* (1968), 223—254.

[3] *K. Borsuk:* Concerning the notion of the shape of compacta. Proc. Internat. Sympos. on Topology and its Applications (Herceg-Novi, 1968). Savez društava Mat. Fiz. i Astronom., Belgrade, 1969, 98—104.

[4] *K. Borsuk:* Fundamental retracts and extensions of fundamental sequences. Fund. Math. *64* (1969), 55—85.

[5] *K. Borsuk:* On movable compacta. Fund. Math. *66* (1969), 137—146.

[6] *K. Borsuk:* On a locally connected non-movable continuum. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *17* (1969), 425—430.

[7] *K. Borsuk:* On the shape of FANR-sets. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *17* (1969), 529—532.
[8] *K. Borsuk:* Some remarks concerning the shape of pointed compacta. Fund. Math. *67* (1970), 221—240.
[9] *K. Borsuk:* A note on the theory of shape of compacta. Fund. Math. *67* (1970), 265—278.
[10] *K. Borsuk:* On the shape of the suspension. Colloq. Math. *21* (1970), 247—252.
[11] *K. Borsuk:* On the concept of shape for metrizable spaces. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *18* (1970), 127—132.
[12] *K. Borsuk:* On the homotopy types of some decomposition spaces. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *18* (1970), 235—239.
[13] *K. Borsuk:* On homomorphisms of homology groups induced by fundamental sequences. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *18* (1970), 353—358.
[14] *K. Borsuk:* A note on the shape of quasi-homeomorphic compacta. Comment. Math. Prace Mat. *14* (1970), 25—34.
[15] *K. Borsuk:* Some remarks concerning the shape of pointed compacta. Fund. Math. *67* (1970), 221—240.
[16] *K. Borsuk:* Remark on a theorem of S. Mardešić. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 475—483.
[17] *K. Borsuk:* Theory of shape. Lecture Notes Series No. *28*. Matematisk Inst., Aarhus Univ., 1971, 145 pp.
[18] *K. Borsuk:* Some remarks concerning the theory of shape in arbitrary metrizable spaces. These Proceedings.
[19] *K. Borsuk and W. Holsztyński:* Concerning the ordering of shapes of compacta. Fund. Math. *68* (1970), 107—115.
[20] *K. Borsuk and W. Holsztyński:* Hopf classification theorem in the shape theory. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 387—390.
[21] *T. A. Chapman:* On some applications of infinite-dimensional manifolds to the theory of shape. Fund. Math. (to appear).
[22] *T. A. Chapman:* Some results on shapes and fundamental absolute retracts. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *20* (1972), 37—40.
[23] *T. A. Chapman:* Shapes of some decomposition spaces. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. (to appear).
[24] *T. A. Chapman:* Shapes of finite-dimensional compacta. Mathematisch Centrum Amsterdam, 1971, mimeographed, 28 pp.
[25] *T. A. Chapman:* Characterizing shapes of compacta. Bull. Amer. Math. Soc. (to appear).
[26] *R. H. Fox:* On shape. Fund. Math. *74* (1972), 47—71.
[27] *S. Godlewski:* Homomorphisms of cohomotopy groups induced by fundamental classes. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *17* (1969), 277—283.
[28] *S. Godlewski:* On shapes of solenoids. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *17* (1969), 623—627.
[29] *S. Godlewski:* Solenoids of comparable shapes are homeomorphic. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *18* (1970), 565—566.
[30] *S. Godlewski:* Homotopy dependence of fundamental sequences, relative fundamental equivalence of sets and a generalization of cohomotopy groups. Fund. Math. *69* (1970), 63—91.
[31] *S. Godlewski:* Cohomotopy groups and shape in the sense of Fox. Fund. Math. *75* (1972), 175—185.
[32] *S. Godlewski and W. Holsztyński:* Some remarks concerning Borsuk's theory of shape. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *17* (1969), 373—376.
[33] *D. W. Henderson:* Applications of infinite-dimensional manifolds to quotient spaces of complete ANR's. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 747—753.

[34] *W. Holsztyński:* An extension and axiomatic characterization of Borsuk's theory of shape. Fund. Math. *70* (1971), 157—168.

[35] *D. M. Hyman:* On decreasing sequences of compact absolute retracts. Fund. Math. *64* (1969), 91—97.

[36] *D. M. Hyman:* A remark on Fox's paper on shape. Fund. Math. *75* (1972), 205—208.

[37] *K. Kuperberg:* An isomorphism theorem of Hurewicz type in the Borsuk's theory of shape. (To appear.)

[38] *R. C. Lacher:* Cell-like spaces. Proc. Amer. Math. Soc. *20* (1969), 598—602.

[39] *S. Mardešić:* n-dimensional LC^{n-1} compacta are movable. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 505—509.

[40] *S. Mardešić:* On the shape of the quotient space S^n/A. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 623—629.

[41] *S. Mardešić:* A non-movable compactum with movable suspension. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 1101—1103.

[42] *S. Mardešić:* Retracts in shape theory. Glasnik Mat. Ser. III. *6* (*26*) (1971), 153—163.

[43] *S. Mardešić:* Decreasing sequences of cubes and compacta of trivial shape. General Topology and its Applications *2* (1972), 17—23.

[44] *S. Mardešić and J. Segal:* Shapes of compacta and ANR-systems. Fund. Math. *72* (1971), 41—59.

[45] *S. Mardešić and J. Segal:* Equivalence of the Borsuk and the ANR-system approach to shapes. Fund. Math. *72* (1971), 61—68.

[46] *S. Mardešić and J. Segal:* Movable compacta and ANR-systems. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *18* (1970), 649—654.

[47] *M. Moszyńska:* On shape and fundamental deformation retracts I, II. Fund. Math.; I: *75* (1972), 145—167; II: to appear.

[48] *M. Moszyńska:* Uniformly movable compact spaces and their algebraic properties. (To appear.)

[49] *M. Moszyńska:* Various approaches to the fundamental groups.

[50] *R. Overton:* Čech homology for movable compacta. (To appear.)

[51] *R. Overton and J. Segal:* A new construction of movable compacta. Glasnik Mat. Ser. III. *6* (*26*) (1971), 361—363.

[52] *H. Patkowska:* A homotopy extension theorem for fundamental sequences. Fund. Math. *64* (1969), 87—89.

[53] *J. Segal:* Shape classification of projective plane-like continua. Glasnik Mat. Ser. III. *6* (*26*) (1971), 365—371.

[54] *R. Geoghegan and R. Summerhill:* Concerning the shapes of finite-dimensional compacta. (To appear.)

[55] *S. Godlewski and S. Nowak:* On two notions of shape. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *20* (1972), 387—393.

[56] *D. Handel and J. Segal:* Shape classification of (projective m-space)-like continua. (To appear.)

[57] *D. Handel and J. Segal:* Finite shape classifications. (To appear.)

[58] *W. Holsztyński:* Continuity of Borsuk's shape functor. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. *19* (1971), 1105—1108.

[59] *S. Mardešić:* Shapes for topological spaces. (To appear.)

[60] *M. Moszyńska:* The Whitehead theorem in the theory of shapes. (To appear.)

[61] *T. J. Sanders:* Shape groups and products. Mimeographed. Univ. of Oklahoma, 1972.

[62] *R. B. Sher:* Realizing cell-like maps in Euclidean spaces. General Topology and its Applications (to appear).