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Summary of the paper σ-algebra generated by analytic sets and applications

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Summary of the paper

σ-algebra generated by analytic sets and applications

by K.Kuratowski [1)]

<u>Definitions</u>. 1. Given a Polish space X , the family of all analytic (called also "Souslin") subsets of X is denoted $S(X)$, or briefly S .

2. The σ-algebra generated by S is denoted $\bar{S}$.

3. A mapping $f : X \longrightarrow Y$ is called $\bar{S}$-measurable, whenever

$$f^{-1}(K) \in \bar{S} \quad \text{for each } K \text{ closed in } Y .$$

4. The graph of $f : X \longrightarrow Y$ is denoted

$$Gr(f) = \{<x,y> : y = f(x)\} .$$

5. The graph of the relation $y \in F(x)$, where F is a closed set-valued mapping $F : X \rightarrow 2^Y$, is denoted

$$G(F) = \{<x,y> : y \in F(x)\} .$$

6. A mapping $F : X \rightarrow 2^Y$ is called upper-Souslin, if

$\{x : F(x) \cap K \neq \emptyset\}$ is Souslin whenever K is closed.

<u>Theorems</u>. 1. If $f : X \longrightarrow Y$ is $\bar{S}$-measurable, then

$$Gr(f) \in \bar{S}(X \times Y) .$$

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1) A large part of the results contained in this paper appeared in [1] .

2. Let $F: X \to 2^Y$. Then

$G(F)$ is Souslin ≡ F is upper Souslin.

3. If $f : Y \to X$ is continuous onto, then the inverse mapping $f^{-1}: X \to 2^Y$ is upper-Souslin.

4. Under the assumption of Theorem 3, the mapping $F(x) = f^{-1}(x)$ admits an $\overline{S}$-measurable selector.

Namely, by Theorem 3 and the Kuratowski - Ryll-Nardzewski Theorem [2], the mapping $f^{-1}: X \to 2^Y$ admits an $\overline{S}$-measurable selector $g : X \to Y$, which means that $f \circ g = 1$.

Corollary (comp. v.Neumann [3]). For real-valued mappings, the assumptions of Theorem 3 imply the existence of a Lebesgue-measurable $g : X \to Y$ such that $f[g(x)] = x$.

For, analytic sets composed of reals are Lebesgue measurable.

Bibliography

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3. J.von Neumann, On rings of operators, Ann.of Math., 50 (1949), pp. 401-485.