Adam Mysior

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THE CATEGORY OF ALL ZERO-DIMENSIONAL REALCOMPACT SPACES IS NOT SIMPLE

A. MYSIOR

Gdańsk

For a given Hausdorff space E, a space X is said to be <u>E-compact</u> if it is homeomorphic to a closed subspace of E^{m} for some cardinal m.

Let us use the following notation

I - the closed unit interval,

D - the two-point discrete space,

R - the space of the reals,

N - the discrete space of natural numbers.

Thus we have

I-compact = compact Hausdorff,

D-compact = zero-dimensional compact Hausdorff,

R-compact = realcompact.

It was conjectured in 1958 that

N-compact = zero-dimensional realcompact.

It is clear that every N-compact space is zero-dimensional and realcompact but the converse is, in fact, false. It was showed by P.Nyikos in [6,7] that the Prabir Roy's space Δ, which is zero-dimensional and realcompact, is not N-compact.

There remained an open question /raised by H.Herrlich in [2,3,4]/: is there a space E such that

E-compact = zero-dimensional realcompact ?

In other words : is the category of all zero-dimensional realcompact spaces <u>simple</u>?

<u>Theorem</u>. For every zero-dimensional space E of non-measurable cardinality there exists a zero-dimensional, hereditarily realcompact, locally compact and locally countable space which cannot be embedded as a closed subspace into any topological power of the space E.

Under the assumption that all cardinals are non-measurable it gives the result stated in the title.

For the proof and more detailed information see [5] .

Remark. In the proof of our theorem some minor modification of E. van Douwen's construction of Λ /see this volume/ is used.

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Department of Mathematics
Gdańsk University
Wita Stwosza 57
80952 Gdańsk, Poland