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FINER TOPOLOGIES IN LOCALLY COMPACT GROUPS

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N. W. Rickert [2] raised the following question: "Find the number of topologies between two given locally compact group topologies τ_1 and τ_2 on an abelian group G, and such that $\tau_2 > \tau_1$". In this paper we settle the question when (G, τ_1) is compact. The proof essentially reduces to considering the case when the finer topology τ_2 is obtained by declaring $\Phi(R^n)$ open where $\Phi : (R^n, \tau) \to (G, \tau_1)$ is a 1-1 continuous isomorphism and $\Phi(R^n)$ is dense in (G, τ_1) and $\Phi(R^n)$ receives the homeomorphic topology from (R^n, τ), τ being the usual topology on R^n. We prove the following:

Theorem 1. *If (G, τ_1) is a compact abelian group, (G, τ_2) is locally compact and τ_2 is finer than τ_1, then the number of group topologies τ between them is either finite or uncountable.*

One of the results which is of independent interest and which is used in proving the above theorem is:

Theorem 2. *Let (G, τ_1) be a compact abelian group. Let (G, τ_2) be a locally compact group topologically stronger than τ_1, obtained by declaring $K \times \Phi(R^n)$ to be open as in Theorem 1 of [1]. Let τ_3 be a locally compact group topology of G such that $\tau_2 > \tau_3 > \tau_1$. Then $\tau_1 = \tau_2 = \tau_3$, when restricted to K. So K is compact and closed in τ_3 also. Then $(G/K, \tau_3^*)$ is between $(G/K, \tau_2^*)$ and $(G/K, \tau_1^*)$. Also $(G/K, \tau_2^*)$ is obtained by declaring $\Phi(R^n)$ to be open. Conversely every topology τ_3 between τ_1 and τ_2 is obtained as the inverse of a topology τ_3^* of G/K lying between $(G/K, \tau_1^*)$ and $(G/K, \tau_2^*)$.*

References

[1] *M. Rajagopalan:* Topologies in locally compact groups, Math. Annalen *176* (1968), 169—180.

[2] *N. W. Rickert:* Locally compact topologies for groups, Trans. Amer. Math. Soc. *126* (1967), 225—235.

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