SOME RESULTS ON LOCALLY COMPACT GROUPS

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In an earlier paper [1] we considered the topological group of all p-adic integers and obtained the following:

1. If G is an infinite compact group then the following statements are equivalent.

(a) The closed subgroups of G form a chain ordered by set inclusion.

(b) G is isomorphic to the group of all p-adic integers for some prime p.

(c) G is Abelian and every non-trivial closed subgroup of G is open.

(d) G is Abelian and every closed subgroup of G is nG for some n.

(e) G is Abelian and any infinite collection of closed subgroups of G has trivial intersection.

2. The only locally compact group topologies on the group of all p-adic integers are (1) the natural compact topology and (2) the discrete topology.

3. Any compact Abelian group for which the only other locally compact group topology is the discrete topology must be isomorphic to a topological direct sum $A \oplus B$ where A is the topological group of all p-adic integers for some prime p and B is a finite discrete Abelian group.

We report hereunder generalizations of some of these and conclude with some open problems.

1. A locally compact group G in which the closed subgroups form a chain ordered by set inclusion must be isomorphic either to discrete $C(p^\infty)$ or to the compact group of all p-adic integers for some prime p or to the locally compact group of all p-adic numbers for some prime p.

2. A locally compact Abelian group in which every non-trivial closed subgroup is open is either discrete or is isomorphic either to the compact group of all p-adic integers for some prime p or to the locally compact group of all p-adic numbers for some prime p.

3. A locally compact Abelian group in which every closed subgroup is nG for some n must be either discrete cyclic or must be isomorphic to the compact group of all p-adic integers for some prime p.

4. A locally compact Abelian group in which any infinite family of closed subgroups has trivial intersection is either finite or discrete cyclic or is the compact group of all p-adic integers for some prime p.

5. A compact group in which every non-trivial closed subgroup is open is either finite or is torsion-free and is the extension of the compact p-adic integer group by a finite group.

6. A compact group in which any infinite family of closed subgroups has trivial intersection is either finite or is torsion-free and is the extension of the compact p-adic integer group by a finite group.

Questions

I. Characterize all locally compact groups for which the only other locally compact group topology is the discrete topology. Even for the Abelian case the answer is not yet known.

II. Characterize Abelian groups which allow of only a finite number of locally compact group topologies. The answer here for the case of only one topology is easy.

References

[1] *Soundararajan, T.:* The topological group of p-adic integers. Publ. Math. Debrecen, *16* (1969), 75–78.

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