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A Class of Competing Models with Discrete Delay.

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Abstract. In this paper we consider a predator-prey models with discrete time lag. The prey, is assumed to regenerate in the absence of predators by logistic growth with carring capacity K. Two competing predators feed over the prey without interference between them. We assume the functional response of the predator population in the Michaelis-Menten forms. We show that the system is pointwise dissipative and the existence of a global attractor for the solutions of the model.

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This is the preliminary version of the paper.

1 Statement of the model

In 1978, Hsu, Hubbel and Waltman in the papers [4,5], have introduced the model

$$
\begin{aligned}
S'(t) &= \gamma S(t)(1-\frac{S(t)}{K}) - \frac{m_1X_1(t)S(t)}{a_1+S(t)} - \frac{m_2X_2(t)S(t)}{a_2+S(t)} \\
X_1'(t) &= \frac{m_1X_1(t)S(t)}{a_1+S(t)} - D_1X_1(t), \\
X_2'(t) &= \frac{m_2X_2(t)S(t)}{a_2+S(t)} - D_2X_2(t).
\end{aligned} \tag{1}
$$

where $S(t)$ is the number of the prey at time t, $X_i(t)$ is the number of the ith predator at time t, It is assumed that in the absence of predation, the prey growth logistically with carring capacity K. The predators are assumed to feed on the prey with saturing functional response to prey density. Specifically, we assume that the Michaelis-Menten Kinetics describes how the predators feed on the prey. The parameter m_i is the maximun birth rate of the ith predator, D_i is the death rate for the ith predator, a_i is the half-saturation constant for the ith predator i.e., the prey density at with the functional respose of the predator is half maximal, the parameter γ is the intrisic rate of increase, while K is the carrying capacity for the prey population.

In this model it is assumed that there are no significant time lags in the system.

A more realist situation occur if considered the past history of species are considered. This is , consider that prey population growth instantaneously but the dynamic of the predators depend on the prey density in the past by mean of a discrete delay. We get the following system

$$
\begin{aligned}
S'(t) &= \gamma S(t)(1-\frac{S(t)}{K}) - \frac{m_1X_1(t)S(t)}{a_1+S(t)} - \frac{m_2X_2(t)S(t)}{a_2+S(t)} \\
X_1'(t) &= \frac{m_1X_1(t)S(t-\tau_1)}{a_1+S(t-\tau_1)} - D_1X_1(t), \\
X_2'(t) &= \frac{m_2X_2(t)S(t-\tau_2)}{a_2+S(t-\tau_2)} - D_2X_2(t).
\end{aligned} \tag{2}
$$

with initial condition $S_0(\theta) = \phi(\theta)$, $\theta \in [-\tau, 0]$, $\phi \in C([-\tau,0], R^+)$ and $\tau = \max\{\tau_1, \tau_2\}$, $\tau_1 > 0$, $\tau_2 > 0$, $S(0) = \phi(0) \geq 0$, $X_{10}(\theta) = X_{10} \geq 0$, $X_{20}(\theta) = X_{20} \geq 0$.

2 Main Results

We define the parameters μ_1 y μ_2 as follow

$$\mu_i = \frac{a_i D_i}{m_i - D_i}, \quad i = 1, 2$$

and we suppose that $\mu_1 \neq \mu_2$.

In the following result we show that the solutions of system (2) is one positive and the pointwise dissipativity is established.

Theorem 1. *Let*

$$E = \{\phi = (\psi_1, \psi_2, \psi_3) \in C([-\tau, 0], R^3_+) : \psi_i(\theta) \geq 0, \quad \theta \in [-\tau, 0], i = 1, 2, 3\}$$

then, E is positively invariant under the flow induced by the system (2). Futhermore, the system (2) is point dissipative and the absorbent set; that is, the set where all the solutions eventualy enters and remains is $B = [0, K] \times [0, M_1] \times [0, M_2]$, where $M_1 = \gamma \frac{a_1 + K + 1}{m_1}$ and $M_2 = \gamma \frac{a_2 + K + 1}{m_2}$.

Corollary 2. *The systems (2) have a global attractor in $C([-\tau, 0], R^3_+)$.*

If $\mu_1 \neq \mu_2$ the point of equilibrium of the system (2) are

$$\begin{aligned}
&E_0 = (0, 0, 0), \qquad E_K = (K, 0, 0), \\
&E^* = (s^*, \frac{\gamma}{m_1 K}(K - s^*)(a_1 + s^*), 0), \ s^* = \frac{a_1 D_1}{m_1 - D_1}, \\
&E_* = (s_*, 0, \frac{\gamma}{m_2 K}(K - s_*)(a_2 + s_*)), \ s_* = \frac{a_2 D_2}{m_2 - D_2}
\end{aligned}$$

where $m_i > D_i$, $0 < s^* < K$ y $0 < s_* < K$.

Lemma 3. *If $X_i(t)$ survives then $0 < \mu_i < K$.*

Theorem 4. – *$E_0 = (0, 0, 0)$ is unstable.*
– *If*

a) $m_1 - D_1 \leq 0$ or $\frac{a_1 D_1}{m_1 - D_1} > K$, and

b) $m_2 - D_2 \leq 0$ or $\frac{a_2 D_2}{m_2 - D_2} > K$, then

$\lim_{t \longrightarrow \infty} S(t) = K$ *y* $\lim_{t \longrightarrow \infty} X_i(t) = 0, \quad i = 1, 2.$

The following lemma gives us a necessary condition for the extintion of X_1 and X_2.

Lemma 5. *If $\lim_{t \longrightarrow \infty} X_i(t) = 0, \quad i = 1, 2,$ then*

$$\frac{m_i - D_i}{a_i D_i} \leq \frac{1}{a_i + K}, \quad i = 1, 2.$$

Lemma 6. *$m_i - D_i \leq 0$ or $0 < K < \dfrac{a_i D_i}{m_i - D_i}$ if and only if $\dfrac{m_i - D_i}{a_i D_i} < \dfrac{1}{a_i + K}$.*

Lemma 7. *$0 < \dfrac{a_i D_i}{m_i - D_i} < K$ if and only if $\dfrac{m_i - D_i}{a_i D_i} > \dfrac{1}{a_i + K}$.*

Lemma 8. *Let $0 < \dfrac{a_1 D_1}{m_1 - D_1} < K < a_1 + \dfrac{2a_1 D_1}{m_1 - D_1}$. If $m_2 - D_2 \leq 0$ or if $\dfrac{a_1 D_1}{m_1 - D_1} < \dfrac{a_2 D_2}{m_2 - D_2}$. Then critical point $(s*, x_1^*, 0)$ is local asymptotically stable, where $s^* = \dfrac{a_1 D_1}{m_1 - D_1}$, $x_1^* = \dfrac{\gamma}{mK}(K - s^*)(a_1 + s^*)$.*

Theorem 9. *Let*

a) $0 < \dfrac{a_1 D_1}{m_1 - D_1} < K$, and

b) $m_2 - D_2 \leq 0$ or $\dfrac{a_2 D_2}{m_2 - D_2} > K$. If $K < a_1 + \dfrac{2a_1 D_1}{m_1 - D_1}$, Then

$$\begin{aligned}
&\lim_{t \longrightarrow \infty} S(t) = S^* = \frac{a_1 D_1}{m_1 - D_1},\\
&\lim_{t \longrightarrow \infty} X_1(t) = \frac{\gamma}{K m_1}(K - s^*)(a_1 + s^*),\\
&\lim_{t \longrightarrow \infty} X_2(t) = 0.
\end{aligned}$$

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