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Inequalities for solutions of systems with "pure" delay

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Abstract. A result concerning the estimation of a solution of a linear system with "pure" delay is formulated.

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Let us consider the following system of differential equations with delay

$$\dot{x}(t) = B(t)x(t-\tau), \quad x \in \mathbb{R}^n,\, t \geq t_0,\, \tau > 0, \tau = \text{const}. \tag{1}$$

We suppose that the corresponding system without delay (i.e. the system (1) with $\tau = 0$) has the fundamental matrix of solutions $\Phi(t, t_0)$, normed in $t =$

This is the preliminary version of the paper.

t_0. Investigation of the system (1) is performed with the aid of nonautonomous Liapunov function of quadratic form

$$v(x,t) = x^T H^*(t,t_0)x$$

with

$$H^*(t,t_0) = [\Phi(t,t_0)^{-1}]^T \Phi^{-1}(t,t_0).$$

Theorem 1. *Let matrix $B(t)$ be bounded, i.e. $\|B(t)\| \le \bar{B}$. Suppose, moreover, that there exists a constant $k > 0$ such that*

$$\gamma(t,t_0,k) \ge 0 \quad \textit{for} \quad t \ge t_0 + \tau$$

with

$$\gamma(t,t_0,k) =$$
$$= \sqrt{\lambda_{\max}[H^*(t,t_0)]} \times \left\{ \frac{k\lambda_{\min}[H^*(t,t_0)]}{\lambda_{\max}[H^*(t,t_0)]^{3/2}} - 2\bar{B}^2 \int_{t-2\tau}^{t-\tau} \frac{e^{-k(s-t)/2}\, ds}{\sqrt{\lambda_{\min}[H^*(s,t_0)]}} \right\}.$$

Then for solution $x(t)$ of the system (1), determined by continuous initial function $\varphi(t)$ on initial interval $[t_0-\tau, t_0]$, the following inequalities hold:

$$|x(t)| < [1 + \bar{B}(t-t_0)]\|x(t_0)\|$$

if

$$t_0 \le t \le t_0 + \tau$$

and

$$|x(t)| < [1 + \bar{B}\tau]\|x(t_0)\| \times \sqrt{\frac{\lambda_{\max}[H_0^*]}{\lambda_{\min}[H^*(t,t_0)]}} \exp\left\{ k\tau - \frac{1}{2}\int_{t_0+\tau}^{t} [\gamma(s,t_0,k) - k]\, ds \right\}$$

with

$$\|x(t_0)\| = \max_{t\in[t_0-\tau,t_0]} \|\varphi(t)\|, \quad \lambda_{\max}[H_0^*] = \max_{t_0-\tau\le s\le t_0+\tau} \{\lambda_{\max}[H^*(s,t_0)]\}$$

if $t \ge t_0 + \tau$.

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