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MATHEMATICAL SOLUTION OF DIRECT AND INVERSE PROBLEM FOR TRANSONIC CASCADE FLOWS

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The work deals with numerical solution of direct and inverse problem of transonic cascade flows based on potential model. Governing equation of a direct problem is full potential equation, governing equation of an inverse problem is equation for Mach number in hodograph plane (Φ,ψ), Φ-velocity potential, ψ-stream function. Both equations are partial differential equations of second order, mixed elliptic-hyperbolic type. In the solution of direct problem one can consider discontinuity of the first derivatives along some curves called shock waves, in the inverse problem one must find classical solution.

Numerical solution of both problems is based on using finite difference method and Jameson's rotated difference scheme. The system of difference equations is solved iteratively using succesive line relaxation method.

The work presents results of numerical solution of transonic flows in cascade of compressor and turbine type and one example of numerical solution of inverse problem.

I: Direct problem

A steady irrational isoentropic flow is fully described by the quasilinear partial differential equation of mixed elliptic-hyperbolic type for a velocity potential:

$$(a^2-\Phi_x^2)\Phi_{xx} - 2\Phi_x\Phi_y\Phi_{xy} + (a^2-\Phi_y^2)\Phi_{yy} = 0 \ , \qquad (1)$$

where Φ is velocity potential and $a = a(\Phi_x^2 + \Phi_y^2)$.
We assume the existence of weak shock waves as curves of discontinuity of the first derivatives Φ_x, Φ_y. The weak solution is assumed in a class $K(\Omega)$, where Ω is a domain of solution (see [1]).

The mathematical formulation of transonic cascade flows is some combination of Dirichlet's, Neuman's and periodic boundary value problem. On the inlet boundary we prescribe a Dirichlet's condition $(\bar{w} = \bar{w}_\infty)$, on profile contour a Neuman's condition of non-permeability $(\partial\Phi \backslash \partial\bar{n} = 0)$ and on the outlet boundary also a Neuman's condition $(\bar{w} = \bar{w}_2)$, where $\bar{w}_2$ is a constant determined uniquely by the value of circulation of velocity around the one profile of the cascade γ. Potential Φ still satisfies a Kutta-Youkovski condition on the trailing edge of the profile. The value of γ, unknown in advance, is determined during iteration process of the numerical solution.

Equation (1) is possible to locally transform to the form

$$(1 - M^2)\Phi_{ss} + \Phi_{nn} = 0 \qquad (2)$$

that is similar to equation (1), M-Mach number, $M = M(\Phi_s^2)$, M-given function, s - streamline direction, n - normal.

Consider (x,y) coordinate system and regular orthogonal grid. Jameson's concept of stable difference scheme is based on central difference approximation of second order for Φ_{ss} using Φ_{xx}, Φ_{xy}, Φ_{yy} in elliptic point $(1 - M^2 < 0)$ and backward approximation of first order for Φ_{ss} in hyperbolic point $(1 - M^2 < 0)$. Central approximation of second order in both cases is used for Φ_{nn} (details see [1]).

The system of difference equations is solved by a SLOR method. It is solved in one step of iteration for grid points lying on line x_i = const., succesive in the direction of flow. The relaxation parameter is chosen 1.7 for all mesh points in line x_i = const., if all this points does not lie on profile contour and if their local Mach number in computed iteration is less than 1; and equal to 1 in other cases.

II: Inverse problem

Solving inverse problem of transonic flow over an airfoil or through a cascade the following governing equation in hodograph plane has been used

$$AM_{\Phi\Phi} + BM_{\psi\psi} + CM_\Phi^2 + DM_\psi^2 = 0, \qquad (3)$$

$$A = M(1 - M^2)P^{\frac{2}{\varkappa-1}}, \quad P = 1 + \frac{\varkappa-1}{2}M^2,$$

$$B = M$$

$$C = -(1 + 3\frac{\varkappa-1}{2}M^2 + \frac{3-\varkappa}{2}M^4)P^{\frac{3-\varkappa}{\varkappa-1}},$$

$$D = -(1 + \varkappa M^2 P^{-1})$$

M-Mach number, Φ - velocity potential, ψ - stream function. Smooth solution is considered in this case due to regularity of transformation $(x,y) \to (\Phi,\psi)$. Boundary value problem is based on eq. (3) and Dirichlet's conditions for an airfoil or combination of Dirichlet's, Neuman's and periodicity conditions for a cascade.

The details are described in [2]. Numerical solution of the problem is a similar to the solution of eq. (1). Knowing $M(\Phi,\psi)$ we find angle ϑ (oriented angle of the flow in (x,y) system)

$$\vartheta = \int_{\Phi_0}^{\Phi} P^{-\frac{\varkappa}{\varkappa-1}} M_{\psi} M^{-1} d\tau$$

and then streamline coordinates ("zero" streamlines)

$$x(\Phi,\psi) = x(\Phi_0,\psi) + \int_{\Phi_0}^{\Phi} \frac{\cos\vartheta}{q(M)}\, d\tau,\quad y(\Phi,\psi) = y_0(\Phi_0,\psi) + \int_{\Phi_0}^{\Phi} \frac{\sin\vartheta}{q(M)}\, d\tau,$$

$q = (u^2 + v^2) = F_1(M)$, F_1 - given function.

III: Numerical results

Fig. 1 shows the iso-Mach lines of transonic flows calculation for compressor cascade with upstream Mach number $M_\infty = 0.83$. We can see the typical choked fows with so called closed sonic line ($M = 1$). It means that first end of the sonic line is situated on lower profile surface and the other end is situated on the upper profile surface.

Fig. 2 shows the iso-Mach lines of transonic flows calculation for turbine cascade with upstream Mach number $M_\infty = 0.337$ and downstream Mach number $M_2 = 0.803$. Small supersonic region ($M > 1$) is situated near lower profile surface. This cascade is more cambered and therefore the problem of numerical solution of transonic flows through this cascade is very complicated. The comparisons of our numerical results and experimental data is published in [4].

Fig. 3 shows results of inverse problem for given Mach number along upper (M_h) and lower (M_d) profile surface (fig. 3a); fig. 3b showes geometry of found cascade corresponding given distribution of Mach number along profile surface and other parameters.

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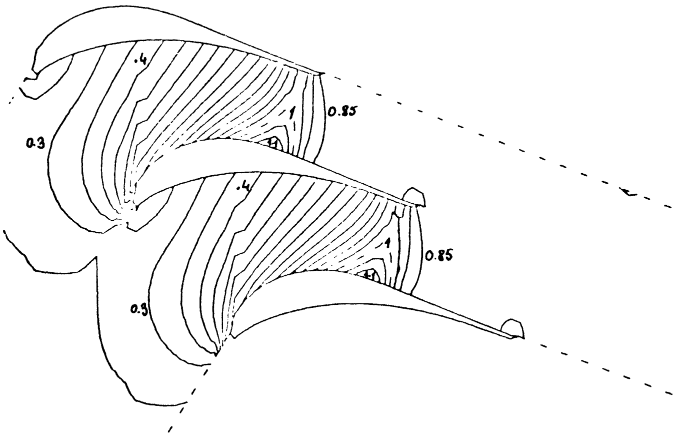

Fig. 1 : Compressor cascade. Iso-Mach lines of computed flow field, increment $\Delta M = 0.05$, $M_\infty = 0.83$

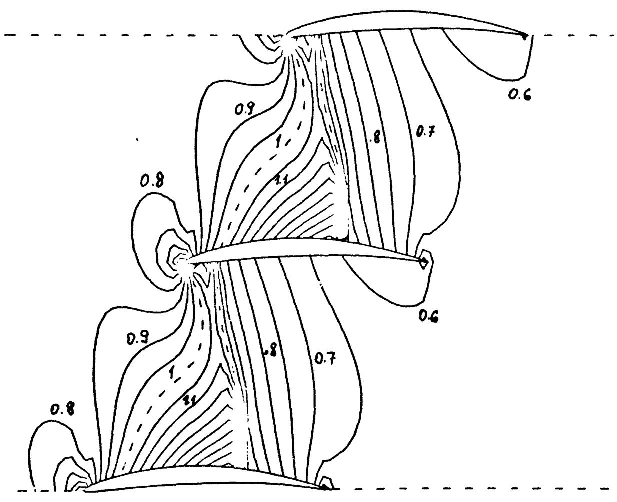

Fig. 2 : Turbine cascade. Iso-Mach lines of computed flow field, increment $\Delta M = 0.05$, $M_\infty = 0{,}337$, $M_2 = 0.809$

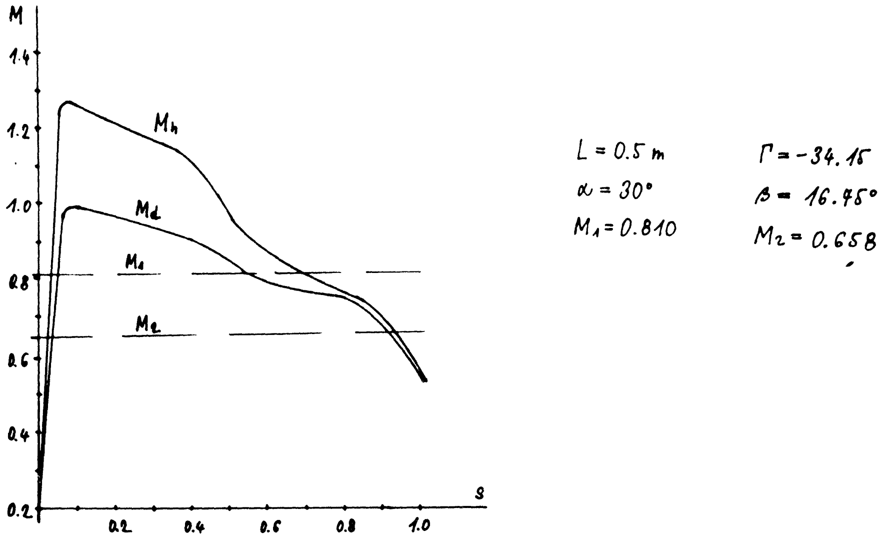

Fig. 3a: Inverse problem. Distribution of Mach number along upper and lower side of profile (M_h, M_α).

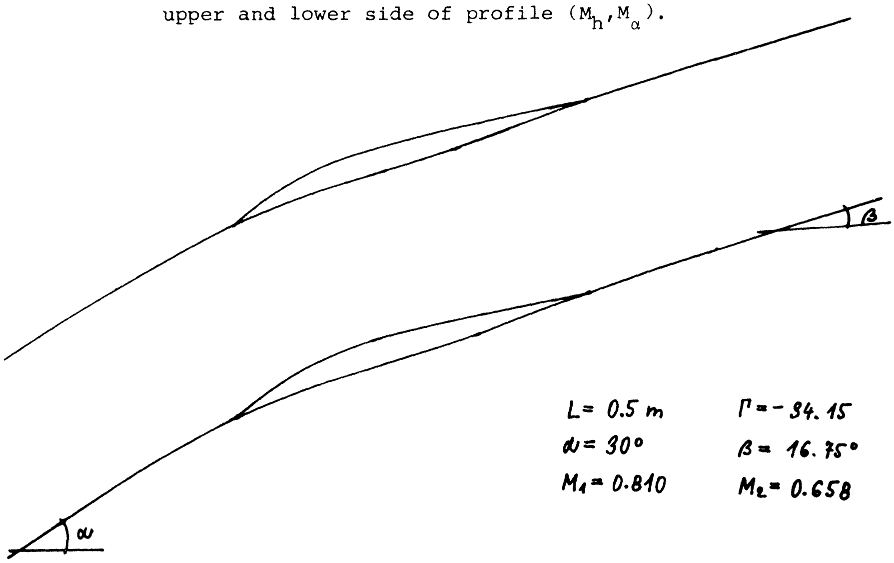

Fig. 3b: Inverse problem. Cascade geometry for given distribution of Mach number along profile.