Herbert Gajewski

On uniqueness and stability of steady-state carrier distributions in semiconductors

In: Jaromír Vosmanský and Miloš Zlámal (eds.): Equadiff 6, Proceedings of the International Conference on Differential Equations and Their Applications held in Brno, Czechoslovakia, Aug. 26 - 30, 1985. J. E. Purkyně University, Department of Mathematics, Brno, 1986. pp. [209]--214.

Persistent URL: http://dml.cz/dmlcz/700156

ON UNIQUENESS AND STABILITY OF STEADY-STATE CARRIER DISTRIBUTIONS IN SEMICONDUCTORS

H. GAJEWSKI
Karl-Weierstraß-Institut für Mathematik der Akademie der Wissenschaften der DDR 1086 Berlin, Mohrenstraße 39, DDR

In this paper we establish a simple smallness condition guaranteeing the basic equations for carrier distributions in semiconductors to possess a unique steady-state solution. Under this condition arbitrary perturbations of the steady state decay exponentially in time.

1. Introduction

Let G be a bounded Lipschitzian domain in R^d, $d \le 3$. Let the boundary S of G be the union of two disjoint parts S_1 and S_2, S_1 closed in S, mes $S_1 > 0$. A familiar model of carrier transport in a semiconductor device occupying G is given by the system [10,13]

$$-\Delta u = (\alpha/\varepsilon)(f + p - n), \tag{1.1}$$

$$qn_t = \nabla . J_n - qR, \; J_n = q\mu_n(k\nabla n - n\nabla u), \tag{1.2}$$

$$qp_t = -\nabla . J_p - qR, \; J_p = -q\mu_p(k\nabla p + p\nabla u), \tag{1.3}$$

$$u = U_s, \; n = N_s, \; p = P_s \text{ on } R^+ \times S_1, \; \nu.\nabla u = \nu.\nabla n = \nu.\nabla p = 0 \text{ on } R^+ \times S_2 \tag{1.4}$$

$$n(0,x) = n_0(x), \; p(0,x) = p_0(x), \; x \in G . \tag{1.5}$$

Here

- u is the electrostatistic potential,
- n and p are the mobile electron and hole densities,
- J_n and J_p are the current densities,
- f is the net density of ionized impurities,
- q is the electron charge,
- ε is the dielectric permitivity of the semiconductor material,
- $R = (np - n_i^2)/(\tau(n+p+2n_i))$ is the recombination rate,
- n_i is the intrinsic semiconductor carrier density,
- τ is the electron and hole lifetime,
- μ_n and μ_p are the (constant) electron and hole mobilities,
- U_s, N_s and P_s are given boundary values,
- ν is the outward unit normal at any point of S_2.

In the expressions for the current densities the Einstein relation $D_{n.p} = k\mu_{n,p}$ between diffusion coefficients and mobilities is used. ($k = k_B T/g$, k_b= Boltzmann constant. T = absolute temperature.)

The carrier transport equations (1.1)-(1.3) were derived by Van Roosbroeck [11] in 1950 and are now generally accepted. The first significant report on using numerical techniques to solve these equations for carriers in an operating semiconductor device structure has been published by Gummel [6] in 1964. Since then, the numerical modelling of semiconductor devices proved to be a powerful tool for device designers (see [13]).

In spite of their physical and technological relevance, the device equations received relatively little attention from the side of mathematical analysis. To our knowledge, the first matematical paper devoted to these equations appeared in 1972. In this paper Mock [7] proved the solvability of the steady-state equations associated to (1.1)-(1.5) supposing that $\mu_n = \mu_p$ and R = 0. More recently, Seidman [12], the author [3] and Gröger [5] have published more general existence theorems for steady states. All these results are based on maximum principle and compactness arguments.

As to the instationary problem (1.1)-(1.5), again Mock [8] was the first to prove a global existence and uniqueness result in a special situation. Recently, the author [2] and Gajewski&Gröger [4] could show the existence and uniqueness of global solutions under rather general assumptions. Of course, the crucial step in these papers consists in finding appropriate a-priori estimates. Such estimates are obtained by means of a physically motivated Liapunov function and an iteration technique due to Moser and Alikakos.

One of the essential open questions arising from the Van Roosbroeck equations is that of the uniqueness and stability of steady states. General answers to this question are not to be expect by physical reasons [1,10]. A special result in this direction [7] concerns the case of small perturbations of the thermal equilibrium which results from the assumption

$$U_s - k\log(N_s/n_i) = U_s + k\log(P_s/n_i) = c = \text{const. on } S_1$$

and is given by

$$N = n_i \exp((U - c)/k), \quad P = n_i \exp((c - U)/k),$$

where U is the (unique) solution of the nonlinear boundary value problem

$$-\Delta U = (q/\varepsilon)(f - 2n_i \sinh((U - c)/k)) \text{ in } G,$$

$BU = U_s$, where $Bv = \{v$ on S_1, $\nu.\nabla v$ on $S_2\}$ and $U_s = 0$ on S_2. The thermal equilibrium has been shown to be globally asymptotically stable (comp. [9] for the special case $S = S_2$ and [2,4] for more general situations). In fact, it was proved in [4] that for reasonable initial values the solution $(u(t),n(t),p(t))$ of (1.1)-(1.5) converges to the corresponding thermal equilibrium (U,N,P) exponentially in time. The proof of this result heavily upon the observation that the function

$$L(t) = \int_G (kq(n(\log(n/N)-1)+N+p(\log(p/P)-1)+P)+(\varepsilon/2)|\nabla(U-u)|^2)dx$$

is monotonously decreasing.

The main purpose of the present paper is to state another kind of smallness condition implying uniqueness as well as global asymptotic stability of stationary solutions. Our smallness condition involves the essential physical parameters and can be easily checked.

2. Results

Let L_2, L_∞, H_2^1 be the usual space of functions defined on G. We use the following notations

$$|v|^2 = \int_G v^2 dx, \quad |v|_\infty = \text{vrai max } v, \quad \|v\|^2 = \int_G |\nabla v|^2 = dx ,$$

$$v = \{v \in H_2^1 / \ v = 0 \text{ on } S_1\}, \ W = \{v \in (H_2^1 \cap L_\infty)^3 / \ v_2, v_3 \geq 0 \text{ in } G\}.$$

We assume that $f \in L_\infty$ and that the boundary values can be represented by functions $(U_s, N_s, P_s) \in W$. Let λ be the smallest eigenvalue of the problem

$$-\Delta v = \lambda v \text{ in } G, \quad Bv = 0 \text{ on } S ,$$

such that we have

$$\lambda |v|^2 \leq \|v\|^2, \quad v \in V . \qquad (2.1)$$

Now we can state our results.

Theorem 1. Let $(U,N,P) \in W$ be a stationary solution of (1.1)-(1.4) such that

$$r(Q) = \frac{\lambda}{2\lambda k}\left(\frac{q}{\varepsilon}(F + Q) + \frac{1}{2\mu\tau}\left(1 + \frac{Q}{2n_i}\right)\right) < 1$$

where

$$F = |f|_\infty, \quad Q = 4(|N|_\infty + |P|_\infty), \quad \mu = \min(\mu_n, \mu_p) .$$

Then (U,N,P) is unique in W.

Remark. As to existence results for steady states $(U,N,P) \in W$ we refer to [3]. In this paper also explicit bounds for $|N|_\infty$ and $|P|_\infty$ can be found which involve only f and the boundary values.

Theorem 2. Suppose $0 \le n_0,\ p_0 \in L_\infty$. Let (u,n,p) be the solution of (1.1)-(1.5) and let (U,N,P) be a stationary solution satisfying the hypotheses of Theorem 1. Then for $t \ge 0$ the following estimates are valid with $a = 2k\lambda\mu(1 - r(Q))$

$$\mu_p|n(t)-N|^2 + \mu_n|p(t)-P|^2 \le e^{-at}(\mu_p|n_0-N|^2 + \mu_n|p_0-P|^2)\ ,$$

$$\sqrt{\lambda}\,|u(t)-U| \le \|u(t)-U\| \le (q/(\varepsilon\sqrt{\lambda}))(|n(t)-N| + |p(t)-P|)\ .$$

Remark. The existence and uniqueness of the time-dependent solution (u,n,p) is guaranteed by [4], Theorem 1.

3. Proofs

We denote by $(.,.)$ the L_2-scalar product as well as the pairing between the Hilbert space V and its dual $V^* \subset L_2$. We introduce the set

$$M = \{[N,P] \in (H_2^1 \cap L_\infty)^2,\ N,P \ge 0 \text{ on } G,\ N=N_s,\ P=P_s \text{ on } S_1\}\ .$$

Finally, we define an operator $A \in (M \to (V^*)^2)$ by

$$(A[N,P],[h_1,h_2]) = \mu_p((\mu_n(k\nabla N-N\nabla U),\nabla h_1) + (R,h_1)) +$$
$$+ \mu_n((\mu_p(k\nabla P+P\nabla U),\nabla h_2) + (R,h_2)) \quad \forall h_1,h_2 \in V\ ,$$

where $R = R(N,P)$ and $U = U(N,P)$ is the solution of the boundary value problem

$$-\Delta U = (q/\varepsilon)(f + P - N),\quad BU = U_s \quad \text{on } S\ .$$

The main tool for proving our results is the following monotonicity property of the operator A.

Lemma. Let $[N_j,P_j] \in M$, $j=1,2$, $N_2 \le \overline{N}$, $P_2 \le \overline{P}$ in G, $\overline{N},\overline{P}$=cons. Set $Q=4(\overline{N}+\overline{P})$. Then it holds with $m=\mu_n\,\mu_p k(1-r(Q))$, $N=N_1-N_2$, $P=P_1-P_2$,

$$(A[N_1,P_1] - A[N_2,P_2],[N,P]) \ge m(\|N\|^2 + \|P\|^2)\ .$$

Proof. Setting $U_1= U(N_1,P_1)$, $U_2= U(N_2,P_2)$, $U=U_1-U_2$ and using (2.1) we get

$$\|U\|^2 = (q/\varepsilon)(P - N,U) \le (q/\varepsilon)|P - N|\,|U| \le (q/(\varepsilon\lambda))\|P - N\|\,\|U\|$$

and consequently

$$\|U\| = (q/(\varepsilon\sqrt{\lambda}))|P - N| \le (q/(\varepsilon\lambda))\|P - N\|,\ |U| \le (q/(\varepsilon\lambda))|P-N| \quad (3.1)$$

Thus we find

$$(k\nabla N-N_1\nabla U_1+ N_2\nabla U_2,\nabla N) + (k\nabla P + P_1\nabla U_1- P_2\nabla U_2,\nabla P) =$$
$$= k(\|N\|^2+ \|P\|^2) - (N\nabla U_1+ N_2\nabla U,\nabla N) + (P\nabla U_1+ P_2\nabla U,\nabla P) =$$
$$= k(\|N\|^2+ \|P\|^2) + (q/(2\varepsilon))(P^2-N^2,f + P_1-N_1)+(P_2\nabla P-N_2\nabla N,\nabla U) =$$
$$= k(\|N\|^2+ \|P\|^2) + (q/(2\varepsilon))(((N-P)^2,N_1+P_1) - (N^2,f + 2P_2) +$$
$$+ (P^2,f - 2N_2) + 2(NP,N_2+ P_2)) + (P_2\nabla P - N_2\nabla N,\nabla U) \ge$$

$$\geq k(\|N\|^2 + \|P\|^2) - (q/(2\varepsilon\lambda))((F + \overline{N} + 3\overline{P})\|N\|^2 + (F + 3\overline{N} + \overline{P})\|P\|^2 + $$
$$+ 2(\overline{N}\|N\| + \overline{P}\|P\|)\|P - N\|) \geq$$
$$\geq k(1 - (q/(2\varepsilon\lambda))(F + Q))(\|N\|^2 + \|P\|^2) .$$

On the other hand, setting $a_j = \tau(N_j + P_j + 2n_i)$, we get

$$(R_1 - R_2, N) = ((1/a_1)(NP_1 + N_2P - ((N_2P_2 - n_i^2)/a_2)\tau(N + P)), N) \geq$$
$$\geq -(1/\lambda)((\overline{N}/(2a_1))(\|N\|^2 + \|P\|^2) + (Q/(16a_1))(\|N\| + \|P\|)\|N\| +$$
$$+ (1/(8\tau))(\|N\|^2 + \|P\|^2)) \geq$$
$$\geq -(1/(4\lambda\tau)((\overline{N}/n_i)(\|N\|^2 + \|P\|^2) + (Q/(16n_i)(3\|N\|^2 + \|P\|^2) =$$
$$+ (1/2)(\|N\|^2 + \|P\|^2)).$$

Evidently, an analogous estimate holds for $(R_1 - R_2, P)$. Now the lemma is an immediate consequence from these estimates.

Proof of Theorem 1. Using the operator A we can rewrite the stationary problem as follows.

$$A[N,P] = 0, \quad [N,P] \in M . \qquad (3.2)$$

From this it becomes clear that the theorem follows easily from the lemma.

Proof of Theorem 2. We can write (1.1)-(1.5) in the compact form

$$[\mu_p n_t, \mu_n p_t] + A[n,p] = 0, \ [n(t),p(t)] \in M, \ n(0) = n_0, \ p(0) = p_0.$$

Hence, using (3.2) and the lemma, we get

$$0 \geq \tfrac{1}{2}(\mu_p|n - N|^2 + \mu_n|p - P|^2)_t + k\lambda\mu(1 - r)(\mu_p|n - N|^2 + \mu_n|p - P|^2).$$

Applying a well-known differential inequality and (3.1) we obtain the theorem.

Remark. Our lemma can also be used in order to find relaxation parameters b such that the iteration sequence $([N_j,P_j])$ defined by

$$-\Delta[h_1,h_2] = b\,A[N_j,P_j], \quad h_1,h_2 \in V, \ j = 0,1,\ldots$$
$$N_{j+1} = N_j + h_1, \ P_{j+1} = P_j + h_2, \ [N_0,P_0] \in M$$

converges to a stationary solution.

References

[1] BONČ-BRUEVICH,V.L., ZVJAGIN,I.P., MIRONOV,A.G., *Spatial electrical instability in semiconductors* (russian), Moscow 1972.

[2] GAJEWSKI,H., *On existence, uniqueness and asymptotic behavior of the basic equations for carrier transport in semiconductors*, ZAMM 65, (1985), 101-108.

[3] GAJEWSKI,H., *On the existence of steady-state carrier distributions in semiconductors*, In: Probleme und Methoden der Mathematischen Physik, Teubner-Texte zur Mathematik 63. (Ed. V. Friedrich u. a.).

[4] GAJEWSKI,H., GRÖGER,K., *On the basic equations for carrier transport in semiconductors*, J. Math. Anal. Appl., to appear.

[5] GRÖGER,K., *On steady-state carrier distributions in semiconductor devices*, to appear.

[6] GUMMEL,H.K., *A selfconsistent iterative scheme for one-dimensional steady state transistor calculations*, IEEE Trans. Electron Devices ED-11 (1964), 455-465.

[7] MOCK,M.S., *On equations describing steady-state carrier distributions in a semiconductor device*, Comm. Pure Appl. Math. 25 (1972), 781-792.

[8] MOCK,M.S., *An initial value problem from semiconductor device theory*, SIAM J. Math. Anal. 5 (1974), 597-612.

[9] MOCK,M.S., *Asymptotic behavior of solutions of transport equations for semiconductor devices*, J. Math. Anal. Appl. 49 (1975), 215-225.

[10] MOCK,M.S., *Analysis of mathematical models of semiconductor devices*, Dublin 1983.

[11] VAN ROOSBROECK,W., *Theory of the flow of electrons and holes in Germanium and other semiconductors*, Bell Syst. Tech. J 29 (1950), 560-623.

[12] SEIDMAN,T.I., *Steady state solutions of diffusion-reaction systems with electrostatic convection*, Nonlinear Analysis 4 (1980), 623-637.

[13] SELBERHERR,S., *Analysis and simulation of semiconductor devices*, Wien-New York 1984.