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Vladimír Sladek; Ján Sladek

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MULTIPLE RECIPROCITY METHOD FOR PLATE BENDING VIBRATION, SOUND PRESSURE VIBRATION AND COUPLED STRUCTURAL–ACOUSTIC PROBLEMS

VLADIMÍR SLADEK — JÁN SLADEK

ABSTRACT. A unique approach is applied to solution of boundary value problems for the Helmholtz equation in three dimensions, the plate bending vibration equation, and finally for coupled fields governed by these two equations. The conventional boundary element method is reformulated by using the frequency independent fundamental solutions and the multiple reciprocity method is applied to transform the domain integrals into the boundary ones. The singular integrals are regularized before the discretization of the boundary integral equations.

1. Introduction

In recent years there has been an increasing need to study control and reduction techniques of noise emanating from vibrating structures. The noise in most cases is caused by a coupled vibrating effect between the structure and its ambient medium (air). Therefore, such structural-acoustic coupling problems must be analysed to investigate the noise properties correctly. Several attempts were made on the coupling problems from both the analytical and numerical standpoints. The analytical approach [1, 2] is restricted to simpler systems. At present, there are available the numerical approaches in which the ambient medium is treated by the boundary element method and the structure is discretized either by the finite element method [3–6] or by the boundary-domain elements [7].

In this paper, we shall be interested in the coupling problem for a finite cavity-backed finite plate. The sound pressure field is governed by the scalar wave equation in three dimensions. Making use of the Laplace (or Fourier) transform with respect to time or concerning with harmonic vibrations, the governing equation is reduced to the Helmholtz equation. We confine ourselves to harmonic

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excitations. The motion of the thin elastic plate is governed by the plate bending vibration equation, and the interaction is described by the equilibrium and compatibility conditions.

The convectional BEM formulation [8] of solution of the three-dimensional boundary value problems for the Helmholtz equation is based on the use of the fundamental solution $G = \exp(ikr)/4\pi r$. Hence, both the real and imaginary parts of the boundary integrals are to be evaluated and all the integrals must be recompute once again for any value of the frequency. Moreover, a very fine discretization mesh is needed for large values of the wavenumber due to the oscillatory character of the fundamental solution. All these disadvantages of the conventional formulation can be removed by using the MRM–BEM formulation [9, 12, 13], while still preserving the pure boundary character of the formulation.

A similar approach is employed also in the BEM formulation for solution of harmonic vibrations of plate bending problems including viscous damping. Making use of the static fundamental solutions one can convert the domain integrals once again into the boundary ones using the multiple reciprocity method. Note that the infinite series of the boundary integrals converge uniformly provided that the frequency of harmonic oscillations and the size of the domain are finite. Thus, we obtain fully-boundary formulations for solution of plate bending vibration, sound pressure vibration and coupled structural-acoustic problems. Moreover, the derived formulation is nonsingular what made us possible to develop sufficiently accurate numerical algorithms. The C^1–continuous Overhauser elements have been employed in the numerical implementation of the "plate–BIE", in order to satisfy the continuity of the derivative of deflection amplitudes occurring in the nonsingular BIE. As all the integrals in this formulation are frequency independent, previously evaluated integrals can be reemployed in computations for any frequency. Several numerical examples [9, 11] illustrate high accuracy and efficiency of the developed BEM–MRM formulation which can be used as powerful tool in eigenvalue and optimization problems where many computations for different frequencies are required.

2. MRM–BEM formulation for sound vibration

It is known that the amplitude of the sound pressure field harmonically varying in time is governed by the Helmholtz equation

$$\nabla^2 p(\boldsymbol{x}) + k^2 p(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Omega, \tag{1}$$

in which the wavenumber k is related with the angular frequency of harmonic oscillations, ω, by $k = \omega/c_0$, if c_0 is the velocity sound in air.

Making use of the sequence of the fundamental solutions

$$u^*_{(a)}(r) = \frac{1}{4\pi}\frac{r^{2a-1}}{(2a)!} \qquad \text{for } a = 0,1,2,\ldots, \nabla^2 u^*_{(a+1)} = u^*_{(a)} \text{ and } \nabla^2 u^*_{(0)} = -\delta(r),$$

one can derive the integral representation of the pressure field (strictly speaking its amplitude) given by

$$p(\boldsymbol{y}) = \sum_{a=0}^{n}(-k^2)^a \int_{\partial\Omega}\Big[\frac{\partial p}{\partial \boldsymbol{n}}(\eta)u^*_{(a)}(|\eta-\boldsymbol{y}|) - p(\eta)q^*_{(a)}(\eta,\boldsymbol{y})\Big]dS(\eta) + \widetilde{R}_n(\boldsymbol{y}), \quad (2)$$

where

$$q^*_{(a)}(\eta,\boldsymbol{y}) = \frac{2a-1}{4\pi(2a)!}r^{2(a-1)}r_{,k}n_k(\eta), \quad (a = 0,1,2,\ldots).$$

The rest term

$$\widetilde{R}_n(\boldsymbol{y}) = -(-k^2)^{n+1}\int_{\Omega} p(\boldsymbol{x})\, u^*_{(n)}(|\boldsymbol{x}-\boldsymbol{y}|)\, d\,\Omega(x)$$

converges once again to zero as $n \to \infty$, if k and Ω are finite [9].

In order to eliminate the r^{-2} singularity contained in the kernel $q^*_{(0)}$, one might superpose the solution (2) with a homogeneous solution. Then, approaching the interior point $\boldsymbol{y}$ to the boundary one, $\zeta \in \partial\Omega$, we obtain the nonsingular BIE

$$\int_{\partial\Omega}[p(\eta) - p(\zeta)]\, q^*_{(0)}(\eta,\zeta)\, dS(\eta) + \\ + \int_{\partial\Omega}\Big[p(\eta)\hat{q}^*(\eta,\zeta) - \frac{\partial p}{\partial \boldsymbol{n}}(\eta)\hat{u}^*(|\eta-\zeta|)\Big]\, dS(\eta) = 0, \quad (3)$$

in which

$$\hat{u}^*(r) = \sum_{a=0}^{n}(-k^2)^a u^*_{(a)}(r), \quad \hat{q}^*(\eta,y) = \sum_{a=1}^{n}(-k^2)^a q^*_{(a)}(\eta,\boldsymbol{y}).$$

Thus, unknown are localized on the boundary surface $\partial\Omega$ alone, and the boundary integrals, which are to be calculated, are independent of the wavenumber. Both the need of complex arithmetic in integration and the spatial oscillations (due to the fundamental solution of the conventional formulation) are avoided.

3. MRM–BEM formulation for plate vibration

Consider a homogeneous, isotropic, thin and linear elastic plate of surface S and perimeter Γ which is subjected to the lateral dynamic load with a harmonic variation in time. Then, the amplitude of forced vibrations of the plate, $w(\boldsymbol{x})$, is governed by the equation

$$D\nabla^4 w(\boldsymbol{x}) - \kappa w(\boldsymbol{x}) = p(\boldsymbol{x}), \tag{4}$$

where

$$\kappa = \omega^2 \rho h - i\omega g, \quad i = \sqrt{-1}, \tag{5}$$

$p(\boldsymbol{x})$ is the amplitude of the external loading per unit area, ω is the angular frequency of harmonic oscillations, ρ is the mass density, h is the uniform thickness, and $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate, with E and ν being the Young modulus of elasticity and the Poisson ratio, respectively. Note that the external viscous damping is included with g being the damping coefficient.

It is known that the fundamental solution of equation (4) can be expressed in terms of complex Bessel or Kelvin functions [10]. Since this fundamental solution is frequency dependent, all the integrals in such a BEM formulation are to be recalculated for any value of the frequency. In order to avoid this shortcoming, it is possible to employ the MRM–BEM formulation [11], in which all the integrals are frequency independent.

Making use of the sequence of the fundamental solutions

$$\begin{aligned} w^*_{(a)}(r) &= \frac{r^{2(2a+1)}}{4^{(2a+1)} 2\pi D \big((2a+1)!\big)^2} \big(\ln r - S_{(a)}\big), \\ S_{(a)} &= \sum_{l=1}^{2a+1} \frac{1}{l}, \quad (a = 0, 1, 2, \ldots) \end{aligned} \tag{6}$$

satisfying the equations

$$\nabla^4 w^*_{(a+1)} = w^*_{(a)}, \quad \nabla^4 w^*_{(0)}(r) = \frac{\delta(\boldsymbol{r})}{D}, \tag{7}$$

one can derive the BEM formulation [11], in which the domain integrals are replaced by infinite series of boundary integrals with frequency independent kernels. Since the rests of truncated series converge to zero (uniformly, as long as the frequency and the size of the domain are bounded) by increasing the number of terms considered in the series to infinity, one may truncate these series with neglecting the rest terms. Moreover, the derived singular BIE can be regularized before their discretization. Note that all the integrals involved are frequency independent and, hence, it is sufficient to evaluate them only once and store in the computer memory in order to be used in the computations for other values of frequency.

4. Structural–acoustic coupling system

Let us consider a cavity Ω with the boundary $\partial\Omega + S$, where S is an elastic thin plate and $\partial\Omega$ is acoustically rigid surface. The cavity is assumed to be filled with air and the plate is excited harmonically in time by an external loading of the amplitude $p_e(\boldsymbol{y})$, $\boldsymbol{y} \in S$. Taking into account the equilibrium condition, we may write the governing equations for the amplitudes of the sound pressure $p(\boldsymbol{x})$ and the deflection $w(\boldsymbol{y})$ as

$$\nabla^2 p(\boldsymbol{x}) + k^2 p(\boldsymbol{x}) = 0\,, \quad \boldsymbol{x} \in \Omega\,, \tag{8}$$

$$D\nabla^4 w(\boldsymbol{y}) - \kappa w(\boldsymbol{y}) = p(\boldsymbol{y}) - p_e(\boldsymbol{y})\,, \quad \boldsymbol{y} \in S\,. \tag{9}$$

The boundary conditions for the plate bending vibrations are not influenced by the interaction with air, and we shall not yet specify them. On the other hand, bearing in mind the compatibility condition, the boundary condition for the sound pressure field on the interface is determined by the velocity of the vibrating plate. Thus,

$$\frac{\partial p}{\partial \boldsymbol{n}}(\boldsymbol{y}) = \omega^2 \rho_0 w(\boldsymbol{y})\,, \quad \boldsymbol{y} \in S\,, \tag{10}$$

where ρ_0 is the density of air. On the acoustically rigid walls, $\partial\Omega$, the normal derivative of the sound pressure is equal to zero.

The set of the boundary integral equations derived for this problem involves the "*plate unknowns*" on the boundary contour Γ and the *sound pressure amplitude* on $\partial\Omega + S$, since the amplitude of the deflection at interior points $\boldsymbol{y} \in S$ can be expressed in terms of the "*plate unknowns*" and the pressure $p - p_e$ on the interface S.

Numerical example.

In order to compare the numerical results by the proposed MRM–BEM formulation with the analytical ones, we have considered the same model as that investigated analytically and experimentally by Guy and Bhattacharya [2].

This model consists of a rectangular box with one simply supported brass panel. The other walls of the box are acoustically rigid.

Since all the integrands are bounded, we employed the standard Gaussian quadrature with 12 integrations points on each contour element and 6×6 integration points on the surface elements.

We have evaluated the variation of the pressure ratio P_1/P_2 versus the frequency $f \in \langle 10, 1000 \rangle$ [Hz], where P_2 is the sound pressure at the midpoint of the back wall of the box, opposing the brass panel, and P_1 is the external exciting pressure applied to the brass panel. In the analytical computation [2], we employed the step $\Delta f = 1$ [Hz], while in the numerical computation, the step $\Delta f = 10$ [Hz], and near the resonant frequencies, we decreased the step to 0.01 [Hz]. Having stored the calculated integrals in the computer memory, we could perform a lot of numerical solutions of the boundary value problem for different values of the frequency. In order to satisfy the convergence of the MRM–BEM formulation for large values of the frequency, we employed $n = 15$ terms in the expansion series.

Figure shows the comparison of the numerically computed pressure ratio P_1/P_2 with the analytical solution. In Table 1, we present the resonant frequencies for the coupled system predicted from the analytical solution by G u y and B h a t t a c h a r y a [2] as well as those founded from the numerical solution by the MRM–BEM formulation.

TABLE 1. Resonant frequencies [Hz] for the cubic cavity-backed the simply supported panel

	f_1^{res}	f_2^{res}	f_3^{res}	f_4^{res}
analytical	91.	391.	696.	853.
MRM–BEM	87.7	390.	680.12	854.
% error	3.62	0.26	2.28	0.12

5. Conclusions

The developed MRM–BEM formulation for solution of the structural-acoustic coupling problems can be characterized as:

(i) pure boundary formulation with the unknowns localized on the boundary alone,

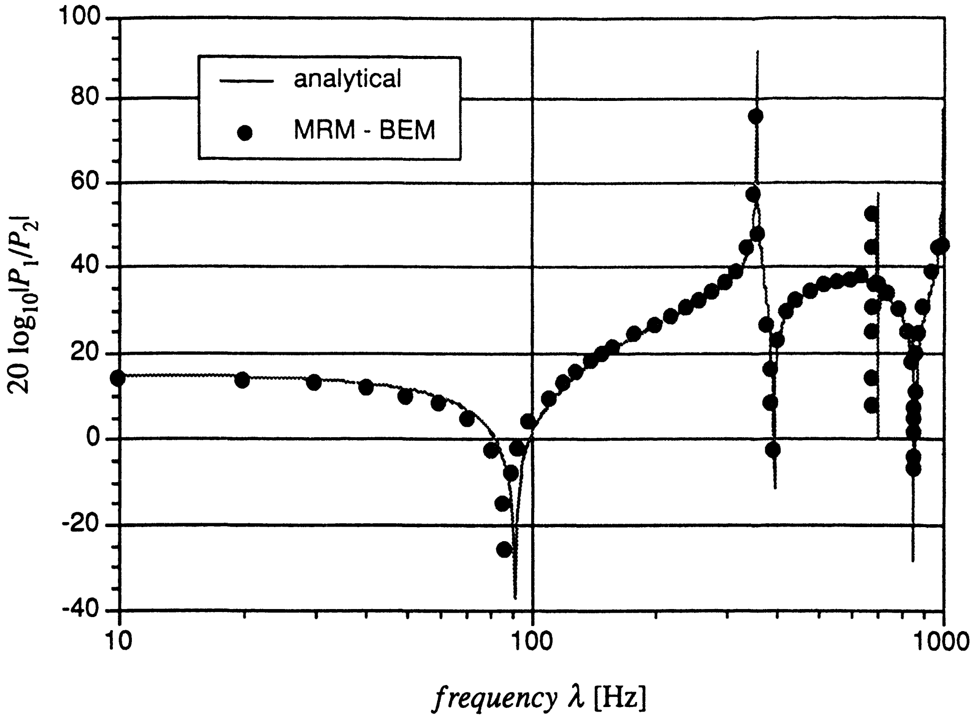

FIGURE 1.

(ii) the boundary integrals are frequency independent and can be calculated by using real arithmetic,

(iii) the boundary integral equations are nonsingular, hence, the standard Gaussian quadrature can be used for numerical integrations,

(iv) the static fundamental solutions are employed, so the spatial oscillations of the integral kernels are eliminated and no special requirements are imposed on fine discretization of the boundary.

The efficiency of the proposed formulation appears to be expressively remarkable, if the same boundary value problem is to be solved repeatedly many times for different values of frequency.

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Institute of Construction and Architecture
Slovak Academy of Sciences
842 20 Bratislava
SLOVAKIA