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Accessibility and homology

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Accessibility and homology

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The letters F and p denote everywhere a given closed subset of the Euclidean n-space E_n and a given point of it.

We say that F is totally accessible in p if every neighbourhood U of p contains a neighbourhood V of p, such that $D+(p)$ is a semicontinuum for every component D of $U-F$, such that

$$DV \neq 0.$$

We say that F is semitotally accessible in p if, given any two neighbourhoods U and Z of p, there exists a neighbourhood V of p, such that $D+(p)$ is a semicontinuum for every component D of $U-F$, such that

$$DV \neq 0 \neq D-Z.$$

We write $\alpha(p, F)=0$ if every neighbourhood U of p contains a neighbourhood V of p, such that $U-F$ has a finite number of components D, such that $DV \neq 0$.

We write $\alpha'(p, F)=0$ if, given any two neighbourhoods U and Z of p, there exists a neighbourhood V of p, such that $U-F$ has a finite number of components D, such that $DV \neq 0 \neq D-Z$.

If $\alpha(p, F)=0$, then F ist totally accessible in p; if $\alpha'(p, F)=0$, then F is semitotally accessible in p. The converse statements are false.

It follows from the local duality theorem (Alexandroff and Čech) that the equation $\alpha(p, F)=0$ expresses a topological property of the space F in the point p. The same thing is true for $\alpha'(p, F)=0$.

Alexandroff proved that the total accessibility of F in p is a topological property of F in p. The same thing is true for the semitotal accessibility.

Borsuk proved that $\alpha(p, F)=0$ if F is locally contractile in every point. It is possible to prove a more general theorem. Let m designate either $n-1$ or $n-2$. Suppose that, given any $\varepsilon>0$, there exists a $\delta>0$ such that, if C^k is a k-cycle $(0 \leqslant k \leqslant m)$ situated in a compact subset S of E_n-F, such that $d(S)<\delta$, where d is the diameter, then there exists a compact subset T of E_n-F such that $d(T)<\varepsilon$ and $C^k \backsim 0$ in T. Then F is totally accessible if $m=n-1$ and semitotally accessible if $m=n-2$.

Let $\mu(p, F)$ denote the number of those complementary domains D of F for which $D+(p)$ is a semicontinuum. Then the number

$$\max[1, \mu(p, F)]$$

is a topological property of F in p.

Достижимость и гомология

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(Резюме)

Различные понятия достижимости ставятся в связь с локальными гомологическими свойствами, локальными теоремами двойственности и локальной стягиваемостью.