SUMMARY

The present monograph extends the available surveys of philosophical interpretations of probability (the treatise [320] by I. Saxl, the book [120] by D. Gillies and other papers cited in the references) by a look into the Czech lands. In this respect, it also supplements the book [225] by K. Mačák, which describes the development of probability theory in this region before 1938.

Recall that although most mathematicians agree with the axiomatic definition of probability in the sense of Kolmogorov's foundations presented in his *Grundbegriffe der Wahrscheinlichkeitsrechnung* [196], a disagreement concerning the most convenient *interpretation* of this concept still prevails. Usually, two groups of interpretations are distinguished: *epistemic* or *personal interpretations* that associate probability with the knowledge or belief of human beings, and *objective interpretations* that consider probabilities to be human-independent features of the objective material world. The domain of epistemic interpretations covers the so-called *logical* and *subjective* theories. The first one identifies probability with a degree of rational belief in a hypothesis or a prediction; given the same evidence, all rational human beings would entertain the same probability. This conception goes back to G. W. Leibniz and B. Bolzano with his extensive work *Wissenschaftslehre* [B10], where probability theory is explicitly developed as an extension of deductive logic and an integral part of the entire logical theory. From other representatives of this conception, let us mention J. von Kries, W. E. Johnson, J. M. Keynes, L. Wittgenstein, F. Waismann (see e.g. their works [199], [178], [193], [390] and [383], respectively). The subjective interpretation identifies probability with the degree of belief of a given individual in the occurrence of some event or in the validity of some hypothesis. W. F. Donkin, B. de Finetti, F. P. Ramsey and L. J. Savage are amongst its most famous proponents (see [297], [98]–[101] and [317]). Two principal objective interpretations are *frequency theory* that defines the probability of an outcome as the limiting frequency with which it appears in a long series of similar events, and the *propensity theory* that takes probability to be a propensity inherent in a set of repeatable conditions. The former is connected mainly with R. L. Ellis, J. Venn, G. T. Fechner, G. F. Helm, H. Bruns and, above all, R. von Mises (see [87], [380], [93], [141], [47] and [241]–[243]); the latter was introduced by K. Popper (see [292]).

The first chapter of this book recounts the development of the definition of probability from the 17$^{\text{th}}$ to the 20$^{\text{th}}$ centuries. It also presents a survey of the main interpretations of this concept and points out their importance for motivating students to study probability in school curriculums. Special attention is paid to the subjective interpretation that deals with real concepts, with a subjective acceptance or rejection of hypothesis, and thus corresponds to everyday considerations. Czech textbooks from the 19$^{\text{th}}$ century are also discussed; in most of them, applications of probability theory, especially in the field of insurance and betting, were highlighted. Further chapters are devoted to the contributions of particular personalities to probability theory with the emphasis on interpretations.

The first is Bernard Bolzano (1781–1848), the pioneer of logical conception. In order to defend the Holy Scripture, he incorporated probability calculus into a religious textbook *Lehrbuch der Religionswissenschaft* [B8] published in 1834. From a mathematical point of view, the above-mentioned treatise *Wissenschaftslehre* [B10], where Bolzano builds probability theory as an extension of deductive logic, might be more interesting. He defines the *probability* or the *relative validity* of a proposition M with respect to propositions A, B, $C, \ldots$ and variables i, j, $k, \ldots$ as the ratio of the number of cases in which all the propositions A, B, $C, \ldots$ as well as M are true to the number of cases in which only the propositions A, B, $C, \ldots$ are all true. In other words, Bolzano's probability expresses the degree of justification of the hypothesis M on the basis of the evidence $E = A \wedge B \wedge C \wedge \ldots$. If we denote with $m(X)$ the measure for the set of the variables for which a proposition X is true, we can write $P(M|E) = m(M \wedge E)/m(E)$.

Many of Bolzano's ideas contained in [B10] were spread thanks to the first edition of the textbook of philosophical propaedeutic for grammar schools [401], published in 1853 by Bolzano's former student R. Zimmermann.[42] Towards the end of the 19^{th} and beginning of the 20^{th} centuries, [B10] was appreciated by F. Brentano and his pupils, especially B. Kerry, E. Husserl, K. Twardowski and his student J. Łukasiewicz (see [192], [158], [377] and [219], respectively). Later it was held in high regard by the representatives of logical empiricism. Bolzano's contribution to probability theory was cited, for example, by P. Frank, F. Waismann and W. Dubislav at the *First Conference on the Epistemology of Exact Sciences*, which took place in Prague in 1929. Their contributions were published in the first volume of Erkenntnis, a publication series of the Vienna Circle whose program declaration was read for the first time at the Prague conference. In the introduction to the Stuttgart edition of *Wissenschaftslehre*, J. Berg compared the theories of Bolzano, Wittgenstein and Carnap and highly appreciated Bolzano's contribution by designating him the first philosopher who drew up the concept of inductive probability.

The third chapter of the book concerns another proponent of the logical conception of probability, Tomáš Garrigue Masaryk (1850–1937). For his inaugural lecture at Charles-Ferdinand University in Prague, Masaryk chose the theme *Probability Calculus and Hume's Scepticism*; he developed the topic further in the Czech treatise [M3] and in its shortened German variation [M6]. Although all of Masaryk's other publications were devoted to philosophy, sociology and later politics, and, although the character of the mentioned treatises was primarily philosophical, it is remarkable that they display Masaryk's wide acquaintance with the development of probability theory, especially in the connection with inductive logic. The cited treatises were conceived as an answer to Hume's ideas formulated in his *Enquiry Concerning Human Understanding* [150], especially to the idea that inductive inferences are solely based on habits, and since the concept of causal connection does not correspond to any impression of the

[42] On the recommendation of Bolzano himself, Zimmermann did not explicitly cite his name, since his treatises were on the *Index Librorum Prohibitorum*.

external or internal experience, it is completely blank. Masaryk characterises the principle of Hume's scepticism with the following: *Only mathematics deserves our confidence, empirical sciences are uncertain, since the recognition of causal connections of facts evades us; because we can gain reliable knowledge only on the basis of an evident relation between the cause and an effect* ([M3], p. 24).

Masaryk describes the history of philosophical attempts to disprove Hume's scepticism, and he finds all of them insufficient. He starts with the ideas of philosophers of the Scottish School, T. Reid, J. Beattie and J. Oswald, and then he comes to I. Kant and F. E. Beneke. He continues with the first attempts to disprove Hume's scepticism with the help of probability theory, namely the contributions of J. G. Sulzer, M. Mendelssohn, J. M. de Gérando, S.-F. Lacroix and S. D. Poisson. Then he turns to inductive logic and its history. He discusses the work of G. W. Leibniz, J. Bernoulli, P. S. Laplace, A. Quetelet, R. Herschel, J. Venn etc. He concludes with the remark: *All these recent contributions lack an explicit relation to Hume; hereby they lack, I would say, the real point* ([M3], p. 14). It seems that Masaryk did not know the relevant treatises of B. Bolzano yet: in [B8] Hume is explicitly cited, in [B10] Bolzano systematically builds inductive logic as the extension of deductive logic, based on probability theory.

The next chapter deals with the contribution of the Czech priest and mathematician Václav Šimerka (1819–1887), who can be classed as an advocate of subjective theory. In the remarkable treatise, *Power of Conviction* [Š11], he thoroughly investigated the numerical expression of the strength of conviction by probability. Let us briefly remark that to assemble more convictions together, Šimerka introduces the concept of an *imperfection of a conviction* as a difference between complete knowledge and the given conviction v. Consider convictions v, v', $v'', \ldots$ and the corresponding imperfections. The resulting power of conviction V is given by the formula $1-V = (1-v)(1-v')(1-v'')\ldots$, which can be expressed as follows: the imperfection of a human conviction is a product of imperfections of its grounds. For $v = v' = v'' = \cdots = 0$ we have $V = 0$; according to Šimerka's words: *empty grounds provide no belief.* For $v' = v'' = \cdots = 0$ we obtain $V = v$ and the characterisation: *in an empty mind every ground enroots with its full power.* Šimerka's paper was appreciated by Masaryk, but even though it was published also in German (see [Š12]), it remained without any substantial influence on the later development of the subjective interpretation of probability.

Philosophical interpretations, including the contributions of Masaryk and Šimerka were criticised by Karel Vorovka (1879–1929), to whom the fourth chapter is dedicated. Vorovka was mainly interested in philosophy and the philosophical problems of mathematics. With respect to the topic of this book are especially interesting his papers *Philosophical Reach of Probability Theory* [V5] and *On Probability of Causes* [V7], where he criticised the efforts of basing the theory of logical induction on probability theory, challenged the caution when using probability theory in real situations and insisted that it cannot solve the problem of causality; he stressed that the concept of cause and effect should be replaced by the concept of correlation. Vorovka also clarified the most substantial problem of logical interpretation, namely, the determination of prior probabilities in Bayes' formula for the probability of certain hypothesis, conditioned by available evidence. Unlike Masaryk, Vorovka claims that Hume's objections are justified and they cannot be disproved by probability theory. He insists that probability calculus and Hume's scepticism belong to completely different intellectual areas and it is not possible to bring them into a rational relation. He compares the application of probability calculus to Hume's scepticism as to cutting an atom with a knife and the introduction of Hume's scepticism into probability calculus as to sharpening the atoms in the knife.

A separate chapter is devoted to geometric probability which played an important role for extending the classical definition and introducing the concept of a set and measure to probability theory. Recall that when we pass from the investigation of properties of finite populations to the geometric properties (e.g., volume, area, length, shape) of geometrically describable objects (e.g., human beings and their organs and blood vessels, animals, plants, cells in a tissue, rivers, rocks etc.), replacing random population samples by probes of a lower dimension (sections or microscope images, linear or point probes), and instead of "classical" probability we base our inferences on geometric probability, then we pass from statistics to the domain of stereology. Since we are not only surrounded, but even formed, by such structures, stereology and hence also geometric probability are of great importance to our lives and represent a substantial tool for exploring and understanding the world around us. The text calls to mind the origins of geometric probability and then the contributions of E. Czuber, B. Hostinský and J. Baťa are discussed in more detail.

The personality of Emanuel Czuber (1851–1925) is the subject of the next chapter. He discussed the problem of the foundations of probability theory and other philosophical questions in various works – e.g., in a monograph on geometric probability [C23], in a textbook of probability theory and statistics [C37], treatises [C32], [C34], [C38], [C41] and [C42] and in the monograph *Die philosophischen Grundlagen der Wahrscheinlichkeitsrechnung* [C51], solely devoted to the philosophical foundations of probability theory. He admitted various approaches as meaningful and profitable in different situations. For repeated events, he considered the frequency approach useful; for unique unrepeatable events, he preferred the logical theory. He also emphasised the great importance of probability theory for epistemology and natural philosophy.

The final chapter is devoted to Otomar Pankraz (1903–1976), who published two papers ([P28] and [P31] with the supplement [P32]) dealing with the foundations of probability theory. Here he discussed Kolmogorov's axioms and argued that the fundamental concept of probability theory should be rather a conditional probability. He provided its axiomatic definition and studied its properties and various interpretations, including frequency and logical interpretations. These treatises remained practically unknown up to the present time; independently of them, similar axiomatic definitions were proposed in the 1950's by A. Rényi and K. Popper (see [300]–[304] and [291]–[292]).