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LIFE AND WORK OF PROFESSOR VOJTĚCH JARNÍK (22. 12. 1897 – 22. 9. 1970)

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On March 16, 1998 the Czech mathematical community commemorated the centenary of the birthday of Prof. RNDr. Vojtěch Jarník. The commemorative meeting was organized by the Faculty of Mathematics and Physics of Charles University together with the Union of Czech Mathematicians and Physicists and the Mathematical Institute of Academy of Sciences of the Czech Republic. A welcoming address was delivered by the Dean of the Faculty Prof. Bedřich Sedlák, further speakers being Professors Ivan Netuka (life and work of Vojtěch Jarník), Jaroslav Kurzweil (diophantine approximations), Břetislav Novák (analytical number theory, geometry of numbers), Jaroslav Nešetřil (graph theory), Luděk Zajíček (theory of real functions) and Jiří Veselý, CSc. (educational activities). In the conclusion several participants presented their recollections of V. Jarník.

Prof. V. Jarník influenced by both his scientific work and his teacher's activities several generations of Czech and Slovak mathematicians. He was probably the first Czechoslovak mathematician whose scientific works received wide and lasting international response and have been cited till now. The personality of Prof. Jarník represents a link between the classical and the modern mathematics. He was a real expert in traditional fields of mathematical analysis and simultaneously one of the first Czechoslovak mathematicians to master the set theory, topology, theory of measure and integral. He was known as a lecturer *par excellence* and was without doubt one of the best teachers at Charles University in the twentieth century.

Vojtěch Jarník was born on December 22, 1897 in the family of Jan Urban Jarník, Charles University professor of Romanic philology. He studied mathematics and physics in the years 1915–1919 at Faculty of Humanities (Philosophical Faculty) of Charles University. The degree RNDr. he received at the then newly founded Faculty of Science of Charles University in 1921 on the basis of his thesis "On the roots of Bessel functions". During his studies as well as later on V. Jarník was influenced by his teacher Karel Petr, Professor of Charles University during

1909–1938. In 1919–1921 Jarník was Assistant Professor at Technical University in Brno. Since 1921 he was continuously, with the exception of the forced interruption caused by the closing of Czech institutions of higher education by the Nazi occupants, member of staff of Charles University. His extraordinary mathematical erudition was first demonstrated already in his paper On Bolzano's function (1922) in which he examined in detail the just discovered manuscript of Bernard Bolzano from the thirties of the 19th century. V. Jarník proved among other that Bolzano's function is in fact the oldest example of a continuous nowhere differentiable function. In the years 1923–1925 and 1927–1929 Jarník paid two long-term visits to Göttingen, then the most important centre of European mathematical life. During the twenties this was the place of work for Professors D. Hilbert, R. Courant, E. Landau, C. Runge, G. Herglotz, P. Bernays, E. Noether and O. Neugebauer. Jarník was apparently influenced most by Edmund Landau, an outstanding specialist in mathematical analysis and number theory. After his return from the first visit Jarník defended in 1925 his habilitation thesis (devoted to lattice points). The first course lectured by V. Jarník as Associated Professor (Dozent) after his habilitation concerned the Lebesgue integral (the lecture notes of the course have unfortunately not been found). In 1929 V. Jarník was appointed Professor Extraordinary of Charles University, in 1934 he becomes member extraordinary (and in 1946 regular member) of Czech Academy of Sciences and Arts and in 1935 is appointed full Professor of Charles University. (At that time professors were K. Petr and B. Bydžovský, M. Kössler and E. Schönbaum, those appointed later but before 1950 were V. Hlavatý, V. Kořínek and E. Čech. Besides, F. J. Studnička, E. Weyr, J. Sobotka and V. Láska were professors at Charles University after 1900 and before 1935.)

In 1931 V. Jarník published in Petr's book Integral Calculus as an appendix the very first Czech text on (naive) set theory. Of great importance is Jarník's assessment of the work Funktionenlehre which was discovered in Bolzano's inheritance and published with a hundred-year delay. In Čech's book Point Sets an appendix "On derived numbers of functions of one variable" appears, in which V. Jarník summarized his then new results on differentiability of typical continuous functions. In the late thirties Jarník started publishing his books on differential and integral calculus. These textbooks of monographical character influenced generations of Czech mathematicians. The four-volume work was concluded in 1955 by publishing Integral Calculus II.

V. Jarník was member of the Union of Czechoslovak Mathematicians and Physicists from his student's years, several decades he was actively working in its Central Committee. In the years 1935–1950 he was Chief Editor of the mathematical part of Journal for Cultivation of Mathematics and Physics, and he succeeded

in raising the journal to international level. For extraordinary scientific results V. Jarník was awarded State Prize in 1952.

Professor Jarník held numerous offices at Charles University. In 1947–1948 he was Dean of the Faculty of Sciences of Charles University, 1948–1949 Vice-Dean of the same Faculty and 1950–1953 Vice-Rector of Charles University. In 1952 he was among the founding members of Czechoslovak Academy of Sciences, 1952–1955 chairman of its Mathematical-Physical Section and 1964–1966 Chairman of the Scientific Board for Mathematics of the Academy. V. Jarník ended his active career at the University in 1968. He died on September 22, 1970.

In spite of ever lasting big load by various offices, Jarník engaged himself with extraordinary assiduity to his teaching activities. I personally was lucky enough to belong to the last class to which Jarník during the years 1962–1964 lectured an extensive course of mathematical analysis (successively 5, 4, 4 and 3 hours a week). V. Jarník further lectured to us courses on differential equations in the complex domain and on special functions; I also attended his lectures on probabilistic methods in number theory. All his lectures were perfectly thought out, scrupulously prepared, read with equanimity and pedagogical mastership. Attending Jarník's lectures has been an experience for the whole life, I still enjoy remembering them.

Jarník's view of young mathematicians' education is well described by the following paragraph from B. Novák's paper In Memoriam Prof. Vojtěch Jarník (Za prof. Vojtěchem Jarníkem (Czech), Pokroky Mat. Fyz. Astronom. 16(1971), 1–5):

"An outsider could get the impression that Prof. Jarník has few students of his, that he did not establish his own school as is usual with mathematicians of his stature. However, the fact is that practically all our living mathematicians can be considered directly or indirectly Jarník's students, and everybody who in Czechoslovakia is engaged in Mathematics, has been to a greater or smaller extent influenced by Jarník's personality, by his example. Prof. Jarník was aware of the fact that it was of much greater importance for our Mathematics to pass to a great number of young people the affection for Mathematics and give them firm fundaments both in the knowledge and the methods of scientific work than to educate a few narrow specialists in his field. In the same way, he preferred filling in the gaps in our mathematicakl literature to writing himself a ten or twenty research papers."

The departure of V. Jarník from active University life coincided with my start of the teacher's career at the Faculty, and therefore I will use the opinions of V. Knichal and Š. Schwarz from their paper Academician Vojtěch Jarník Sexagenarian (Akademik Vojtěch Jarník šedesátníkem (Czech), Časopis pěst. mat. 82(1957), 463–492) to describe his personal qualities:

"Academician Jarník is a modest man, sometimes too modest, who always and under any circumstances behaves with the utmost tact. Never and to nobody does he show his professional superiority. He is patient, sometimes even too patient. In situations when much younger and much less experienced colleagues lose their temper, Jarník keeps his head. In addition to high intelligence and capability to assess all matters in a wider context this requires also a piece of trained personal discipline and willpower which is not given to everybody. For these rare qualities, Jarník is not only respected and appreciated by all his friends but also really very popular. He has broad knowledge of culture, ardent relation to music and has been regular visitor of Prague concert halls for several decades. He plays well tennis, is a fair skier, loves Nature and hiking."

After many years Prof. Štefan Schwarz from Bratislava recalls his encounters with Prof. Jarník during his frequent visits in Prague in his paper Some recollections of Academician Vojtěch Jarník (Niekolko vzpomienok na akademika Vojtěcha Jarníka (Slovak), Pokroky Mat. Fyz. Astronom. 35(1990), 340–345):

"When it was only possible I never missed an opportunity to talk with him, to ask him for an opinion. It was always an enjoyable event for me. Even now, being myself well over seventy, I see in fromt of me a man with deep humane feeling, a man of clear character, such as I had rarely met in my life."

In the conclusion let us briefly mention the scientific papers of V. Jarník. Their list includes 90 items, almost a third of which is devoted to problems of lattice points, another third to diophantine approximations and geometry of numbers, about twenty papers concern the theory of real functions. During the twenties Jarník published 31 papers, in the subsequent decades 38, 12, 6 and 3, respectively.

Number theory, real functions as well as graphs are accounted for in more extensive papers in the present publication. Left aside have been three older and less known papers [11], [13], [14] (see the list of literature) concerning the rearranging of infinite series. I would like to say several words of the paper [14] for at least three reasons: the result is, apparently, not generally known, at the same time being elegant, interesting, "Jarník-wise" perfect and definitive. Secondly, the problems of series and their rearranging have become lately a very actively developing part of functional analysis; see, e.g., M. I. Kadets, V. M. Kadets: Series in Banach spaces. Conditional and unconditional convergence, Birkhäuser, Basel, 1997. And finally, it is the rearranging (for Cesaro's summing method) to which the recent research has led my PhD. student Roman Lavička who has studied the properties of the Laplace operator on a Hilbert space.

It is generally known that a nonabsolutely converging series can be rearranged to an arbitrary sum (B. Riemann, 1867). In other words: for series with real summands the set of all sums of the rearranged series is either empty, or a singleton,

or the whole real axis. For the series with complex summands, more generally for series of elements from a finitedimensional space, the analogue of Riemann's theorem reads as follows:

Theorem (P. Levy 1905, E. Steinitz 1913). *Let $v_1, v_2, \dots$ be elements from $\mathbb{R}^k$ and let S be the set of all sums of convergent rearrangements of the sum $\sum_{n=1}^{\infty} v_n$. Then either $S = \emptyset$ or there exist $w \in \mathbb{R}^k$ and a linear subspace $L \subset \mathbb{R}^k$ such that $S = w + L$. In particular, for a series with complex summands the set S is either empty or a singleton or a line in $\mathbb{C}$ or the whole of $\mathbb{C}$.*

The proof of the Levy-Steinitz theorem is not so easy as could be judged at first sight from its formulation. This is nicely expressed by P. Rosenthal in his paper The remarkable theorem of Levy and Steinitz (Amer. Math. Monthly 94(1987), 342–351):

"I was told of the Levy-Steinitz Theorem by Israel Halperin. The first few times that he started to explain the proof to me, I didn't listen; I assumed that I could prove the theorem in some easier way. Finally, after I realized I couldn't prove it, I let him describe the proof. The exposition that follows is mainly based on these private lectures, for which I am extremely grateful."

The contribution of V. Jarník consists in including also divergent rearrangements of series of complex numbers in his analysis. To be able to formulate his result, let us introduce the following notation:

Let $x = \{x_n\}_{n=1}^{\infty}$ and $y = \{y_n\}_{n=1}^{\infty}$ be sequences of complex numbers. We will write $y \sim x$ if there is a permutation π of positive integers such that $y_n = x_{\pi(n)}$, $n \in \mathbb{N}$. Let us denote by $H(x)$ the set of all accumulation points of the sequence of partial sums of the series $\sum_{n=1}^{\infty} x_n$. Let us set $J(x) = U\{H(y);\ y \sim x\}$.

Theorem (V. Jarník). *Let x be a bounded sequence of complex numbers. Then the following cases can occur for $J(x)$:*

- *the empty set,*
- *a singleton,*
- *a line,*
- *an equidistant system of parallel lines,*
- *an arithmetic sequence,*
- *lattice points,*
- $\mathbb{C}$.

V. Jarník further proved that there exists $y \sim x$ such that $J(x) = H(y)$. Let us note that the paper [11] concerns series of real numbers and its result represents a

variant of Riemann's theorem. The paper [13] contains a characterization of subsets M of the closed Gauss plane for which there exist $y \sim x$ satisfying $H(y) = M$ provided x is a sequence converging to zero.

The commemorative meeting has again demonstrated how essentially Jarník's contribution influenced mathematical analysis, number theory as well as some other branches of Mathematics. It has also shown the depth of roots of Jarník's legacy as a teacher, which is present under the name of Jarník style even thirty years after Jarník's retirement from active teaching.