The growth of mathematical culture in the Lvov area in the autonomy period (1870–1920)

Gymnasia

In: Stanislaw Domoradzki (author): The growth of mathematical culture in the Lvov area in the autonomy period (1870–1920). (English). Praha: Matfyzpress, 2011. pp. 17–79.

Persistent URL: http://dml.cz/dmlcz/402155

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CHAPTER I

GYMNASIA

1.1. Introduction

Reorganization of the state of Habsburgs, which took place during 1860–1867, embraced the school system. The Constitution of 1867 and acts following it from 1868 to 1873 had an impact on public education. In 1860 the Ministry of Religion and Enlightenment ceased to exist, some of the powers were taken over by the Ministry of State, and the initiative to change the education system was passed to the national seyms and the parliament in Vienna. Parliaments of Upper and Lower Austria, Moravia and Carinthia fought for the nationalization of the educational system, the Parliament of Galicia fought for something more: to ensure that the society used to influence education and to pattern the school system after the tradition of the Commision of National Education.[12,13]

In the years 1867–1914 the number of Galician gymnasia increased from 19 to 130 – almost seven times. Also the number of teachers increased from about 309 to 2045 in state schools and to about 1000 teachers in private schools.

The number of the staff in some state gymnasia changed depending on the size and resilience of the specific environment. The pedagogical set of every higher gymnasium (with classes from I to VIII) consisted of one headmaster and from 10 to as many as 40 teachers, depending on the number of students. The teachers' staff included also catechists of all denominations, as the religion was a mandatory subject. By the end of the 19th century, St. Anna's gymnasium in Cracow had the biggest staff – today named I Comprehensive Lyceum named after B. Nowodworski; in the second place there was the IV Gymnasium in Lvov.

1.2. Gymnasia in the memories of Hugo Steinhaus and Franciszek Leja

Children's freedom ended when they had to go to school. Then, the headmaster of the school was Klemens Sienkiewicz, a jovial Ruthenian with gray hair, who liked Pilsner and resembled a landowner, rather than a teacher. [...] *I was nine years old. In the First class there was Latin and German. Studying*

[12] In Polish Komisja Edukacji Narodowej

[13] R. Dutkowa, *Oświata Polityka szkolna w Galicji 1866–1890* [Education and School Policy in Galicia 1866–1890], in: A. Meissner, J. Wyrozumski (eds.), *Galicja i jej dziedzictwo* [Galicia and its heritage], volume 3*, Nauka i Oświata* [Science and education], WSP Rzeszów, 1995, pp. 137–149.

was treated seriously, and history gave me considerable difficulties, because I did not know how to learn by heart, and I had not seen a better way. At least one third of my colleagues were of peasant homes (another proof for the wealth of the county), the rest were recruited from the sons of lower railway men, postmen, tenants, craftsmen, townspeople from Kołaczyce, Dembowiec, Pilzno and Krosno. They were also the sons of Jewish merchants and the Ruthenian, the sons of priests, and only a small percentage came from the so-called intelligentsia. Some people – especially young professors – saw fit to ignore students ex cathedra. Those teachers lost the whole authority in my eyes.[14] In this way Hugo Dionizy Steinhaus, a Ph.D. of Göttingen University, recalls the beginnings of learning in a gymnasium. (Steinhaus also had *veniam legendi* of c.k. Lvov University in the era of autonomy, a professor of Polish University of Jan Kazimierz in Lvov, after World War II Professor of the University and the Polytechnics of Wrocław.)

In the final class in gymnasium in Jasło, where Steinhaus was a schoolboy, the following manuals were used:[15]

— P. Dziwiński, *Zasady algebry dla wyższych klas gimnazyów i szkół realnych* [Rules of algebra for higher classes of gymnasia and real schools], 1st ed., Lvov, 1891, 384 pages, 2nd ed., 1898, Lvov, 3rd ed. Lvov, 1907, approved by the order of School National Committee in 1906, 448 pages,
— F. Močnik, G. Maryniak, *Geometrya dla szkół średnich*, cz. 2 [*Geometry for secondary schools*], part 2, 5th ed., Lvov 1903; part 3 and 4, 5th ed., Lvov, 1903, 6th ed., Lvov 1906, 328 pages,
— I. Kranz, *Zbiór zadań matematycznych. Podręcznik dla wyższych klas szkól średnich, zastosowany do instrukcyi ministeryalnych* [Collection of mathematical tasks. The manual for higher classes of secondary schools, followed to ministerial instructions], Cracow 1902, 2nd ed., 1905, pages 176,
— I. Kranz, *Tablice pięciocyfrowe logarytmów liczbowych i funkcyi trygonometrycznych do użytku szkolnego* [Tables of five-digit numerical logarithms and trigonometric functions for school use], 1st ed., Cracow 1900, 126 pages.

[14] H. Steinhaus, *Wspomnienia i zapiski*, publ. Aneks, Londyn, 1992, p. 16.

[15] Information about some authors:

Grzegorz Maryniak (1853–1896) finished gymnasium in Sambor. He studied at the Lvov University, and passed the examination for teaching mathematics and physics there in 1880. He translated and adapted for F. Močnik's textbooks on geometry for secondary schools, which were widely used at secondary schools in Galicia till 1906.

Placyd Zasław Dziwiński (1851–1936) – for more information about him see Chapter III and others.

Ignacy Kranz (1854–1924) finished gymnasium in Rzeszów in 1874. Then he studied mathematics and physics at the Jagellonian University. In two stages (1879 and 1880) he passed examination for teacher of mathematics and physics. He was a teacher at gymnasium of St. Anna in Cracow. He elaborated some textbooks for secondary schools, a collection of tasks and mathematical tables, some of them were renewed many times till 1930.

Below we put final school exam exercises which were solved by H. Steinhaus when graduating from school (according to the school Report of Headmasters i. c. Gymnasium in Jasło in 1905).

6. *Matematyczny:* 1) Rozwiązać równanie:

$$\log\left(10 + 5\sqrt[3]{x} - 4\sqrt[4]{x}\right) = 2{\cdot}80277.$$

2) Krawędzie prostopadłościanu tworzą postęp geometryczny, objętość tego prostopadłościanu wynosi 1000 cm ^{3} a powierzchnia 700 cm ^{2}. Jak wielkie są krawędzie?

3) Pierwiastki równania: $2 \operatorname{tg} x + 3 \operatorname{cotg} x = 5$ wyznaczają szerokości geograficzne dwu miejsc na ziemi. Obliczyć powierzchnię pasa sferycznego między równoleżnikami tych dwu miejsc ($R = 6730$ km).

Solving these exercises demanded knowledge of logarithmic equations, metrical combinations in the cuboids, trigonometric equations and regular relations in the sphere. They demanded from students consistency in thinking, beautifully uniting two worlds – algebraic and geometrical, and highlighted the application of mathematics.

Students faced rather high requirements. The best evidence is the results of maturity examination (according to the school Report of Headmasters c.k. Gymnasium in Jasło in 1905).

Wynik egzaminu dojrzałości.

Do egzaminu ustnego zgłosiło się:	
a) uczniów publicznych	72
b) eksternistów	11
razem	83
Z tych uznano za:	
a) dojrzałych z odznaczeniem	5
b) dojrzałych	43
Pozwolono po wakacyach powtórzyć egzamin z jednego przedmiotu uczniom	19
Reprobowano na rok:	
a) uczniów publicznych	9
b) eksternistów	6
Od egzaminu odstąpił	1
razem	83

Steinhaus graduated from gymnasium in Jasło with high school diploma in 1905. The following information, taken from the report for the 1904–1905 school year, shows that Steinhaus received the certificate of maturity with distinction.

KLASA VIIIa.

Stopień pierwszy otrzymali:

1. **Steinhaus Hugon** (z odzn.) 3. Bock Eugeniusz
2. **Węgrzyński Jan** (z odzn.) 4. Boczar Jan

Świadectwo dojrzałości otrzymali:

W VIII. a: Bock Eugeniusz, Dudek Henryk, Gajewski Jan, Gonet Józef, Grzyb Wincenty, Jeleń Franciszek, Kmicikiewicz Roman, Knebel Maryan, Kobryn Alexander, Kopyściański Stefan, Kucharski Karol, Marczak Henryk, Maritczak Alexander, Matuszewski Paweł, Sanakowski Marcin (eksternista), Skwara Paweł, Sołek Michał, Steinhaus Hugon (z odznaczeniem), Wawrzkowicz Stanisław, Weinstein Mojżesz, Weis Ignacy, Węgrzyński Jan (z odznaczeniem), Zossel Ignacy, Głód Ludwik.

Among 83 people taking an examination together with Steinhaus, 48 people passed in the first term. After retaking exams finally 16 students failed.

As Steinhaus mentions further: *first of all I was dressed in a formal uniform, in accordance with that famous order of c.k. order of School National Committee, which ruled that the blouse has to be navy blue, trousers grey, while the soul of pupil is pure and immaculate. The other matter is that only an insignificant fraction of gymnasia boys complied with these regulations; especially grey trousers were not popular and they were made of the same material as blouse.*

Franciszek Leja (1885–1979), professor of the Jagiellonian University remembers gymnasia like this:[16]

In eight-year Galician gymnasia in the Austrian annexation, in the years 1896-1904, just before the First World War humanities reigned all-powerfully. The Polish literature, especially the three great poets, and Latin, Greek and German literature dominated over other subjects. Subjects like mathematics or physics, especially in provincial gymnasia, i.e. outside Lvov and Cracow, were rather merely tolerated; they were generally believed to be of little use.

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[16] *Dawniej było inaczej* [It was different in the old days], memoirs, manuscript deposited in Institute of Mathematics, the Jagiellonian University.

Such an opinion may be due to professor Leja character traits: *I was rather introverted and taciturn, which was not conducive to the development of literary talents.*

Interestingly, in another place in his memoirs we read: *In 1900 I passed to the 5th class. There was a new professor of mathematics who was an excellent lecturer and from whom I learned a lot by solving of the so-called construction exercises. Today unfortunately I do not remember the name of this professor; he taught us for only a year. Studying in higher gymnasia classes went well, especially the learning of mathematics. Being in the 6th class I lived with two 8th class schoolboys, of which one was distinguished in his own class as the extremely talented mathematician and at home he often bombarded me with exercises to solve, and added: „I am sure that you will not solve it", or: „I bet my life that you will not work out this task". Such things fuelled my ambition and this way in the 6th class I became acquainted with mathematics of upper classes, which influenced the improvement of my financial condition, because in upper classes I gave some paid private mathematics lessons.*

Professor Leja's opinion on teaching mathematics in provincial gymnasia needs to be seen in a wider social context. Of course changes of teachers were frequent, attitude to mathematics was rather negative, especially in families of important leading opinion, in other words, in families of rich landowners and townspeople.

Striving for independence, as F. Leja notices in his memoirs, was present in every Galician gymnasium. *Being in the 6th class I was accepted to the secret pupil organization, forbidden by school authorities, and which existed surely in upper classes of all colleges of Galicia. The aim of this organization was to familiarize the young with the true history of parted Poland and to organize celebrations of the anniversaries of important events of our history, as, for example, the Constitution of May 3rd .*

Poles, Jews and Ruthenians existed together in the gymnasium. As F. Leja mentions further *in lower classes there were no national differences between us. They appeared when we were in the 6th class, and it was mostly between Poles and Ruthenians. The latter began to be called Ukrainians.* […] *This enmity appeared mostly by singing the new Ukrainian song starting with the words: „It's no time, it's no time to serve to Poles (old name – Lachy) …"*

The development of school system and education influenced greatly the economic, social, political and cultural position of people of Lvov and Galicia.

1.3. Programs of teaching[17]

Teaching curricula were stipulated by the School Council in Vienna. In Lvov there was a national school council, which was formally subject to the government, but in fact it was autonomous in supervising education within the framework of laws. The council dealt with education independently of universities, which were subject to Department of Education in Vienna. The council consisted of two representatives of the clergy, appointed by the Emperor and elected members, namely: one member of the National Department, one deputy from municipal council of Lvov and another one from Cracow; and two scientists appointed by the Emperor, elected upon National Department's application. Members of the School Councils were lieutenancy councilors and government supervisors. The latter had only 3 votes in the Council. Thus self-government factors had more power than government factors. The School Councils' staff was changed in 1905 for the benefit of social factors.

The bases for secondary educational system were eight-class gymnasia, whose role in propagating education, culture and patriotism was immense. They consisted of classical gymnasia where literature and ancient languages dominated. During the eight years, lessons of Latin took place 5–6 hours every week, and lessons of Greek 4 hours every week. Since 1867, Polish was also taught and successively it was becoming the language in which the classes were conducted. History, geography, mathematics, sciences, religion and propaedeutic of philosophy were also taught. Education in gymnasium ended with the final school examination. Completing a classical gymnasium made it possible to enter the university without examinations. The second type of secondary school was the real gymnasium, where considerable attention was paid to mathematics and natural science subjects. The real (secondary technical) educational system had two stages. Lower schools, three forms, with time were transformed into vocational schools with the agricultural, forest, industrial and commercial courses. Higher schools of six forms, and afterwards seven forms, allowed to enter only technical colleges. The real gymnasium was related to social needs and focused on the trade training. In 1871 the real gymnasia were transformed into the six-year real schools. At the beginning of the 20th century discussions on educational programs and on the organizational shape of the educational system escalated. Main goals of upbringing were defined. From the end of the 19th century pedagogy has been identified as a scientific discipline. First there were herbartists (the followers of Herbart, who developed the foundations of the scientific pedagogy), who

[17] See author's works, *Programy nauczania matematyki w sprawozdaniach szkolnych gimnazjów galicyjskich* [Teaching mathematical programs in annual reports of gymnasiums in Galicia], Antiquitates Mathematica 3(2009), pp. 223–241.

in Galicia followed the direction based on ethics, which defined the aims of activities, and on psychology which qualified resources of their realization. Galicia understandably remained under the influence of German science. In spite of the autonomy, Austrian and German school system was imposed on the Galician school in a "natural" manner. At secondary school the classical philology and the Herbart pedagogy reigned supreme. From the end of the 19th century a teachers' movement also was developed in Galicia. The first teacher's organization was the Pedagogical Society. In turn, the Society of Teachers of Colleges, the National Association of People's Teachers, the Society of the People's School came into being together with pedagogical periodicals (among others Szkoła [the School], Muzeum [the Museum], Przegląd Pedagogiczny [the Pedagogical Movement]), where the discussions concerning methods and programs of teaching were held.

It seems proper to notice the unusual activity of Józef Puzyna (1856–1919), professor of the Lvov University and the precursor of the Lvov Mathematical School. He could notice, value, improve, show strong points and encourage further work, as it can be seen in the example of a short thesis about determinants by J. Korczyński. Also the role of Stanisław Zaremba (1863–1942), professor by the Jagellonian University and the founder of the Cracow Mathematical School, who worked hard to improve the methods of teaching in secondary schools is not duly valued and recognized. Zaremba was the author of the book *Zarys pierwszych zasad teorii liczb całkowitych* [The Outline of first rules of the theory of whole numbers] (1907) dedicated to future teachers of mathematics.

It is worth putting special emphasis and the didactic reflection to Chapter XII of the manual entitled: *The opinion on guilds of the mathematical accuracy. Combined difficulties with study and the cognition of mathematical theories. Advice of the pedagogical nature.* Considering these two university professors from Lvov and Cracow, it highlights the contemporary role of a professor who performs scientific research, sets schedules of research, educates students and future research workers, actively participates in committees responsible for teachers, supports teachers in their scientific aspirations, is interested in school programs, takes active participation in the organization of the school life, publishes for students and teachers, reviews even minor works, (which certainly was important for their authors) takes part in scientific conventions, international congresses etc. The two professors beautifully supported teachers and headmasters, explained to parents and school students the necessity to learn mathematics, its importance in formation of the young person, the need of its use.

In the Appendices the list of publications related to an instruction of mathematics in the periodical Muzeum [Museum] is given. Some of the mentioned publications were written by chief mathematicians from Cracow, Lvov and Warsaw.

1.3.1. Teaching programs of mathematics in classical gymnasium

From the School Reports (see Bibliography) we extract teaching programs of mathematics, topics of final school examinations (see Appendices) and other matter related to mathematics. On the ground of the management reports of chosen gymnasia from Lvov, we will pay a special attention to changes taking place in teaching programs or school-leaving tasks.

Gymnasia reports are documents showing the activity of Galician gymnasia. Earlier these were class Cards, which included the list of professors and school students and marks obtained by them in each subject. Later gymnasia began to publish the school Programs. They appeared in the whole Austrian-Hungarian Monarchy and in all German countries. After 1871, they were published under changed name of school Reports by all secondary schools, and also lower gymnasia, real schools and teachers colleges. Every report consisted of dissertations and school news. Analyses of the material content of reports cleary shows that they are an important source of information which permits to qualify the vision of a Galician gymnasium.

One of the part included, among other things, papers, also those concerning mathematics.

The editors of Reports were school managers. The official part of reports contained school news worked out by the manager, teachers and pupils. There was information about: the teachers' staff, programs, numbers of hours of each subject, topics of written compositions, the list of textbooks, school-leaving tasks, lists of school-assistances, school activities in the range of the physical development of young people or the help given to poor schoolchildren[18].

We will consider teaching programs of mathematics on the example of one gymnasium in Lvov: c.k. the Francis Joseph I classical gymnasium. The institution was founded in 1850 as parallel forms I–IV with the Polish language of instruction at the II Gymnasium. From 1857 the change of name took place: it was the III gymnasium named after the emperor Franz Josef I with the Polish language of instruction, as an expression of gratefulness for the recognition of the rights of Polish people; this happened in the period of introducing rules of the autonomy. In 1892 there appeared a branch which gave the beginning for the V gymnasium.

After 1919, in free Poland the name changed to the III Lvov State Gymnasium named after King Stefan Batory.

Mathematics was taught in the following quantity:

Years of studying	Form								Together
	I	II	III	IV	V	VI	VII	VIII	
1876–1904	3	3	3	3	4	3	3	2	24
1905–1908	3	3	3	3	4	3	2	2	23

[18] A. Meissner, *Galicja i jej dziedzictwo* [Galicia nad its heritage], volume 3, Nauka i oświata [Science and education], WSP Rzeszów 1995.

We will show in details the mathematical contents from 1876–1878.

FORM I
3 hours every week. In the first term only **arithmetic**, in the second – 2 hours of geometry, 1 hour of **arithmetic**. Decimal system of numbers, four operations with integer and decimal numbers. Calculus with mixed numbers, divisibility of numbers. Geometrical figures. Angles, triangles.
Notes: Numerous exercises at home, in-school assignments fortnightly.

FORM II
3 hours every week. **Arithmetic**: mathematical operations with simple fractions, ratios, proportions, weight and coin measures; geometry: properties of triangles and polygons – calculation of the surface. Assignments as in form I.

FORM III
3 hours every week. A**rithmetic**: beginnings of algebra, calculus on letters, raising integer numbers and fractions to a power; the square and cubic extraction of a root; important rules on permutation and combination. **Geometry**: similitude of triangles; properties of circle.
Notes: frequent homework exercises; in-school assignments fortnightly.

FORM IV
3 hours every week. **Arithmetic**: ratios and proportions, simple and compound percentages; calculus of companies and mixtures, first-degree equations. Geometry: basic knowledge about solids – **solid geometry**.

FORM V
4 hours every week. **Algebra**: the numeral system; definitions of mathematical operations and quantities; four operations; the divisibility of numbers; simple and decimal fractions, continued fractions.
Geometry: planimetrics.
Notes: in-school assignments monthly.

FORM VI
3 hours every week. **Algebra**: ratios, proportions, powers, roots, logarithms. **Geometry**: solid geometry, from trigonometry: goniometry.
Notes: in-school assignments monthly.

FORM VII
3 hours every week. **Algebra**: The revision of logarithms, equations, combinations, binomial theorem. **Geometry**: the revision and the completion of trigonometry and analytics (analytic geometry).

FORM VIII
2 hours every week. The revision, arrangement and use of examples from the whole subject.
Notes: in-school assignments monthly.

The above presented programme was elementary. In 1880–1882 the change of the content in the teaching programmes of mathematics for several forms took place.

FORM I
3 hours a week. In the first term only the **arithmetic**, in the second the **arithmetic** and **geometry** alternately. Decimal system of numbers, four operations on integer and decimal numbers, calculus with mixed numbers. Geometrical figures. Angles, triangles.
Notes: Numerous exercises at home, in-school assignments fortnightly.
Programme for this form was not changed.

FORM II
3 hours every week. **The arithmetic**: mathematical operations with simple fractions, ratios, proportions, weight and coin measures. **Geometry**: properties of triangles and polygons – calculation of the surface, the equality and similitude of figures (polygons).
Notes: Frequent home exercises, in-school assignmentss every month.

FORM III
3 hours a week. **The arithmetic**: four operations with letters, raising integer numbers and fractions to a power; the square and cubic extraction of root; **Geometry**: Similitude of triangles and polygons; properties of circle.
Notes: Frequent homework exercises; in-school assignments every month.
The programme was completed with similitude of polygons, but contents of permutations and combinations were not included.

FORM IV
2 hours a week. **The arithmetic**: ratios and proportions, simple and complex per cents; calculus of the union (firm), the rule of the term, mixed calculus; equations of the first degree with one or two unknowns (diophantine).
There were fewer lessons, with no geometrical contents.

FORM V
4 hours a week. **Algebra**: the number system; definitions of different mathematical operations and quantities; four operations; the divisibility of numbers; simple and decimal fractions, continued substituted fractions.
Geometry: all planimetrics.
Notes: in-school assignments monthly.

Forms V–VIII belonged to the so-called higher gymnasium. In smaller towns there were lower gymnasia (I–IV). That is why geometrical contents appeared here only in the V form.

FORM VI
3 hours a week. **Algebra**: ratios, proportions, powers, roots, logarithms. **Geometry**: solid geometry, from trigonometry: goniometry.
Notes: in-school assignments monthly.

FORM VII
3 hours a week. **Algebra**: The revision of logarithms, equations, combinations, Newton's binomial theorem. **Geometry**: the revision and the completion of trigonometry and analytic geometry.
Combination was transferred from form III to VII.

FORM VIII
2 hours a week. The revision, arrangement and the use of examples from the whole material.

In the years 1883–1885 some changes occurred. In the form I the following conceptions were added: the divisibility of numbers, spatial quantities. In the form II systems of measures, weights and coins were deleted. But in form IV geometry returned, in the form VI the program contained information an irrational numbers.

FORM I
3 hours a week. In the first term only **arithmetic**, in the second **arithmetic** and **geometry** alternately. Decimal system of numbers, four operations integer and decimal numbers, calculus with mixed numbers. The divisibility of numbers. Concept of spatial quantities. Angles, triangles, congruence.
Notes: Numerous exercises at home, school assignments every month.
Relation of congruence, its elementary properties were realized in the form I.

FORM II
3 hours a week. **Arithmetic**: the divisibility of numbers, operations with common fractions, intercourse, proportions, the rule of three single. **Geometry**: properties of triangles and polygons: calculation of the surface of polygons, the transformation of polygons.
Notes: Frequent homework exercises. School assignments monthly.

Nowadays term "equivalence by distribution" was already realized in the II form of gymnasium. It can be noticed, especially at geometrical contents of trying to form mathematical culture through solving geometrical problems.

FORM III
3 hours a week. **Arithmetic**: four operations with general numbers, powers of integer numbers and fractions; the square and cubic extraction of a root. **Geometry**: similitude of triangles, polygons, circle.
Notes: Frequent homework exercises; school assignments monthly.

FORM IV
3 hours a week. **Arithmetic**: ratios and proportions, simple and complex per cents; calculus of company, the term rule, calculus of the mixture, first-degree equations of one and two unknowns. **Geometry**: solid geometry, interposition of lines and surfaces, the calculation of the surface and volume of solids.
Notes: School assignments monthly.
According to the previous program this one was completed with the chosen contents of and with measurable units of solids.

FORM V
4 hours a week. **Algebra**: the number system; conception of different mathematical operations and quantities; four operations; the divisibility of numbers; simple and decimal fractions, decimal continued fractions. **Geometry**: all planimetrics.
Attentions: School assignment monthly.

FORM VI
3 hours a week. **Algebra**: ratios, proportions and their use, powers, roots, irrational and pure numbers, logarithms. Geometry: solid geometry, from trigonometry – goniometry.
Notes: School assignments monthly

FORM VII
3 hours a week. **Algebra**: The revision of powers and logarithms, imaginary numbers, equations, arithmetic and geometric progressions, calculus component percentage, combinations, Newton's formula. Geometry: the revision and the completion of trigonometry and analytic geometry.
Notes: school assignments monthly.

FORM VIII
2 hours a week. The revision, arrangement and use of examples from the whole subject.

In 1886–1892 some other changes followed in programs for particular classes. The weekly number of hours did not change, while the program was extended and the following contents were added: second degree curves (ellipse, parabola and hyperbola).

FORM I
3 hours a week. **Arithmetic**: The decimal system of numbers, four operations with integer non-concrete and concrete numbers and mixed numbers. The divisibility of numbers. Common fractions, decimal fractions. **Geometry**: initial conceptions: lines, angles and triangles including congruence.
Notes: Homework exercises from lesson to lesson. School assigments monthly. Necessity of regularity in teaching of mathematics was noticed.

FORM II
3 hours a week. **Arithmetic**: The shortened multiplication and division, proportions, the single rule.
Geometry: The congruence of triangles with applications. Important properties of the circle and polygons.
Notes: Frequent homework exercises. School assigments monthly.

FORM III
3 hours a week. **Arithmetic**: Four operations with algebraic expressions, powers of integer numbers and fractions, the square cubic extraction of a root.
Geometry: The calculation of plane figures surface, similitude of triangles and polygons; ellipse, parabola, hyperbola.
Notes: Homework exercises from lesson to lesson. School assigments monthly.

FORM IV
3 hours a week. The arithmetic: Equations of first-degree with one and two unknowns, the rule of three, composition, the chain rule, compound percentages.
Geometry: Solid geometry, the mutual position of line and surfaces. Calculation of surface and volumes of solids.
Notes: Frequent homework exercises; school assignments monthly.

FORM V
4 hours a week. **Arithmetic**: First four operations with algebraic numbers, negative numbers, fractions, numerical systems, proportions and first-degree equations with one and more unknowns. Geometry: Planimetrics.
Notes: Homework exercises; school assigments monthly.

FORM VI
3 hours a week. **Algebra**: Powers, roots, irrational and imaginary numbers, logarithms. Equations of the second degree. **Geometry**: Solid geometry and trigonometry.
Notes: Homework exercises; school assigments monthly.

FORM VII
3 hours a week. **Algebra**: The repetition of powers and logarithms, imaginary numbers, equations, arithmetical and geometrical progressions, calculus of component percentage, combinations, Newton's formula. **Geometry**: the Revision and the completion of trigonometry and analytic geometry.
Notes: Homework exercises; school assignments monthly.

FORM VIII
2 hours a week. The revision, arrangement and use of examples of all subjects.

In the years 1893–1907 further changes in teaching program of mathematics occurred before 1918. The weekly number of hours did not change, only in 1905 in the form VII teaching of mathematics was reduced by an hour every week. The program was greatly extended.

FORM I
3 hours a week. In the first term only **Arithmetic**: The decimal system of numbers; Roman numbers; four operations with non-concrete and concrete, integer and decimal, one and multi-kind numbers, the divisibility of numbers, simple fractions, the least multiplicity and the biggest measure. In the second term **geometry**: initial conceptions, study about lines, angles and triangles.
Notes: Short homework exercises, and school assigments monthly.

FORM II
3 hours a week. **Arithmetic**: the supplement of study about multiplicities and measure, the exact study about common fractions, change of decimal fractions into ordinary ones and vice versa; proportions, the rule of single three with the use of proportions and deduction; calculus of simple percentage. **Geometry**: perpendicular to a segment and angle; congruence of triangles with applications; important properties of circle and polygons. Exercises as in form I.

FORM III
3 hours a week. **Arithmetic**: shortened multiplication and division; four operations with general numbers; rising integer numbers and fractions to the second power, the extraction of a root; incomplete numbers. **Geometry**: The transformation and the division of figures; the calculation of the surface of plane figures; Pythagoras theorem; similitude of triangles and polygons.
Notes: School and homework exercises as in form I.

FORM IV
3 hours a week. **Arithmetic**: The rising to cube and the cubic extraction of a root: equations of first-degree with one and several unknowns, equations of the

second and the third degree with applications to geometry; the complex rule of three. **Geometry**: solid geometry, the mutual position of the line and surfaces, the calculation of the surface and volumes of blocks with the exclusion of the truncated cone.
Notes: School and homework exercises as in form I.

FORM V
4 hours a week. **Arithmetic**: First four operations on algebraic numbers. Foundations of the most important theorems about the divisibility of numbers. The greatest common divisor and least common multiplier of numbers and some polynomials. Common and decimal fractions. Ratios and proportions and their use. Determined equations of first-degree with one and several unknowns. **Geometry**: basic formations: theory of parallels; properties of triangle and cases stopping. Theorems about quadrangles and the polygon, about angles and chords in circle – interior and circumscribed circles. Proportionality of sections and similitude of figures. The equality of the areas and their calculation.
Attentions: Short homework exercises. Three school assignments in each term.

FORM VI
3 hours a week. **Algebra**: powers, roots. Conception of irrational and mutual unit. Logarithms. Equations of II degree with one unknown. **Geometry**. From the solid geometry the major theorems about straight lines and surfaces in the space; about the angle. The partition and more general properties of solids. The surface and the volume of prisms, full and truncate pyramids; the cylinder, the cone, the truncated cone and ball. Goniometrical functions, solution of right triangles; the simplest goniometrical equations.
Notes: Short homework exercises. Three school assignments in a term.
One should notice that for the first time the word "function" appeared in the teaching programmes, it was the beginning of the XX century.

FORM VII
2 hours a week. **Algebra**. Equations of higher degrees with one unknown which can be solved by means of equations of the second degree. Simpler examples of equations of 2 degree with two unknowns. Arithmetical and geometrical progressions. Calculus of the compound interest and calculus of pensions. Foundations of combinations. Newton's formula for entire, positive exponents. **Geometry**: from trigonometry the solving of oblique triangles and some using. Beginnings of the analytics geometry in plane. Equation of straight line, circles and cone-shaped cuts.
Notes: Exercises and tasks as in form V.

FORM VIII
2 hours a week. The revision, arrangement and use of examples from the whole subject.

In order to make the issues presented above more familiar (in the printed programs there were just headlines), we will present the content of the textbook *Zasady Algebry dla Wyższych Gimnazyów i Szkół Realnych* [The Rule of Algebra for higher Classes of Gymnasia and Real Schools] by P. Dziwiński (1898).

The textbook contains 351 pages and includes introduction, 12 chapters and conclusion.

In the introduction the author explains basic concepts: calculating, particular number, general number, axiom and theorem. The author notes that any letter can stand for a certain number of units and calls it the total number. Today we would say we mean any fixed integer.

In chapter I *Główne działania proste* [Main simple operations] he describes. the addition, the multiplication and raising to a power. Formulas for the square and cube the sum of two terms were introduced, as well as multi-term square of the sum.

For example, the square of the four-term sum is written in a symbolic way: $(a+b+c+d)^2 = \sum(a^2) + \sum(ab)$, where Σ (read sum) points out, that from the numbers *a, b, c, d,* should be created the sum of all terms similar to the one which is in the brackets after Σ. By "similar terms" the author understands the terms constructed according to the given rule.

In the II chapter *Główne działania odwrotne* [Main inverse operations] there were discussed the subtraction, division and square roots. Simultaneously with the actions in the set of real numbers, the actions on polynomials were discussed.

Chapter III shows *Stosunki i proporcje* [Ratios and proportions]. The author introducted proportions and their different kinds. Similarly, as in the most algebra textbooks from that period, the following concepts were carefully discussed: the rule of three straight lines, the rule of three compound, the chain- rule, the rule of the company and the compound rule of the company.

This information was especially useful for school students who after the graduation started work in the trade, banking or exchange offices. In contemporary programs these topics are not included.

In the following chapter IV *Układy liczb* [Number systems] the author describes a decimal system and a method of conversion of numbers written in decimal system to another system, and vice versa.

In the chapter V *Równania stopnia pierwszego z jedną, i wieloma niewiadomymi. Równania n-tego stopnia z jedną niewiadomą* [Equations of the first degree with one, and many unknowns]. The equations of n-th degree with one unknown, there were discussed methods for solving the two equations of 1st degree with two unknowns.

In the Chapter VI *O potęgach, pierwiastkach i logarytmach* [On powers, roots and logarithms] there were derived formulas to calculate the square and the cube of polynomial on the basis of which there were derived formulas for the square root of the polynomial.

In Chapter VII *Równanie stopnia drugiego, zupełne i niezupełne, dwukwadratowe, układy równań jednorodnych stopnia drugiego.* [The equation of the second degree, complete and incomplete, two-squared, homogeneous equations of the second degree] all the models are carefully derived and a lot of examples for each type of equation are solved.

In Chapter VIII *Równania nieoznaczone* [The undetermined equations] the author does not use the name of a diophantine equation, but all the examples of undetermined equations in this manual have integer coefficients and their solutions are integers.

Chapter IX *Procent składany i renty* [Compound interest and pensions], the author accounts for compound interest and pensions account to introduce them to the student.

In Chapter X *Ułamki ciągłe* [Continued fractions] ways are given to convert the ordinary fractions into continued and vice versa.

In Chapter XII *Rachunek prawdopodobieństwa* [Probability calculus] next to the basic concepts of probability calculus, the student is acquainted with the concept of mathematical hope, for example, he knows the life insurance account, the method of securing capital for survival.

In the end we find an extending information about structures and operations on complex numbers and the theory of equations. The last paragraph of the conclusion is a very beautifully designed *Pogląd historyczny na rozwój algebry* [Historical view on the development of algebra].

1.3.2. Teaching programs of mathematics in gymnasium of real type

We will represent the teaching programme of mathematics from the Higher Real Gymnasium named after Adam Mickiewicz. From 1908 to 1915 gymnasium was named after Mickiewicz in Lvov, from 1916 the name was changed to the Higher and Real Gymnasium named after Adam Mickiewicz in Lvov.

Gymnasium of real type in the school year 1908/09 included six classes. The next class was already added in 1910, so teaching included 7 classes, and since 1911 gymnasium of the real type was an eight-year school.

Schedule of hours teaching of mathematics was the following:

Years of studying	**F o r m**								**Altogether**
	I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	
1908–1909	3	3	3	5	4	3	–	–	21
1910–1911	3	3	3	5	4	2	3	–	23
1911–1913	3	3	3	5	4	2	3	2	25

During 1908–1909 the mathematics programme included the following contents:

FORM I
3 hours a week. **Arithmetic**: System of metrical measures; the decimal system of numbers; four operations with integers, decimals and mixed numbers; the divisibility of numbers; the simple factorization; beginnings of study of common fractions. **Geometry**: The general study of space quantities. The right line, circle, angle, parallels, the triangle.
<u>Notes</u>: Frequent homework exercises. 3 school assignments each term.

FORM II
2 hours a week. The arithmetic: The measure and the multiplicity; operations with common fractions; changing decimal fractions into ordinary ones and vice versa; ratios and proportions; the simple rule of three with the use of proportion and conclusions; calculus simple per cent. **Geometry**: Axes of symmetry of sections and angles; the congruence of triangles; properties of the circle, quadrangles and polygons.
Notes: Tasks as in form I.

FORM III
3 hours a week **Arithmetic**. Four operations mostly with general integer numbers, and with fractions. The square and the extraction of the second root; approximate numbers and operations on them. **Geometry**: The equality, the exchange and the partition of figures; the measurement of the line and the surface; similarity of figures.
<u>Notes</u>: Three assignments every term.

FORM IV
5 hours every week. Equations of the first degree with one and several unknowns; pure equations of the second and third degree. The rising to the cube and the extraction of the third root. The complex rule of three, the rule of the partition, calculus of the compound interest. **Solid geometry**.
<u>Notes</u>: Assignments as in form III

FORM V
4 hours a week. **Algebra**: four operations; negative numbers; the divisibility, the measure, the multiplicity, fractions, proportions, simple equations of one and several unknowns. **Geometry**: Planimetrics.
<u>Notes</u>: 3 assignments a term.

Let us note that in the teaching of geometry the solid geometry contents were covered first, then the planimetrics ones. Reference was made to the fact that the student saw the first three-dimensional objects, then the abstract two-dimensional and one-dimensional by models of cords, ropes and the like.

FORM VI
3 hours a week. **Algebra**: Powers, roots, logarithms, and solving simple equations. **Geometry**: revision of planimetrics. Solid geometry.
Notes: 3 assignments a term.

At the turn of 1909 the changes in teaching mathematics followed in gymnasium of the real type. In the VI form one produced a separate subject descriptive geometry. The teaching programme of mathematics was extended by the introduction of new concepts, such as: quantitiess directly and inversely proportional, properties of plane figures,
Considerable pressure was put on teaching geometry. In 1910 students completed gymnasium of the real type after the seventh class.

FORM I
3 hours a week. **Arithmetic**: System of metrical measures; system of decimal numbers; four operations on integer and decimal dimensional and non-dimensional numbers; beginning of study of common fractions on simplest concrete examples. **Geometry**. Beginning of study of simple geometrical forms, of a cube and ball on the basis of sight. Exercises in using compasses, rulers, triangles, scales. The measurement and drawing of objects from the environment. Study of the property of the simplest concrete spatial formations as well as relationships among them. Area of a square, a rectangle, the volume of a cube and a column as the use of the metric system.

FORM II
3 hours a week. **Arithmetic**: the measure and multiplicities; operations with common fractions changing decimal fractions to common ones and vice versa. Quantities directly and inversely proportional in calculus with deducing. Continual exercises in dimensional computing with decimal numbers. The simplest examples from calculus of the percentage.
Geometry. The preview of study of symmetry of solid and planar figures. Study of sufficient elements to determine the planar figure by construction (instead of proofs of congruence). The various use of measurement in the schoolroom, if possible also outdoors. Triangles, quadrangles, polygons (particularly regular), circle. Cuboids, pyramids, cylinderes and cones. Ball, pointing out the correlation with the study of geography. The variability of objects (changes in their shape and dimensions with the change of defining elements).

FORM III
3 hours a week. **Arithmetic**: Four operations mainly with general and integer numbers and with fractions. Raising to the square and extraction of the second root; approximate figures and operations of them. **Geometry**: The equality, the exchange and the partition of figures; the measurement of the line and the surface; similarity of figures.

FORM IV
5 hours every week. **Arithmetic**. Equations of the first degree with one and several unknowns; pure equations of second and third degree. The measure; the multiplicity; fractions. Geometry: Planimetrics.
Notes: Tasks as in form I.

FORM V
4 hours a week. **Algebra**: Four operations; negative numbers; divisibility, measure, multiplicity, fractions, proportions, simple equations with one and several unknowns. **Geometry**: Planimetrics.
Notes: 3 assignments a term

FORM VI
2 hours a week. The arithmetic: Powers, elements, logarithms and solving equations of the first and second degree with one unknown. Pictures of geometric algebraic equations.
G*raphic g*eometry. **General solid geometry**. The measurement of angular, round and regular blocks. The beginning of trigonometry.
Notes: 3 assignments a term.

FORM VII
3 hours a week. Equations of the second and higher degrees; calculus of the percentage; the simplest kinds of permutation, variation and combination. Trigonometry and the analytics.

Since 1911 real gymnasium had eight forms. Programme contents were enriched, in particular in application of mathematics and, as today we would say, a holistic attempt at revision.

FORM VIII
2 hours a week. Finishing and revision of the school study from all the range of study of mathematics, especially equations and series, solid geometry, trigonometry and analytic geometry. Applications of problems to different areas of the school study and the practical life.
Notes: 3 papers a term.

In the following years the programme did not change considerably, in 1912 trigonometry and the analytic geometry was already entirely covered in the seventh form.

In the following years 1914–1915 the weekly number of hours was the following:

SPRAWOZDANIE

wo.en.

GIMNAZYUM im. A. MICKIEWICZA

WE LWOWIE

ZA ROK SZKOLNY 1914/15.

Non mirari, non indignari
sed intelligere.

II.

Plan nauk, podręczniki, lektura, zadania.

Wobec konieczności wprowadzenia 5 godzin nauki języka rosyjskiego **w każdej klasie**, okazała się konieczność redukcyi godzin niektórych innych przedmiotów.

Wobec tego Rozkład godzin przedstawiał się następująco:

Klasy	I	II	III	IV	V	VI	VII	VIII	Razem
Religia	2	2	2	2	2	2	2	2	16
Język polski	3	4	3	3	3	3	3	4	26
Język łaciński	4	4	4	4	4	4	4	4	32
Język grecki**)	—	—	—	3	3	3	3	3	15
Język niemiecki	4	4	3	3	3	3	3	3	26
Język rosyjski	5	5	5	5	5	5	5	5	40
Język francuski*)	—	—	3	3	3	3	2	—	14
Język angielski*)	—	—	—	—	2	2	2	—	6
Historya	2	2	2	2	3	3	3(4)	3	20(21)
Geografia	2	2	2	2	1	1	—	—	10
Matematyka	3	3	3	3	3	3	3	2	23
Geometrya wykreślna*)	—	—	—	—	2	2	—	—	4
Historya naturalna	2	2	—	—	3	2	1*)	—	10
Chemia	—	—	— }	3	—	2*)	2*)	—	4
Fizyka	—	—	2 }		—	—	4	3	12
Propedeutyka filozofii	—	—	—	—	—	—	1	2	3
Rysunki	2	2	2	2*)	—	—	—	—	8

*) Oznaczone przedmioty wyłącznie w gimn. realnem. **) przedm. wyłącznie gimn. klasycznego.

5. Nauce wszechrosyjskiego literackiego języka w zakładzie szkolnym musi się poświęcić niemniej, niż pięć godzin w tygodniu w każdej klasie.

6. Historya, geografia, język polski i historja literatury polskiej, mogą być wykładane tylko na podstawie podręczników, dozwolonych w Cesarstwie Rosyjskiem, albo dopuszczonych specyalnem pozwoleniem mojem, albo osób, przezemnie upełnomocnionych.

Because of the need for five hours of the Russian language in each class, it was necessary to reduce the hours of some other subjects. (Note: the study of Russian was introduced by the pronouncement of the wartime Russian governor of Galicia, on December 4(17), 1914. Before World War I,, Ruthenian, not Russian, was taught as a so-called “relatively compulsory”, i.e., elective subject.) The symbol * marks those subjects that were taught only in real schools; ** those that were covered only in classical gymnasia. Propedeutics of philosophy was covered in Form VII and VIII.

After 1915 in gymnasium of the real type the number of hours of teaching of particular subjects was changed, and remained obligatory to 1919.

Below we present the table with the schedule of hours for gymnasium of the classical and real type. Let us notice that the number of hours in both types of grammar-schools was the same, contents were divided differently, in gymnasium of the real type the descriptive geometry was a separate subject.

The table lists subjects down (the geometry is the fourth) and number of hours in each form and type of gymnasium across (g. means classical gymnasium, r.g. real gymnasium).

Rozkład godzin

(g = oddział klasyczny; rg = oddział realno-gimnazyalny).

	I	II	III g.	III rg.	IV g.	IV rg.	V g	V rg.	VI g.	VI rg.	VII g.	VII rg	VIII g	VIII rg.
Religia	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Język polski	3	4	3	3	3	3	3	3	3	3	3	3	4	4
„ łaciński	6	6	6	6	6	6	6	6	6	6	5	5	5	5
„ grecki	-	—	5	—	4	—	5	—	5	—	4	—	5	—
„ francuski	-	—	—	3		3	—	3	—	3	—	3	—	3
„ niemfecki	5	4	4	4	4	4	4	4	4	4	4	4	4	4
Historya	2	2	2	2	2	2	3	3	3	3	4	4	3	3
Geografia	2	2	2	2	2	2	1	1	1	1	—	1	—	—
Matematyka	3	3	3	3	3	3	3	3	3	3	3	3	2	2
Geometrya wykreślna	—	—	—	—	—	—	—	2	—	2	—	—	—	—
Nauki przyrodnicze	2	2		—		—	3	3	3	3	—	1	—	1
Fizyka	-		2	2	3	3	—	—	—	—	3	3	3	4
Chemia	—	—	—	—	—	—	—	—	—	2	—	2	—	—
Propedeutyka filozof.	—	—	—	—	—	—	—	—	—	—	2	2	2	2
Rysunki	2	2	2	2	3	3	—	—		—	—	—	—	—
Kaligrafia	1	—	—	—	—	—	—	—	—	—	—	—	—	—

Such a number of hours in classical and real gymnasia was compulsory till 1919. Real School named after Jan and Andrzej Śniadecki in Lvov

Real School named after Jan and Jędrzej Śniadecki in Lvov

We will show the teaching programme of mathematics and the descriptive geometry based on Reports from the Real School named after Jan and Andrzej Śniadecki in Lvov. The school came into being in 1899, as a branch of the Real School; in 1903 it was transformed into the II independent State Real School. The presented programme connected whole mathematics with the descriptive geometry and drawings. The publication of programme in the Reports had great influence on the way of teaching mathematical contents in other real gymnasia in Galicia.

The number of hours of teaching mathematics was the following:

Years of studying	Form							Altogether
	I	II	III	IV	V	VI	VII	
Mathematics 1904–1909	3	3	3	3	4	4	4	24
1910	3	3	3	4	4	5	4	26
1911–1917	3	3	3	4	4	4	5	26
Geometry and geometrical drawings 1904–1917	–	2	2	2	3	3	2	14

In 1904–1920 the teaching programme of mathematics and geometry in High Real School in Lvov was the following.

FORM I

3 hours a week. The decimal system: How Romans wrote the numbers. First four operations on integers and decimal fractions, dimensional and non-dimensional. Explanation of the metric system of measures and weights. Exercises in simple deduction. Divisibility of numbers, factorization into primes; the greatest common measure and least common multiple. First 4 operations on simple fractions. Changing common fractions into decimal ones and vice versa. Calculations with mixed numbers. Beginning of study of geometrical forms. Fundamental concepts of geometry and their explanation from the viewing of elementary solids: the cube, the prism, the pyramid, the cylinder, the cone and the ball. Explanation of the most important forms of planar geometry and their main features on the visual basis.
Tasks: four school assignments a term, moreover smaller homework exercises.

FORM II

3 hours a week. Revision of common fractions. Calculus with incomplete numbers. Shortened multiplication and division. Solving exercises in a simple and compound rule of three with deducing. The most important information about measures, weight and money. Study of relations and proportions with the

applications to solving problems with a law of three, simple and compound. Calculus of simple percentage, commission and the discount. Tasks as in I form.

FORM III

3 hours a week. Beginning of the general arithmetic. Study of four main operations on general numbers with one or two digits written using letters, with the exclusion of fractions calculus. The rising to the square and to the cube of algebraic one- and multi-digit expressions, also of decimal numbers. Extraction of square and cubic root from decimal numbers. Continual exercises in computing with specific numbers to memorize arithmetical information from previous forms: exercises in calculus of partition. Written compositions as in form I.

Geometry and geometrical drawings (2 hours a week)

Geometry (1 hour). Continuation and completion of planimetrics. The equality and the transformation of the area of planar figures. The calculation of the area, proportionality and similarity in the relationship with suitable material of study of mathematics in this form.

Geometrical drawings (1 hour). Extension of constructions introduced in the second form to the scientific material presented above.

FORM IV

3 hours a week. **The general arithmetic**: Revision, justification and extension of study of the first four operations on general and special, integer and fractional numbers. The justification of the simplest rules of the divisibility of numbers of the decimal system. Theory of the greatest common measure and the least common multiplicity, the application to polynomials. Equations of the first degree with one and more unknowns with applications to solving major practical problems. Study of relations and proportions between general numbers with applications. Tasks as in form I.

Geometry and geometrical drawings (2 hours a week).

Geometry. Rules of solid geometry. The most important theorems about mutual placement of straight lines and surfaces in view of study of projections. The prism, the pyramid, the cylinder, the cone and ball. Calculating the area and volume of these solids. (Examples referring to the ball ought to be given without justification).

Geometrical drawings. Introduction of points, sections of planar figures and selected geometrical solids with two perpendicular projective planes in a visual manner and in the relationship with scientific material of solid geometry.

FORM V

4 hours a week. **General arithmetic**: Indeterminate equations of first degree with two unknowns. Powers and roots: concept of irrationals. Imaginary

unit. Equations of the second degree with one unknown, that can be reduced to quadratic equations. The simplest cases of quadratic equations with two unknowns. Study of logarithms.

Geometry: Planar geometry. Principal formations of planar geometry. Theory of parallels. Theorems about triangle, including congruency: theorems about angles and chords in circle, about inscribed and circumscribed triangles and quadrangles. Proportionality of sections, similarity of figures, as a result of statement about the triangle and circle. Perpendicular bisectors in the triangle, harmonic order of points. The equality of the area, the exchange and the partition of the surface; the calculation of the area. Regular polygons, the measurement of the circle. Some problems on applications of algebra to geometry. Tasks as in I form.

Geometry and geometrical drawings (3 hours a week).

Revision of the most important theorems on mutual position of straight lines and surfaces. Systematic coverage and due training in solving problems of principle geometry of graphic about points, straight lines and surfaces, taking into account also the cross-shaped projective plane. Projections of plane figures and determination of their shadows thrown on projective planes. Drawing of a circle from its projection. Deducing the most important properties of the ellipse from analogous properties of the circle in relationship to its projection.

FORM VI

4 hours a week. **General arithmetic**: Logarithmic, exponential equations. Arithmetical and geometrical progressions. Calculus of component percentage. Calculus of pensions. Revisions.

Geometry: Trigonometry. Goniometrical functions, solving of the right triangle. further goniometrical examples. Solving of regular polygons. Main theorems, helping to solve oblique angled triangles with applications. Easier goniometrical equations.

Solid geometry: The most important statements about mutual position of straight lines and planes in the space. Properties of main angles in general, and particularly of the trihedral wedge (polar wedge). The partition and properties of solids. The congruence and symmetry. Similarity and the symmetrical similarity of solids. Surface area and volume of the prism, the pyramid and the truncated pyramid. The calculation of the volume of the cylinder, the cone, the truncated cone, also the surface of these solids at axes perpendicular to the base. The surface and the volume of the ball, its part of easy limitations. Tasks as in form I.

Geometry and geometrical drawings (3 hours a week)

Rectangular projections of prisms, pyramids, cylinders and cones. Plane sections, nets, the parallel lighting, easier cases of mutual interlacing of these solids. The manner of formation of conical sections in space, their structures and projections. The rising of the most important properties of these curves with their application to determining tangents. Planes tangent to the surface of cylinders and cones. Shadows projected onto the interior of cylindrical and conical surfaces.

FORM VII

4 hours a week. **General arithmetic**: Rules of combinations. Newton's binomial for integer and positive exponents. Rules of study of the probability.

Geometry: spherical trigonometry: The most important properties of the spherical triangle, its surface. The most important examples of solving spherical right and oblique triangles. The use of spherical trigonometry and simple astronomical problems.

Analytic geometry: analytic geometry of the straight line, circles and conical sections on the plane on the basis of rectangular co-ordinates, and in some major cases also polar co-ordinates. Properties of conical sections regarding foci, tangents, normals and diameters. Squaring of the ellipse and the parabola. Revision of all scientific material of upper classes on properly chosen examples. Tasks as in I form.

Geometry and geometrical drawings (2 hours a week). Projections of surface of the ball, its plane sections, tangent surfaces, also cylinders and cones tangent to a ball. Proper shadows and shadows projected on convex and concave sides of the surface of cylinders, cones and sections of the ball. Revision of the most important parts of descriptive geometry on properly chosen problems and examples.

Since 1910 the number of hours of teaching mathematics increased from 24 to 26 hours. While number of hours for the geometry and geometrical pictures did not change.

The teaching programme of mathematics in the real gymnasium was divided into the general arithmetic, the geometry, the solid geometry, the trigonometry, the analytical geometry, the geometry and geometrical pictures. The descriptive geometry was a separate subject. A great emphasis was put on the instruction of the geometry, and the great number of hours intended for it and problems in geometry in the final school examination confirm this.

1.3.3. Business School

The Real-Business School and Business Faculty at the Technical Academy functioned in Lvov in 1817–1876. After the liquidation of the faculty in 1876 and after insistence, the Ministry of Religious Affairs and Education in Vienna approved the existence the University of Economics in Lvov in 1899 (school started 2. X. 1899) together with the Supplementary Business School (I–III forms) and the course of secondary schools leavers.

Higher Business School (later the Academy of Economics) consisted of four annual courses. Curriculum included the compulsory courses (part of which consisted of items related to mathematics – merchant accounts, general and political arithmetic, geometry) and the so-called relatively compulsory subjects.

At the Business Supplementary School trade workers improved their skills. It consisted of three annual courses. To the Business Supplementary School there were admitted shop assistants and students who: showed the school-leaving certificate of at least 5-class high school or a certificate of another school, which replaced it or had other academic equivalent.

The aim of one-year high school course for the school leavers was a comprehensive commercial education.

Polonized Galicia at the threshold of autonomy was practically without trade schools. Under the Austrian law vocational education, including commercial subjects should be covered in one-third by private industrial-commercial institutions. The other two parts were based on local subsidies and authorities in Vienna. Therefore, there was a great disproportion between the networks of vocational schools in Lower Austria and Galicia as well as in relation to the Czech Republic and Moravia.

Mathematical content in the four-class Academy of Trade.

We will present articles related to mathematics: *Merchant Accounts, General and Political Arithmetic,Geometry.* Course contents of ***Merchant Accounts*** were associated primarily with the function of money in time and required advanced knowledge of the percentage calculation. Teaching mathematics at a trade school had two goals. Through the mathematical content student had to understand professional subjects better and stimulate his general development.

General and Political Arithmetic.

Form I

Teaching this subject took place in forms II, III and IV.

Form II

Four operations on general numbers. Fractions. Ratios and proportions.
Powers and roots. Raising to the power and extracting the root. Logarithm.
Every 8 days homework, school assignments – every 3 weeks.

Form III

Equation of the first and second degree with one and more unknowns. Indeterminate equations. Exponential equation. Arithmetic and geometric progress. The science of combinations. The principles of probability theory. The binomial theorem. Homework every week, every 3 weeks – school assignment.

Form IV

Political arithmetic – calculation of compound percentages and pensions. Examples of loan operations. Calculation of redemption plans. Calculating

annuities and inheritances. Some important issues in the life insurance account. Every 14 days – homework, every 4 weeks – school assignment.
Since 1903–1904 algebra was separated in classes I and II. The subject was called The programme of algebra and political arithmetic. The contents of algebra were as follows:

Form I
Four operations on general numbers and fractions (letter calculation). System of decimal numbers. Justification of the simplest rules of divisibility. Greatest common divisor, lowest common multiple. Application of algebra to decimal. Periodic and incomplete fractions, operations on them. First-degree equations with one and several unknowns. Using algebra to study relations and proportions – the rule of three. The theoretical justification for the calculation of a hundred, on hundred or from hundred. Derivation of formulas for percentages. Derivation of a permanent divisor (the key number.) Discussing how to solve problems with an account of the company and the mixture of account rebate, profit and loss, and account of insurance using the proportion. 6 school assignments during the year.

Form II
Powers and roots. Second degree equations with one unknown. Indefinite first-degree equation with two unknowns: using substitution of standard solutions in cases that may happen in the income of the mixture, as it applies to that account. The concept of logarithms in general. General theorem on logarithms (product, quotient, power, roots). Decimal logarithms (Briggs). Logarithmic Tables. Calculation with the use of logarithms. Program of Algebra and political arithmetic was enriched by the economic contents. The geometry was compulsory in the I Form. The 1903–1904 school year was the following:

Form I
The aim of this course was to familiarize students with the most important principles of solid geometry and planimetrics. Completed and extended revision of solid geometry and planimetrics in connection with the curriculum of lower secondary school, or school faculty, was based on known concepts of trying to continue learning as closely as possible to the scientific treatment of the subject. And attention was paid to the application of geometry to solve such tasks, which can develop student's independence. Calculation of the volume should be limited to the solids which occur most frequently in trade, where the volume of solids is good for calculating their weight (marking weight of beams, logs, liquids in barrels, cargo, etc.). In the higher classes, while discussing of particular parts of algebra were meant to modify the relevant tasks in the field of geometry.

1.4. Some statistics on gymnasium

Let us have a look on some statistical data related to secondary education in Galicia from the late nineteenth century. The classification of students in public and private schools in the five-year period 1891/2–1895/6, and during 1986/7 and 1897/8 in the classical gymnasia.

11. Wynik klasyfikacyi uczniów publicznych i prywatystów z końcem roku szkolnego 1891/92 do 1897/98.

Liczba porząd.	Zakłady	W pięcioleciu 1891/2–1895/6 przeciętnie rocznie: z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	% z ogółu uczniów	W roku 1896/7: z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	% z ogółu uczniów	W roku 1897/8: z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	% z ogółu uczniów
	Gimnazya.												
1	Bochnia	22	282	47	13·99	32	316	54	13·49	45	362	28	6·44
2	Brody	25	339	45	10·57	29	307	114	25·94	15	346	74	17·00
3	Brzeżany	22	269	60	17·09	22	288	65	17·34	26	320	56	13·99
4	Buczacz	19	194	47	18·08	18	182	57	22·62	23	178	37	14·15
5	Chyrów OO. Jezuit.	37	238	68	19·83	40	235	45	13·60	46	249	39	10·06
6	Drohobycz	16	208	55	19·71	29	289	57	17·54	19	239	66	20·86
7	Jarosław	21	301	79	19·70	34	364	83	17·26	47	369	106	19·92
8	Jasło	38	369	73	15·20	34	380	78	15·65	54	381	51	10·52
9	Kolomyja kl. polsk.	27	357	91	19·16	19	279	54	15·54	13	286	58	16·16
	" " ruskie					12	122	36	21·18	11	158	27	13·78
10	Kraków Św. Anny	54	402	107	18·35	90	548	68	9·63	64	562	137	17·05
11	" Św. Jacka	29	358	48	11·16	34	398	60	12·20	42	352	44	10·05
12	" gimn. III.	53	373	50	10·89	61	379	41	8·52	97	416	30	5·54
13	Lwów gimn. akademickie	26	346	54	12·68	39	315	41	10·38	34	346	47	11·01
14	" gimn. II. tak zw. niemiec.	26	263	56	16·28	26	269	59	15·14	20	297	59	15·69

Real schools

Liczba porząd.	Zakłady	W pięcioleciu 1891/2–1895/6 przeciętnie rocznie: z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	% z ogółu uczniów	W roku 1896/7: z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	% z ogółu uczniów	W roku 1897/8: z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	% z ogółu uczniów
	Szkoły realne.												
1	Kraków	21	351	77	17·15	32	465	93	15·76	39	533	114	16·62
2	Lwów	27	398	76	17·61	38	390	95	16·16	47	482	108	18·40
3	Stanisławów	12	178	32	14·41	17	231	47	15·77	17	268	35	10·94
4	Tarnopol	6	87	11	10·57	14	156	34	16·67	15	239	65	20·38
5	Tarnów	—	—	—	—	—	—	—	—	5	95	4	9·09
	Razem	66	954	198	16·26	101	1.245	209	16·66	123	1.507	320	16·67
	%	5·42	78·32	16·26	—	6·25	77·09	10·66	—	6·29	77·04	16·67	—
	Ogółem w 35 szkołach średnich	1.077	10.240	2.068	15·45	1.340	11.643	2.259	14·62	1.501	12.733	2.241	13·60
	%	8·04	76·51	15·45	—	8·79	76·39	14·82	—	9·11	77·29	13·60	—

[1]) Zobacz uwagę na str. 96.

Similar number of students were educated in gymnasia of the classical type and the real type in Cracow and in Lvov. In Lvov region (Stanisławów, Tarnopol) because of the existence of Polytechnic school this number was comparatively greater than in Cracow (including Tarnów) at the end of the XIX century.

Apart from that, the gymnasium education was rather expensive and hardly available, as prof. Franciszek Leja[19] expressed in his memoirs. *In the Austrian*

[19] *Dawniej było inaczej*, memoirs, manuscript owned by the author.

annexation gymnasium *was an eight-year college of the humanistic direction. Next to* gymnasia there still were *7-year secondary schools of the technical direction, called real schools. In the latter, Latin was not taught. Gymnasium was divided into lower and higher one. Lower gymnasium* included *the classes from 1st to 4th, higher from 5th to 8th and ended with the school-leaving examination, or with the secondary-school certificate which gave* access *to higher studies without entrance examinations. Contrary to primary schools, the* study *in gymnasia* was not *free. The pupil was obliged to pay the so called tuition fee to the Treasury every year for studying. In case of good progress in* study *the pupil could be free of charge from fee upon showing a certificate of parents' poverty. Each gymnasium pupil was to wear a navy blue uniform with an appropriate number of stripes for a particular class. The stripes were supposed to be fixed to a standing collar. Also, the students were supposed to wear caps with the letter G on it. The Galician village delivered very small percentage of students to secondary schools. The reason was the difficult financial condition of the average country-farm.*

11. Wynik klasyfikacyi uczniów publicznych i prywatystów z końcem roku szkolnego 1891/92 do 1897/98.

Liczba porząd.	Zakłady	W pięcioleciu 1891/2—1895/6 przeciętnie rocznie				W roku 1896/7				W roku 1897/8			
		otrzymało stopień											
		z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	zły lub nie klasyfikowano: % z ogółu uczniów	z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	zły lub nie klasyfikowano: % z ogółu uczniów	z odznaczeniem	dobry	zły lub nie klasyfikowano: razem	zły lub nie klasyfikowano: % z ogółu uczniów
	Gimnazya.												
1	Bochnia	22	282	47	13·30	32	316	54	13·49	45	362	28	6·44
2	Brody	25	389	45	10·57	29	307	114	25·34	15	346	74	17·00
3	Brzeżany	22	269	60	17·00	22	288	65	17·34	26	320	56	13·93
4	Buczacz	19	194	47	18·08	18	162	57	22·62	23	178	37	14·15
5	Chyrów OO. Jezuit.	37	238	68	19·83	46	235	45	13·60	46	249	33	10·06
6	Drohobycz	16	208	55	19·71	29	289	57	17·54	19	239	68	20·86
7	Jarosław	21	301	79	19·70	34	364	83	17·26	47	369	106	19·92
8	Jasło	38	369	73	15·20	34	380	78	15·85	54	391	51	10·52
9	Kołomyja kl. polsk.	27	357	91	19·16	19	279	54	15·84	19	288	58	16·16
	„ „ ruskie					12	122	36	21·18	11	158	27	18·78
10	Kraków Św. Anny	54	402	107	18·35	90	548	68	9·63	84	582	137	17·05
11	„ Św. Jacka	29	353	48	11·16	34	398	60	12·20	42	352	44	10·05
12	„ gimn. III.	58	373	60	10·89	61	379	41	8·52	97	415	80	5·54
13	Lwów gimn. akademickie	26	346	54	12·68	39	315	41	10·38	34	346	47	11·01
14	„ gimn. II. tak zw. niemiec.	26	263	56	16·28	26	269	53	15·14	20	297	59	15·69

The classification of students in public and private schools since 1891/92 till 1897/98 school years.

Fragment from the book concerning the statistical data in Galicia.

In the five years 1891/2–1895/6 (54 + 29 + 58 + 402 + 353 + 373) 1269 people graduated from classical gymnasium in Cracow, in Lvov (26 + 26 + 346 + 263) = 661. And in the school year 1897/8 respectively (84 + 42 + 97 + 582 + 352 + 415) = 1572 in Cracow and (34 + 20 + 346 + 297) = 697 in Lvov. There is a clear upward trend in the education of secondary school in the late nineteenth century. In one year there were more students leaving secondary schools than in five years. In Cracow at that time, twice as many students were finishing classical gymnasium. The opposite was with graduates of real gymnasium, the Eastern Galicia clearly led. In Cracow in period 1891/2–1895/6 372 students graduated from real gymnasium, in Lvov 365, and including data from Stanisławów and Tarnopol in Eastern Galicia, we have 648 people. This data is clearly related to the functioning of Polytechnic School in Lvov. For 1897/8 school year data are the following: Cracow and Tarnow – 612 people, Lvov including Stanisławów and Tarnopol – 1018 people.

	Szkoły realne.												
1	Kraków	21	351	77	17·15	82	465	98	15·76	89	583	114	16·62
2	Lwów	27	338	76	17·61	38	390	95	18·16	47	482	108	18·40
3	Stanisławów . .	12	178	32	14·41	17	234	47	15·77	17	268	35	10·94
4	Tarnopol	6	87	11	10·57	14	156	34	16·67	15	239	65	20·38
5	Tarnów	—	—	—	—	—	—	—	—	5	35	4	9·09
	Razem .	66	954	196	16·26	101	1.245	209	16·66	128	1.507	320	16·67
	% .	5·42	78·32	16·26	—	6·25	77·09	16·66	—	6·29	77·04	16·67	—
	Ogółem w 85 szkołach średnich	1.077	10.240	2.068	15·45	1.340	11.643	2.259	14·82	1.501	12.735	2.241	13·60
	% .	8·04	76·51	15·45	—	8·79	76·39	14·82	—	9·11	77·29	13·60	—

¹) Zobacz uwagę na str. 96.

Since the 1888–1889 school year the number of students that passed the exam in Galicia in the next ten years was as follows: in classical gymnasium – 830, 681, 690, 753, 788, 765, 656, 867, 877, 866 in real gymnasium – 70, 51, 53, 54, 64, 66, 105, 92, 95, 123

12. Egzamina dojrzałości w gimnazyach od r. 1888/89 do r. 1897/98.

Rok szkolny	Wynik egzaminów dojrzałości przy końcu roku szkolnego: Abituryentów publicznych – było ogółem	uznano za dojrzałych	%	reprobowano %	Eksternistów – było ogółem	uznano za dojrzałych	%	reprobowano %	Abituryentów publicznych i eksternistów – było ogółem	uznano za dojrzałych	%	reprobowano %
1888/89	846	784	92·45	7·55	62	46	74·19	25·81	910	830	91·21	8·79
1889/90	794	648	81·30	18·70	48	33	68·75	31·25	842	681	80·88	19·12
1890/91	805	648	80·49	19·51	68	42	61·76	38·24	873	690	78·57	21·43
1891/92	783	688	87·87	12·13	107	65	60·75	39·25	890	753	84·61	15·39
1892/93	845	724	85·68	14·32	102	64	62·75	37·25	947	788	83·21	16·79
1893/93	794	719	90·55	9·45	77	46	59·74	40·26	871	765	87·84	12·16
1894/95	689	599	86·94	13·06	82	57	69·51	30·49	771	656	85·08	14·92
1895/96	904	804	88·94	11·06	90	63	70·00	30·00	994	867	87·22	12·78
1896/97	882	815	92·40	7·60	94	62	65·96	34·04	976	877	89·86	10·14
1897/98	880	803	91·22	8·78	90	63	70·00	30·00	970	866	89·28	10·72
Razem od roku 1889 1898	8.224	7.232	87·94	12·06	820	541	67·92	32·08	9.044	7.773	85·95	14·05
Przyrost od roku 1889–1898	32	19	59·37	40·63	28	17	60·72	39·28	60	36	60·00	40·00
Procent przyrostu	3·77	2·43	—	—	45·16	36·96	—	—	6·59	4·34	—	—

13. Egzamina dojrzałości w szkołach realnych od r. 1888/89 do r. 1897/98.

Rok szkolny	Abituryentów publicznych – było ogółem	uznano za dojrzałych	%	reprobowano %	Eksternistów – było ogółem	uznano za dojrzałych	%	reprobowano %	Abituryentów publicznych i eksternistów – było ogółem	uznano za dojrzałych	%	reprobowano %
1888/89	66	65	98·48	1·52	5	5	100·00	—	71	70	98·59	1·41
1889/90	55	48	87·27	12·73	5	3	60·00	40·00	60	51	85·00	15·00
1890/91	54	44	81·48	18·52	14	9	64·28	35·72	68	53	77·94	22·06
1891/92	56	49	87·50	12·50	12	5	41·67	58·33	68	54	79·41	20·59
1892/93	60	57	95·00	5·00	13	7	53·85	46·15	73	64	87·67	12·33
1893/94	72	59	81·94	18·06	10	7	70·00	30·00	82	66	80·49	19·51
1894/95	99	88	88·89	11·11	22	17	77·27	22·73	121	105	86·78	13·22
1895/96	100	85	85·00	15·00	8	7	87·50	12·50	108	92	85·19	14·81
1896/97	96	86	89·58	10·42	18	9	50·00	50·00	114	95	83·33	16·67
1897/98	120	107	87·85	12·15	18	16	88·89	11·11	138	123	89·13	10·87
Razem od roku 1889–1898	778	688	89·72	10·28	125	85	68·00	32·00	903	773	85·60	14·40
Przyrost od roku 1889–1898	54	42	77·77	22·23	13	11	84·62	15·38	67	53	79·10	20·90
Procent przyrostu	81·82	64·61	—	—	260·00	220·00	—	—	94·37	75·71	—	—

1.5. Teachers of mathematics in c.k. Gymnasium named after Francis Joseph in Lvov

The list of headmasters and teachers of mathematics in the years 1876–1921. Names of teachers who published, were authors of textbooks and held PhD, who were assistant professors or those who worked well in high school are written in bold. In 1876–1889 the headmaster was **Zygmunt Samolewicz**, Doctor of Philosophy, Knight of the Order of Francis Joseph. He taught the Greek language.

Teachers of mathematics who worked in 1876–1889.

Ignacy Petelenz, professor, member of the examination committee for candidates for one-year volunteers, manager of natural lab and library for the youth, he taught mathematics in V and VII forms.

Antoni Filipowski, a teacher and manager of physics lab, the manager of class VIII, he taught mathematics in forms VII, VIII.

Jan Werchratski, a certified assistant teacher, he taught mathematics in forms I and II.

Florian Łoziński, assistant teacher, host of class I, taught mathematics in form III.

Julian Fąfara, a certified assistant *teacher,* manager *of the class* IV, taught mathematics in classes IV, V, VI. *The author of mathematics textbooks.*

Hipolit Parasiewicz, assistant *teacher,* manager *of the class* IV, taught mathematics in classes I, II and IV.

Since 1877:

Franciszek Tomaszewski, a certified assistant *teacher*, manager of the class I, taught mathematics in classes I and VII.

Włodzimierz Szuchiewicz, a certified assistant *teacher*, manager of the class III, taught mathematics in class III.

In 1878

Adolf Zajączkowski, a certified assistant *teacher, taught mathematics in* classes III, IV, V, VI.

After 1880:

1881

Ignacy Petelenz, completed education, extended scientific output, PhD, professor, private leader of zoology at Polytechnic School, member of examination committee for candidates for one-year volunteers, stationmaster of natural class, taught mathematics in VII form. He had 17 hours of classes during the week.

Antoni Filipowski, taught mathematics in I, V, VIII forms. He had 19 hours of classes during the week.

Hipolit Parasiewicz, teacher of mathematics. taught in I, II and III forms. He held 18 hours of classes in a week.

Adolf Zajączkowski, taught in I, II and III forms. He held 19 hours of classes in a week.

Mikołaj Piszkiewicz, a certified assistant teacher, manager of the II class, taught in I, II and IV forms. He had 18 hours of classes in a week.

1882
Michał Służewski, professor, manager of the IV class, *t*aught in IV and V forms.
Ludwik Salo, a certified assistant teacher, manager of the Ic class, taught in I form. He held 18 hours of classes in a week.
Karol Skwarczyński*,* a certified assistant teacher, manager of the Ib class, taught in I, II forms.
Roman Gutwiński*,* taught since 1886. A certified assistant teacher, taught in I, II forms.

During 1890-1904 the headmaster of gymnasium was Wojciech Biesiadzki. The teachers of mathematics changed as well.
Antoni Filipowski, professor, the head of the physical class, taught mathematics In the V, VI, VII and VIII forms.
A. Filipowski was the author of work: *About Cassin's line*, published in Reports of c.k. Gymnasium named by Fr. Joseph in Lvov for 1890/91 academic year.

Józef Limbach, PhD, teacher, head of natural class, taught mathematics in the II form.
Łucyusz Czechowicz*,* assistant teacher, taught mathematics in I, II and IV forms.
Michał Służewski, professor in VIII grade, taught mathematics in V, VI, VII, VIII forms.
M. Służewski was the author of *Lecture of equations in real school classes VII*, published in Reports of Headmasters of Higher Real School in Lvov for 1878/79 academic year.
Zygmunt Schneider, a certified assistant teacher, head of the natural class, taught in I, II forms.
Grzegorz Maryniak, professor, head of physics class, taught in the III, IV forms.
G. Maryniak was the translator of manuals of geometry by F. Močnik.
Ludomir Sykutowski, professor, head of natural class, taught in the II form.
Stanisław Ziobrowski, a certified assistant teacher of Physics Department at Polytechnic School in Lvov, taught mathematics in V, VI forms.
Since 1899
Wincenty Frank, professor, taught in the II, III, V, VI, VI forms. W. Frank is the author of school textbooks.
Tadeusz Wiśniowski, PhD, c.k. teacher, stationmaster of natural classes at the branch. He taught mathematics in the I, II, III, IV forms.
Bolesław Błażek, assistant teacher, taught mathematics in the I, II forms.
Bolesław Błażek was the author *of Detailed plans of mathematics in the gymnasium according to the new ministerial plans* from the *1909 year*, Reports of the Management of c.k. Gymnasium with Polish language of teaching in Przemyśl for 1910/11 academic year.
Michał Służewski c.k. professor in VII grade, Imperial adviser, stationmaster of physical office. He taught mathematics in the IV, V, VI, VII forms.

Since 1903 mathematics was taught by:
Bolesław Błażek in the I, II i III forms.
Konrad Rafałowski, in the III, IV, V, VII forms. 19 hours per week.
Józef Markowski, M. D., examined deputy teachers of natural classes, taught in the II form.

In 1905 **Franciszek Tomaszewski**, PhD, c.k. headmaster in the VI grade, National member of Parliament, Councillor of the City Council in Lvov, became the headmaster of Gymnasium. He taught mathematics in the VIII form.

Since that year the following teachers taught:
Ludwik Frączek, deputy of the teacher, taught mathematics In the I, II, III, V forms.
Konrad Rafałowski, c.k. professor, taught mathematics in the IV, VI, VIII forms. He worked 17 hours during the week.
Wincenty Frank, taught mathematics in the VII, VIII forms.
Zygmunt Klemensiewicz, taught mathematics in the IV form.
Stanisław Łabendziński, taught mathematics in the I, II, III forms.

In 1913 Stanisław Schneider became the headmaster.

Mathematics was taught by:
Adam Maksymowicz, PhD, professor, leader of the Polytechnic School, taught mathematics in III, IV, V forms.
Zygmunt Zawiski, PhD, professor, assighed from c.k. II. gymnasium in Rzeszów, taught mathematics in the I, II, IV, VII forms.

During the 1916–1921 the headmaster was **Konstanty Wojciechowski**, PhD, extraordinary professor of history of Polish Literature at the Lvov University. Member of the Warsaw Scientific Society, member of the Committee Academy of Arts and Literature in Cracow.

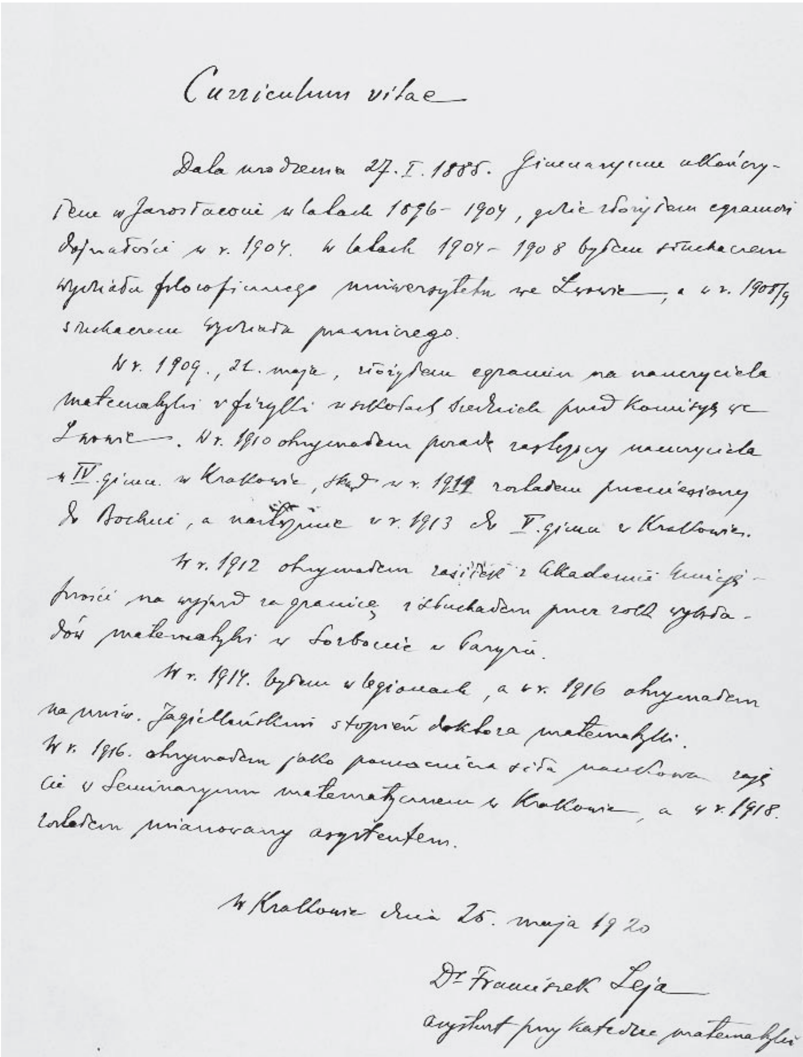
The fact that so many successful teachers have worked in only one school, shows on one hand a high level of teaching, but on the other scarcity of jobs in higher education. This difficult situation has affected the quality of teaching and forming the mathematical culture among secondary school students.

1.6. Preparation of teachers of mathematics

In the 19th and early 20th century the development of universities and other schools of higher educaiton began. As J. Dybiec[20] noticed, such great development was caused by well-performing gymnasia or secondary schools depending on the country. One can mention many excellent schools from Portugal, France through Ukraine and Russia. On the Polish lands the following gymnasia deserve

[20] J. Dybiec, *Nauczyciele krakowskich szkół średnich i ich wkład do rozwoju kultury i nauki (1860–1918)* [Teachers of Cracow's high schools and their contribution to the development of culture and science], in: (ed.) A. Meissner, *Galicja i jej dziedzictwo* [Galicja and its heritage], volume 6, *Nauczyciele galicyjscy* [The teachers of Galicia], pp. 77–95.

the attention: Francis Joseph I gymnasium in Lvov, St. Anna's in Cracow, Mary Magdalene's in Poznań, and Academic Gymnasium in Vilnius. Gymnasia teachers constituted a hierarchical community. The way to become a gymnasium professor was very hard and quite long. Firstly, university studies; we will see their programme on the example of Franciszek Leja, later a famous professor of mathematics at the Jagiellonian University, but initially a gymnasium teacher in Bochnia. F. Leja in the memoir mentioned above, *Dawniej było inaczej*, included memories from a primary school, gymnasium and secondary final school examination.


Curriculum vitae

Data urodzenia 27. I. 1885. Gimnazyum ukończyłem w Jarosławiu w latach 1896–1904, gdzie złożyłem egzamin dojrzałości w r. 1904. W latach 1904–1908 byłem słuchaczem wydziału filozoficznego uniwersytetu we Lwowie, a w r. 1908/9 słuchaczem wydziału prawniczego.

W r. 1909, 21. maja, złożyłem egzamin na nauczyciela matematyki i fizyki w szkołach średnich przed Komisyą we Lwowie. W r. 1910 otrzymałem posadę zastępcy nauczyciela w IV. gimn. w Krakowie, skąd w r. 1911 zostałem przeniesiony do Bochni, a następnie w r. 1913 do V. gimn. w Krakowie.

W r. 1912 otrzymałem zasiłek z Akademii Umiejętności na wyjazd za granicę i słuchałem przez rok wykładów matematyki w Sorbonie w Paryżu.

W r. 1914 byłem w legionach, a w r. 1916 otrzymałem na uniw. Jagiellońskim stopień doktora matematyki. W r. 1916 otrzymałem jako pomocnicza siła naukowa zajęcie w Seminaryum matematycznem w Krakowie, a w r. 1918 zostałem mianowany asystentem.

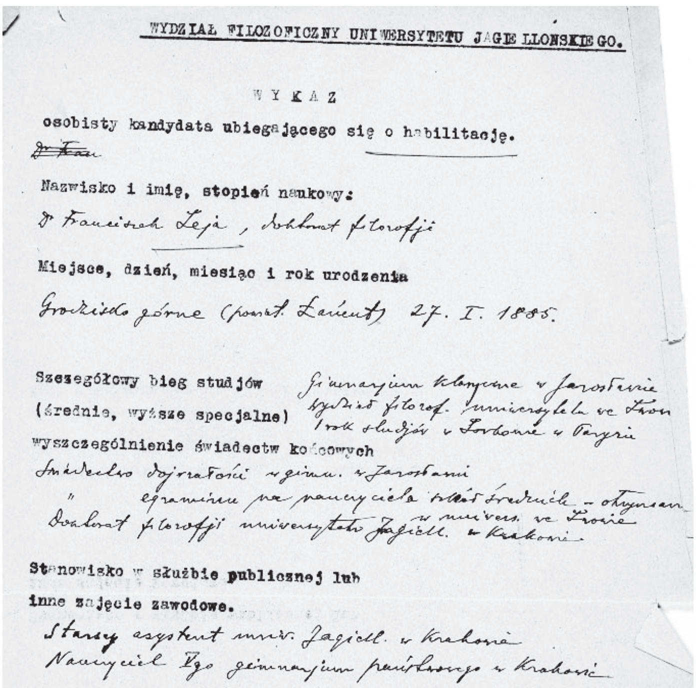
W Krakowie dnia 25. maja 1920

Dr Franciszek Leja
asystent przy katedrze matematyki

Franciszek Leja's CV (from Archive of the Jagiellonian University).

In his Curriculum Vitae from 1920 he mentioned that he studied in gymnasium in Jarosław in the years 1897–1904 and passed the final school examination. In 1904–1908 he studied at the Lvov University, where he listened to mathematics, physics and proper philosophy lectures. In the 1908–1909 academic year he was a student of the Faculty of Law at the Lvov University. In 1910 he passed the exam for teachers of mathematics and physics in secondary schools before the Examination Committee in Lvov. The Committee cooperated closely with the university. This example shows that the University played a central role in not only scientific but also educational, legal, medical and other matters.

The Chronicle of the Lvov University[21] noticed that the Examination Committee for the candidates of the teaching profession in gymnasia and real schools examined candidates who partially or fully completed their studies at the Faculty of Philosophy at the University. Examiners were mostly university professors, the Commission's headquarters were located in the building of the university.



WYDZIAŁ FILOZOFICZNY UNIWERSYTETU JAGIELLONSKIEGO.

WYKAZ

osobisty kandydata ubiegającego się o habilitację.

~~Dr Fran~~

Nazwisko i imię, stopień naukowy:

Dr Franciszek Leja, doktorat filozofji

Miejsce, dzień, miesiąc i rok urodzenia

Grodzisko górne (powiat Łańcut) 27. I. 1885.

Szczegółowy bieg studjów (średnie, wyższe specjalne)

Gimnazjum klasyczne w Jarosławiu
wydział filozof. uniwersytetu we Lwow
1 rok studjów w Sorbonie w Paryżu

wyszczególnienie świadectw końcowych

Świadectwo dojrzałości w gimn. w Jarosławiu
" egzaminu na nauczyciela szkół średnich – otrzymane w univers. we Lwowie
Doktorat filozofji uniwersytetu Jagiell. w Krakowie

Stanowisko w służbie publicznej lub inne zajęcie zawodowe.

Starszy asystent univ. Jagiell. w Krakowie
Nauczyciel Vgo gimnazjum państwowego w Krakowie

Here is F. Leja's register of achivements when tried to get the habilitation. Among others he mentioned the exam for teachers of mathematics and physics in secondary schools and his PhD (from Archive of the Jagiellonian University).

[21] See *Kronika Uniwersytetu Lwowskiego II* [The chronicle of Lvov University], (1898/9–1909/10), Lvov 1912.

One can see how difficult the exam was on the basis of the statistical data from the late nineteenth century and early twentieth century.

Years	Number of candidates who took the examination	Number of approved candidates	Number of failed candidates
1898/99	100	28	4
1899/1900	78	17	6
1900/01	78	20	6
1901/02	81	15	3
1902/03	87	25	3
1903/04	142	29	3
1904/05	151	50	11
1905/06	188	43	11
1906/07	274	71	20
1907/08	344	68	64
1908/1909	430	86	64
1909/10	444	86	62

Every year there were more and more candidates for teachers who took this examination. It was difficult and some retook it several times. Among 538 teachers who passed the examination in the years 1898–1910 94 obtained the right to teach mathematics and physics, 21 could teach mathematics and geometry, 1 person – mathematics. Let us note that 398 candidates were approved to teach in Polish, 14 – in Ukrainian, 66 – in Polish and Ukrainian, 39 – in Polish and German, 4 – in Polish and French, 9 – in Polish, Ukrainian and German, 2 – in Ukrainian and German, one – in German and French. It is worth adding that less than ten women were on the list of approved teachers in this period. To see how the training of teachers was held we will look at individual cases. We will start with the mathematical study of Franciszek Leja on the Philosophy Department at the Lvov University.

1.7. Scientific Publications of mathematics teachers.

A great importance in Galicia was attached to education of gymnasium teachers. There was not only a need for a sufficient number of teaching staff, but primarily for its high quality. Since 1856, university education was required from teachers of secondary schools. It was believed that the education of youth would not improve, though even the best reforms were carried out and the best plans for teaching were prepared, if those who execute them are not adequately prepared for the tasks they face. In a society it was thought that the teacher should be familiar with everything to be able to inform the student, explain and help in every situation.

The primary duty of the candidate for a gymnasium teacher was to take a course of four years of university study: five semesters at the Faculty of Philosophy and three at the same or different faculty, associated with the selected course. Students of foreign philology who stayed half a year abroad, where they mastered the language, could include it in the study period as two semesters. Two years of study for teachers of mathematics, geometry and physics, and three years of study for chemistry teachers were included at the Polytechnic, at the faculties of engineering, civil engineering, mechanical engineering and chemistry. Two more compulsory years they had to spend at the Philosophy Department. Not only a thorough specialized education, but vast general knowledge was required from teachers. Each future teacher was required to pass exams on lectures on philosophy, psychology and pedagogy since 16th century. They should also have attended classes of teaching methodology of the selected subject, school hygiene, physical education and language in which they were going to teach at school. The history of philosophy, history of education, history of schools in Europe and America had a great popularity among students. Graduation from the University was a prelude to work as a gymnasium teacher. Those who wanted to work, had to pass the exam before the c.k. Scientific Examination Committee for Candidates for Secondary School Teachers. It was possible to apply for admission to the examination on the fourth year of studies. Examining Board at each university was appointed by the Minister of Religion and Enlightenment. The first committees were already operating in 1850. Members of the examination were nominated and appointed by the minister. The makeup of the committee was according with the study subjects in a secondary school. Committees consisted of approximately 20 people. Candidates had to take a difficult multi-step test. If they were up to all preliminary requirements, the commission determined the topics of home works from the main and collateral subject. It was also necessary to prepare a written essay of a general philosophical or pedagogical nature or on the specific teaching. Philosophical work was to be written so as to convince the committee that the acquired philosophical education is sufficient for proper understanding and transferring relation of studied subject to other subjects to students in the future process of teaching.

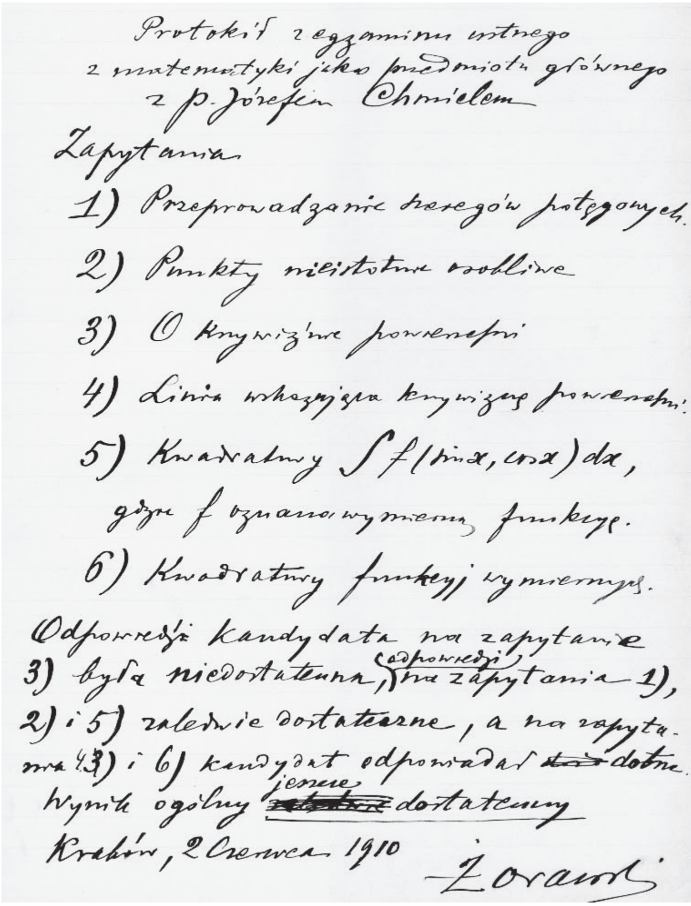
The idea was that the student realized that all the sciences are really closely linked and mutually support and complement themselves. It was very important to understand that no department of science is self-contained, that the discoveries and achievements in one discipline of knowledge are an inspiration and justification for discoveries in the other and working together helps to understand the phenomena of the universe and our role in it. Such understanding of educational goals was to prevent the teachers from concentrating in one discipline of knowledge and equip them intellectually so that they could integrate with and have influence on the educational process. Candidates had three months for writing each essay and this period could be extended for another six months. Most candidates used that possibility. Compulsory dissertations could be supplemented with additional publications and theses, doctoral dissertations, etc. that the committee assessed together. After successful assessing by committee, a future teacher took so-called enclosed examination, which had to be held no later than two years after receiving the home works.

During this examination the candidate had to demonstrate perfect mastering of the material in a particular subject, without using any auxiliary materials. It was not about the details of memory but a great understanding and knowledge of the problems. Enclosed examination of the main subject lasted eight hours, and on collateral course 4 hours. During the exam the committee did not allow any extenuating circumstances and was not in the habit of giving higher marks. It happened that only 15 people passed the exam out of the 54 people who came to the test.

Passing the exam allowed the candidate without experience to undertake so called “extended trial year” in one of the secondary schools that had the public rights. The duties of a student were the inspections of professors’ lessons amounting from 12 to 16 hours per week. After some time the trainee tried to teach independently under the complete control of the leading teacher, and if so called “teaching performances” were satisfying, the trainee could conduct up to two lessons a week, but still under the strict teacher’s supervision. After half a year the same candidate could begin to teach autonomously. Once a week there were meetings for training teachers taking care of trainees. During the year of practice the trainee did not receive a salary. He could receive an allowance, the salary was paid only if he conducted at least 6 hours per week of his own lessons. Obtaining a full teaching qualification certified to work in gymnasium was not an easy task.

The candidate had to possess an extensive specialized and general knowledge, be well-read, demonstrate a broad intellectual horizons and high intelligence. It cannot therefore be surprising that the average age of candidates passing qualifying exams was about 30. Another problem was the cost of exams - the total cost was approximately 100 crowns. For each repeated exam one had to pay an additional fee. In addition, young teachers were obliged to participate in various training courses. The aim of these courses was to familiarize teachers with the progress in the various fields of knowledge. Professors, lecturers and assistants at universities in Cracow and Lvov conducted the lessons. Courses were of a very high level and had great popularity. The cost of meals and accommodation were paid for the commuting teachers. The salaries of teachers and headmasters of gymnasium as well as civil servants were quite high, and these positions were viewed as lucrative. At the end of the 19th century and early 20th century, the headmaster earned with all bonuses around 600 crowns, and a teacher with full qualifications about 450, which was quite a big amount at that time.

Below we set a copy of oral examinations of the Commission in Cracow, put by professor of Jagellonian University Kazimierz Żorawski to Józef Chmiel[22] – later one of the founders of Mathematical Society in Kraków, which since 1920 is called the Polish Mathematical Society.


Protokół z egzaminu ustnego
z matematyki jako przedmiotu głównego
z p. Józefem Chmielem

Zapytania

1) Przeprowadzanie szeregów potęgowych.
2) Punkty nieistotne osobliwe
3) O krzywiźnie powierzchni
4) Linia wskazująca krzywiznę powierzchni.
5) Kwadratury $\int f(\sin x, \cos x)dx$, gdzie f oznacza wymierną funkcyę.
6) Kwadratury funkcyj wymiernych.

Odpowiedź kandydata na zapytanie 3) była niedostateczna, odpowiedzi na zapytania 1), 2) i 5) zaledwie dostateczne, a na zapytania 4) i 6) kandydat odpowiadał ~~[illegible]~~ dobrze.
Wynik ogólny ~~[illegible]~~ jeszcze dostateczny
Kraków, 2 Czerwca 1910
Żorawski

UJ Archive, personal folder of J. Chmiel. Questions from the professor Żorawski to the examined candidate on teacher. Let us note that advanced knowledge, including geometry and analysis was required from the trainee teachers.

[22] Julian Józef Chmiel was born 16 February 1881 in Zagorzyce in ropczycki region, finished gymnasium in Tarnopol in 1902, studied at the Philosophical Faculty of the Jagiellonian University. 1902/03–1905/06. In 1910 he passed the exams for teacher of mathematics and physics.

He studied at the Teacher Seminary in Kęty, in Gymnasium of St. Jack in Cracow (Report 1913/14), in 1915 he was called up for military service, since 1917 he was the teacher In II Real School in Cracow. Since 1921/22 school year. He was the headmaster of State Gymnasium named after Hetman Karol Chodkiewicz in Lida.

Franciszek Leja (1885–1979) mentioned above, published *Pierwsze zasady geometryi nieeuklidesowej* [First Principles of non-euclidean Geometry] as a teacher. Reports of c.k. Management of Gymnasium IV (real) in Cracow for 1910–1911 school year.

This is one of the best works published in the Reports. In this work, Leja lists six Euclidean postulates. Here they are:

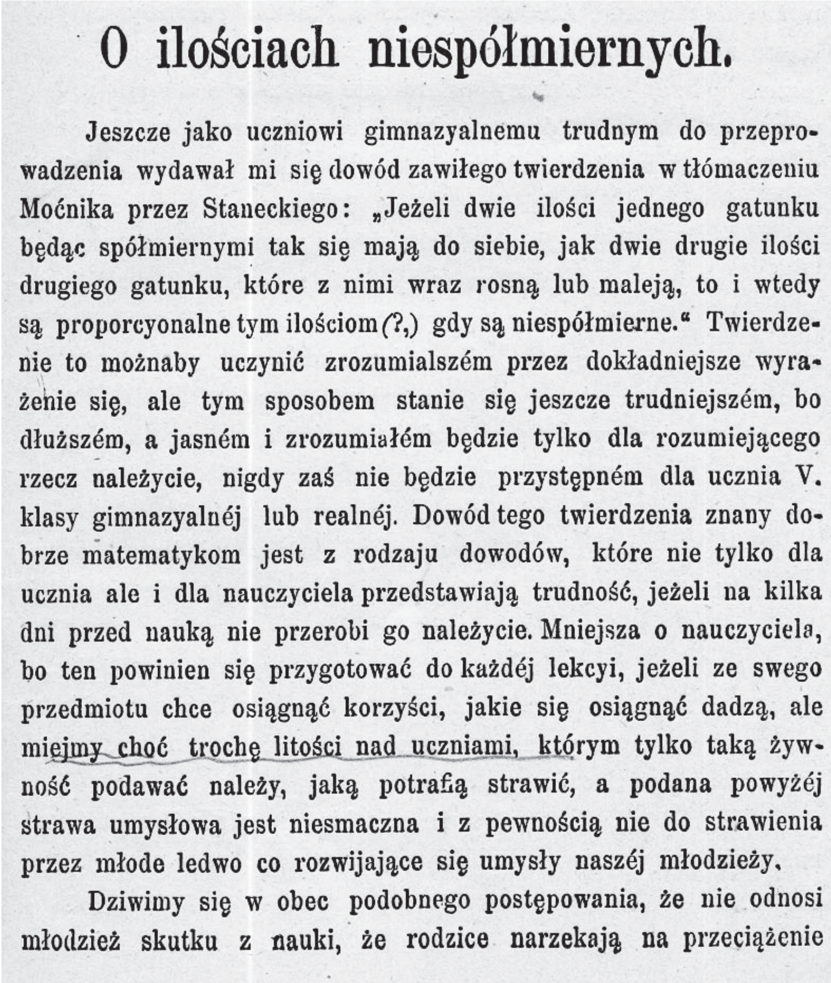
1. To draw a straight line from any point to any point."
2. "To produce [extend] a finite straight line continuously in a straight line."
3. "To describe a circle with any centre and distance [radius]."
4. "That all right angles are equal to one another."
5. The parallel postulate: "That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles."
6. Two lines do not enclose the space.

There are several editions of Elements not completely compatible with each other, hence the discrepancy in the number of postulates. This 6th postulate exists for example in a J. L. Heiberg's publication, Germany – Leipzig 1883. German editions of Elements were not widespread in Poland. The question of validity of the 6th postulate is associated with the so-called concept of the curvature of space introduced by Riemann. Ideas expressed by Riemann were developed by his successors who rejected the postulate VI and formed geometry as logical as the Lobachevsky's or Euclides'. The School Reports present mathematical work of teachers from Western Galicia, which was under the Lvov University[23] impact. Teachers published in the School Reports, theme and presentation of issues authorized to conclude that the mathematical culture of this period is largely due to gymnasium teachers.

Mieczyslaw Arndt (born 1880), *Nauka trygonometrii w klasie IV* [Trigonometry studies in the IV class], Reports of the Executive Branch c.k. Gymnasium in Stryj for1913/14 school year.

Jozef Balon (1849–1885), *O ilościach niewspółmiernych* [On non-commensurable quantities], Reports of Headmastr of c.k. Gymnasium in Jasło for 1882/83 school year.

[23] For a more complete list see S. Domoradzki, *Prace matematyczne w sprawozdaniach gimnazjów galicyjskich,* [Mathematical works in annual reports of gymnasias in Galicia], Antiquitates Mathematicae 3(2009), pp. 243–261.



O ilościach niespółmiernych.

Jeszcze jako uczniowi gimnazyalnemu trudnym do przeprowadzenia wydawał mi się dowód zawiłego twierdzenia w tłómaczeniu Moćnika przez Staneckiego: „Jeżeli dwie ilości jednego gatunku będąc spółmiernymi tak się mają do siebie, jak dwie drugie ilości drugiego gatunku, które z nimi wraz rosną lub maleją, to i wtedy są proporcyonalne tym ilościom (?,) gdy są niespółmierne.“ Twierdzenie to możnaby uczynić zrozumialszém przez dokładniejsze wyrażenie się, ale tym sposobem stanie się jeszcze trudniejszém, bo dłuższém, a jasném i zrozumiałém będzie tylko dla rozumiejącego rzecz należycie, nigdy zaś nie będzie przystępném dla ucznia V. klasy gimnazyalnéj lub realnéj. Dowód tego twierdzenia znany dobrze matematykom jest z rodzaju dowodów, które nie tylko dla ucznia ale i dla nauczyciela przedstawiają trudność, jeżeli na kilka dni przed nauką nie przerobi go należycie. Mniejsza o nauczyciela, bo ten powinien się przygotować do każdéj lekcyi, jeżeli ze swego przedmiotu chce osiągnąć korzyści, jakie się osiągnąć dadzą, ale miejmy choć trochę litości nad uczniami, którym tylko taką żywność podawać należy, jaką potrafią strawić, a podana powyżéj strawa umysłowa jest niesmaczna i z pewnością nie do strawienia przez młode ledwo co rozwijające się umysły naszéj młodzieży.

Dziwimy się w obec podobnego postępowania, że nie odnosi młodzież skutku z nauki, że rodzice narzekają na przeciążenie

The article on non-commensurable quantities deals with one of the most difficult notion of the high school mathematics. Fragment of the first page of the work of J. Balon.

Józef Balon(1849–1885), *O rachunku procentowym* [On calculation of percentages], ibidem, 1884/85.
Szymon Blader (ur. 1887), *Własności biegunowe krzywych algebraicznych i ich zastosowanie w teoryi tych krzywych* [Polar properties of algebraic curves and their theoretical application], Management Reports of c.k. Higher Real School in Krosno for 1913/14 school year.
Bolesław Błażek (ur. 1872), *Szczegółowe plany matematyki w gimnazyum według nowych planów ministeryalnych z r. 1909* [Detailed curriculums of mathematics in gymnasium according to the new ministerial programs], Management Reports

of c.k. Gymnasium with Polish language of teaching in Przemyśl for 1910/11 school year.

Nowe plany ministeryalne z r. 1909 do nauki matematyki w gimnazyum wprowadzają zmiany radykalne tak pod względem wymiaru i rozłożenia materyału, z którego usunięto wiele przestarzałego balastu, zastępując go nowymi, dotychczas zaniedbanymi działami nauki, jak i pod względem sposobu jego ujęcia i metodycznego rozkładu. Położono wyraźniejszy, aniżeli dotychczas nacisk na zastosowanie się do umysłowego rozwoju ucznia, przez ścisłe przestrzeganie zasady, iż jeżeli nauka ma przynieść mu faktyczną korzyść rozwojową, nie śmie wybiegać, czyto w sposobie podawania pewnych problemów, czyto w ich doborze przed naturalną możność ich umysłowego ujęcia. W przeciwnym razie musi zejść na mechaniczną reprodukcyę dźwięków, poza którymi albo zgoła nie będzie żadnej treści, albo w znacznie zmiejszonym wymiarze.

Zewnętrzny objaw nowości stanowi odmienny podział całości nauki na trzy stopnie, niższy, średni i wyższy. Ma on być niejako odbiciem zasadniczych etapów rozwojowych tej żywej jednostki, jaką jest szkoła. Gdyby więc chodziło o scharakteryzowanie tych stopni, to powinne one być równoczesną charakterystyką rozwoju umysłu od wieku dziecięcego do młodzieńczego. Ponieważ zaś każdy z okresów utworzonych nowym planem trwa dwa lub trzy lata, charakterystyka każdego roku nauki, będzie wyspecyalizowaniem ogólnej charakterystyki okresów. Dobór więc materyału, a przedewszystkiem metodyczne postępowanie przy jego udzielaniu, ma tworzyć nieprzerwany z całością wątek treści, zapewniający powodzenie nauce przez dokładne przywłaszczenie sobie przez uczniów tego wszystkiego, co w pewnej klasie stanowi jej przedmiot.

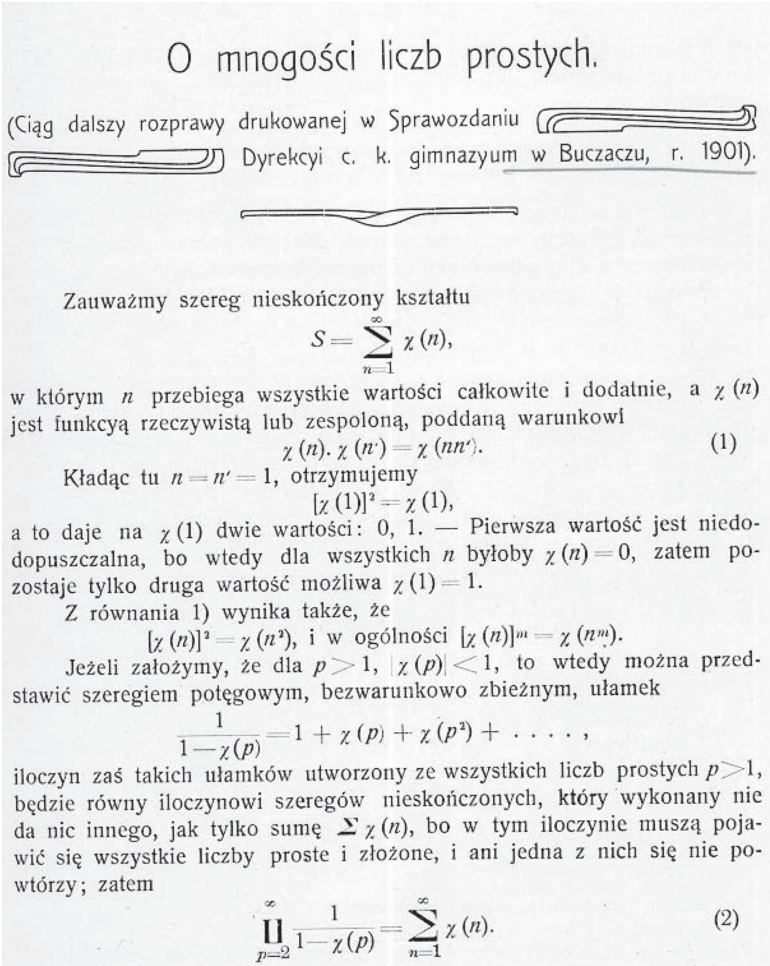
Dla umysłu rozpoczynającego poznawanie, a więc dla dziecka do pewnego wieku istnieje przedewszystkiem czyste postrzeganie, interesowanie się otoczeniem, w zakresie wypełniających je rzeczy i dziejących się w niem zjawisk. Bardzo powoli i bardzo niechętnie zostaje świat zjawisk i rzeczy zastępowany czystymi tworami umysłu, których tworzenie, a raczej zabiegi i usiłowania tworzenia, są znamieniem dojrzewającego, pełne zaś i swobodne nimi władanie, znamieniem dojrzałego umysłu.

I to stanowi zasadę trójstopniowości nauki matematyki w gimnazyum. Oczywiście ścisłe stosowanie się do tego naturalnego podziału jest możliwe tylko przy nauce jednostkowej i przy zupełnem nadto niekrępowaniu się wymiarem zużytego czasu. Wiedy możliwem jest niegwałcenie umysłu t. j. uwzględnienie jego chwilowego stanu przez odpowiedni i ściśle dostosowany dobór i wymiar nauki. Postępuje się wówczas za rozwijającym się umysłem w pełnej gotowości ofiarowania mu tego, czego on w danej chwili pożąda.

A discussion concerning programs in mathematics. The Ministry proposed radical changes.

Kazimierz Bryk (1847–1891), *O najodpowiedniejszych tablicach logarytmicznych przy nauce matematyki w szkołach realnych* [On the best logaritmic tables for teaching mathematics in real schools], Management Reports of c.k. Higher Real School in Jarosław for 1886/87 school year.
Józef Czaczkowski, *O nauce matematyki w wyższem gimnazyum* [About teaching mathematics in higher gymnasium], Management Reports of c.k. Higher Gymnasium in Brzezany for 1878/79 school year.
Karol Czajkowski (1873–1931), *O mnogości liczb prostych* [On the set of prime

numbers], Management Reports of c.k. Gymnasium in Buczacz for 1900/01 school year.



O mnogości liczb prostych.

(Ciąg dalszy rozprawy drukowanej w Sprawozdaniu Dyrekcyi c. k. gimnazyum w Buczaczu, r. 1901).

Zauważmy szereg nieskończony kształtu

$$S = \sum_{n=1}^{\infty} \chi(n),$$

w którym n przebiega wszystkie wartości całkowite i dodatnie, a $\chi(n)$ jest funkcyą rzeczywistą lub zespoloną, poddaną warunkowi

$$\chi(n).\,\chi(n') = \chi(nn'). \qquad (1)$$

Kładąc tu $n = n' = 1$, otrzymujemy

$$[\chi(1)]^2 = \chi(1),$$

a to daje na $\chi(1)$ dwie wartości: 0, 1. — Pierwsza wartość jest niedodopuszczalna, bo wtedy dla wszystkich n byłoby $\chi(n) = 0$, zatem pozostaje tylko druga wartość możliwa $\chi(1) = 1$.

Z równania 1) wynika także, że

$$[\chi(n)]^2 = \chi(n^2), \text{ i w ogólności } [\chi(n)]^m = \chi(n^m).$$

Jeżeli założymy, że dla $p > 1$, $|\chi(p)| < 1$, to wtedy można przedstawić szeregiem potęgowym, bezwarunkowo zbieżnym, ułamek

$$\frac{1}{1-\chi(p)} = 1 + \chi(p) + \chi(p^2) + \ldots\ldots,$$

iloczyn zaś takich ułamków utworzony ze wszystkich liczb prostych $p > 1$, będzie równy iloczynowi szeregów nieskończonych, który wykonany nie da nic innego, jak tylko sumę $\Sigma\,\chi(n)$, bo w tym iloczynie muszą pojawić się wszystkie liczby proste i złożone, i ani jedna z nich się nie powtórzy; zatem

$$\prod_{p=2}^{\infty} \frac{1}{1-\chi(p)} = \sum_{n=1}^{\infty} \chi(n). \qquad (2)$$

The first page of Karol Czajkowski’s article. The title of the article is “On the set of prime numbers”

The author briefly presented the overall research on the famous question: how many simple numbers (prime – *my note*) are located in a finite segment of the natural series of numbers. He used the works of Euclid, Legendre, Chebyshev, Mertens, Lipschitz, Riemann and others. It is worth noting that using the Mertens function, he proved the Lipschitz theorem, and presented Legendre’s formula on prime numbers contained between $\sqrt{x}$ and x.

Karol Czajkowski, *O mnogości liczb prostych,* [On the set of prime numbers], Management Reports of c. k. Gymnasium in Przemyśl for 1903/04 school year.

This dissertation is another part of the mentioned above dissertation.

Placyd Dziwiński (1851–1936), *Ogólne zrównanie walców i stożków stycznych do dowolnej powierzchni drugiego stopnia. W układzie ukośnokątnym na podstawie symbolów profesora dr Żmurki* [General comparison of tangent cylinders and cones with any second degree surface. In oblique system, according to dr Żmurka's symbols], Management Reports of Higher Real School in Jarosław for 1879/80 school year.

Placyd Dziwiński (1851–1936), *Liczby kierunkowe, ich znaczenie i zastosowanie w matematyce* [Directional numbers, their meaning and application in mathematics], Management Reports of c.k. Higher Real School in Jarosław for 1881/82 school year.

The author made and introduction into the theory of complex numbers in an accessible manner, illustrated by numerous examples. The author used the mark a_{α}.

Placyd Dziwiński (1851–1936), *Prawidła podzielności liczb na podstawie teoryi liczb przystających* [The rules of divisibility of numbers according to the theory of congruence of numbers], Reports of Higher Real School in Lvov for 1885 /86 school year.

The author introduces the features of divisibility numbers using the method of congruence (then the functioning name of congruence was "adaptation").

Julian Fąfara (2nd half of the XIX century): *Historyczny zarys matematyki u starożytnych. Część I do Euklidesa* [Historical essay of Mathematics in antiquity, part I before Euclid], Reports of c.k. Real School in Tarnopol for 1882/83 school year.

Zdzisław Fialka (born 1850), *Über einige mit der Schraubenlinie in Zusammenhang stehende krumme Linien.* Zehnter Jahresbericht des k.k. Real- und Ober-Gymnasiums in Brody für das Schuljahr 1888. Brody 1888.

Antoni Filipowski (184–1895), *O linii Cassini'ego* [On Cassini's curve], Reports of c.k. Gymnasium named after Fr. Joseph in Lvov for 1890/91 school year.

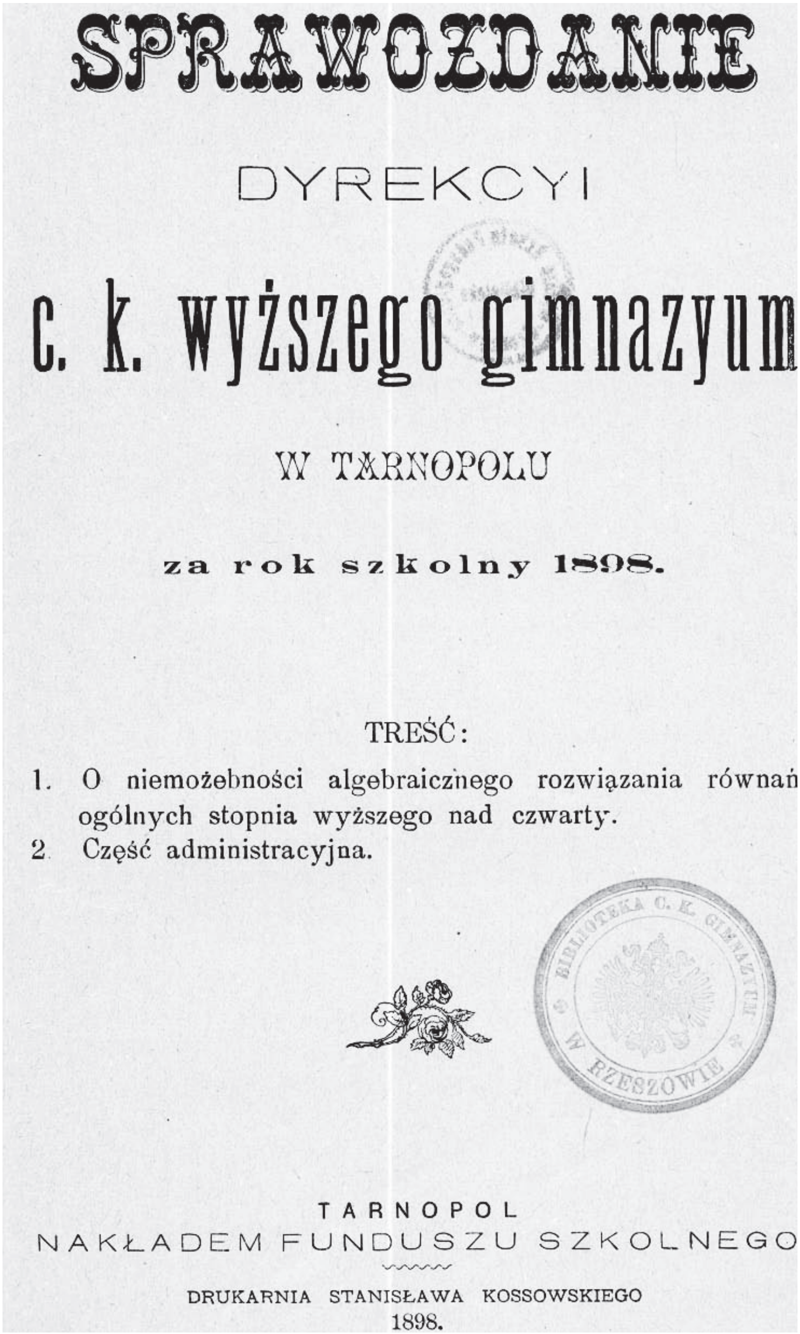
The author presented a monograph on the curve of the fourth degree called the Cassini's line. This line is the geometric place for which the product of distances from two fixed points is constant. He presented the line equation in Cartesian system and the polar coordinate system and examined the shapes of this line in individual cases

Antoni Giedroyć (1841–1909), *Wskazówki dla początkujących do ustawiania równań* [Beginner's tips for equations setting], Management Reports of c.k. Higher Real School in Tarnopol for 1886/87 school year.

Antoni Giedroyć, (1841–1909), *Zestawienie własności figur na powierzchni sfery* [Listing of the figures properties on the sphere surface] ibidem, 1882/83, 1883/84, 1884/85 – work in three reports.

Antoni Giedroyć, (1841–1909), O *metodycznym traktowaniu geometrii elementarnej* [On methodical treatment of elementary geometry], ibidem, 1902/3, 1903/4 – work in two reports.
Kajetan Golczewski, *O funkcjach hiperbolicznych,* [On hyperbolic functions] Management Reports of c.k. Gymnasium in Sanok for 1902/1903 school year.
Grzegorz Grzybowski (1835–1899), *Początki o powierzchniach skośnych (spaczonych). Przyczynek do nauki wykreślnej geometrii w szkole realnej.* [Preliminaries on oblique (warped) surfaces. Contribution for solid geometry learning in real school], Management Reports of c. k. Real School in Tarnopol for 1889/90 school year.
Grzegorz Grzybowski, (1835–1899), *O dotykaniu powierzchni spaczonej obrotowej*, ibidem, [On touching an oblique rotating surface] 1890/91 school year.
The author showed the methods of conducting the tangent plane to the surface of the helix.
Grzegorz Grzybowski (1835–1899), *O dotykalności powierzchni obrotowej,* [About rotating surface's tangibility], ibidem, 1891/92 school year.
Klemens Hlibowicki (1875–1907), *O niemożebności algebraicznego rozwiązania równań ogólnych stopnia wyższego nad czwarty* [On impossibility of solving general equations of degree greater than four], Management Reports of c.k. Higher Gymnasium In Tarnopol for 1897/98 school year.

The author refers in this paper on the Abel work (Oeuvres complètes, v. II), the work of Galois and also quotes Crelle Journal. He introduces the concept of the symmetric group. The conclusion of this article is the statement: The general equation of degree n, $n>4$ cannot be solved algebraically.



SPRAWOZDANIE

DYREKCYI

c. k. wyższego gimnazyum

W TARNOPOLU

za rok szkolny 1898.

TREŚĆ:

1. O niemożebności algebraicznego rozwiązania równań ogólnych stopnia wyższego nad czwarty.
2. Część administracyjna.

BIBLIOTEKA C. K. GIMNAZYUM W RZESZOWIE

TARNOPOL
NAKŁADEM FUNDUSZU SZKOLNEGO
DRUKARNIA STANISŁAWA KOSSOWSKIEGO
1898.

The first page of article *On impossibility of solving general equations of degree greater than four* by Klemens Hlibowcki.

O niemożebności algebraicznego rozwiązania równań ogólnych stopnia wyższego nad czwarty.

NAPISAŁ

KLEMENS HLIBOWICKI.

Po świetnych rezultatach, jakimi uwieńczyły się prace włoskich matematyków na polu rozwiązywania ogólnych równań 3-go i 4-go stopnia, myślał Ferro, Tartaglia, Cardano, Ferrari i inni współcześni, że metoda stosowana dotychczas da się użyć i do rozwiązania równań wyższych stopni, a w pierwszym rzędzie doprowadzi do rozwiązania ogólnego równania piątego stopnia. Ale upłynęło stulecie całe, a kwestya ani krokiem naprzód nie postąpiła. Dopiero w r. 1683 wyszła w „Acta eruditorum" rozprawa Tschirnhaus'a pod tytułem: „Methodus Auferendi Omnes Terminos Intermedios Ex Data Equatione", która zdaniem autora mogła być stosowaną do równań ogólnych wszelkich stopni.

Tschirnhaus umieścił w rozprawie, jako przykład zastosowania swej metody rozwiązanie równania 3-go stopnia.

Metoda Tschirnhaus'a prowadzi rzeczywiście prosto i łatwo do rozwiązania równań 2 go i 3-go stopnia, ale już dla równań dwukwadratowych, robota jest mozolną, gdy tymczasem przy równaniu 5-go stopnia, jest wprost nie do pokonania, i to jest właśnie przyczyną, dlaczego Tschirnhaus nie próbował zastosować swej metody do tych równań, lecz zwyczajem ówczesnych matematyków drogą indukcyi udowodnił prawdziwość swego twierdzenia. Tyle w XVII. stuleciu — W XVIII. natomiast cały szereg uczonych podejmuje na nowo niewdzięczną pracę około rozwiązania wspomnianych równań. Euler, Bézout, Lagrange, Vandermonde i Malfatti poświęcili się ścisłemu badaniu tej kwestyi, ale ich badania, chociaż nie bez wartości, nie doprowadziły do celu. Zaczęto wątpić w skuteczność metody

The initial pages provide a historical outline concerning solving algebraic equations. The author presents Abel's proof, and using some its fragments, provides an exposition of Ruffini's proof.

Klemens Hlibowicki (1875–1907), *Integruvannia rivnan' rižničkovich peršoho rjadu v točkach osoblivich,n-kratnich* [Integration of first order differential equations at singular points of multiplicity *n*]. Management Reports of II c.k. Gymnasium in Peremišl for 1898/9 school year.
Ludwik Hordyński, (1882–1920), *Podstawowe twierdzenie rachunku całkowego* [Fundamental theorem of integral calculus], Reports of Management of II imperial Real School in Lvov 1912/13 school year.

Podstawowe twierdzenie rachunku całkowego

napisał

Dr. Ludwik Hordyński.

The title of the dissertation *Fundamental theorem of integral calculus* by dr Ludwik Hordyński.

I jeszcze cecha wspólna. Najdrobniejsze zdobycze naukowe, jak i życiowe, stają się niekiedy odkryciami epokowemi, rozwiązującemi problematy, nad któremi ludzkość całe wieki się męczyła. Tu za przykład dominujący może służyć rozwiązanie t. zw. kwadratury koła, która przeszło 20-cia wieków zaprzątała umysły wybitnych matematyków. Wskutek przestępności liczby π zagadnienie to jest nierozwiązalne. A czy sam Liouville przypuszczał, że wykazanie istnienia liczb takich, które nie są pierwiastkam równania algebraicznego, zapoczątkuje dowód Hermite'a przestępności liczby e i dowód Lindemann'a przestępności liczby π?

In the introduction the author speculates on some mathematical problems, whose solution turned out to be an intellectual challenge to the humans.

In the introduction, the author ponders upon mathematics achievement. The author talks about the squaring of the circle, which occupied the minds for more than 20 centuries, in the context of the Liouville transcendental numbers.

The author introduces the concept of a definite integral, as well as gives its numerous applications, for example, he shows how the set theory influences understanding and deepens the definition of definite integral.

In the introduction we read what is important for the history of mathematics in Poland:

In Poland the first one who introduced the foundations of the set theory, which for many years started every textbook of the analysis, was dr Józef Puzyna. In the two-volume classical work *Teorya funkcyj analitycznych* [Theory of analytic functions] (Lvov) 1898 he included a whole chapter on the set theory. S. Dickstein also dealt with it in the work *Pojęcia i metody matematyki* [Concepts and methods of mathematics] (Warsaw 1891). The previous year dr Wacław Sierpiński's *Zarys teoryi mnogości* [The Outline of the set theory] appeared (Warsaw 1912), in which in a concise, clear and complete way he introduced everything that had been done till that moment in this branch of knowledge. There is lack of applications to analysis and geometry in this otherwise valuable book, which the author explains in the preface by "the variety" of all these spheres of mathematics. We believe that that was the time that achievements of set theory made an impact onto wider public.

Supporters of the set theory (who can be named with great thoroughness and accuracy), which this study introduces to analysis and geometry, overestimate its value to the disadvantage of other achievements and contributions to mathematics. On one hand,set theory made a lot of issues intelligible, gave them a real scientific rigor, article on the other hand it entered the field of pure philosophy through many postulates it put forward. The mathematical set theory will yield the power of accessibility and will be again an excellent building block for creative mathematical structure. So, in spite of gaining wider rights in the areas of pure mathematics, the theory should not find fault with heuristic methods of exploring certain parts of mathematical knowledge based on certain assumptions, as the way is required by the time economy and consideration of practical applications. And even a smallest question of this theory may in time contribute to the epoch-making discoveries! Therefore, H. Poincaré defended it, being a follower of the great principle of *"science for science"* (*The value of science*, translated by Silberstein, Warsaw, 1908). We believe that it is the time for the achievements of the set theory to penetrate generally wider.

Below we present fragments of a work in order to emphasize the introduction of the new branch of mathematics – set theory – to the awareness of teachers and students.

bierzemy pod uwagę zera. Mnogości bowiem liczb dodatnich i ujemnych są właściwie nieograniczone z góry i z dołu; w elementarnej arytmetyce przyzwyczailiśmy się uważać zero (0) i nieskończoność (∞) za granice ciągu nieskończonego liczb dodatnich. Są to jednak tylko *symbole,* które określić należy w następujący sposób:

Zero jest to granica nieskończonego ciągu liczb dodatnich nieskończenie malejących lub nieskończonego ciągu liczb ujemnych nieskończenie rosnących.

Nieskończoność dodatnia ($+\infty$) jest to granica nieskończonego ciągu liczb dodatnich nieskończenie rosnących.

Nieskończoność ujemna ($-\infty$) jest to granica nieskończonego ciągu liczb ujemnych nieskończenie malejących.

Obie granice dokładniej określimy według Baire'a (Leçons sur les Théories générales de l'analyse — Paris G. V. 1907):

Górna granica M ma następujące własności:

1. Każda liczba mnogości (E) jest mniejsza lub co najwyżej równa M (E).

2. Jeżeli weźmiemy pod uwagę dowolną liczbę

$$\lambda < M,$$

to w mnogości (E) istnieje jeszcze liczba, która jest większa od λ.

Powiemy zatem:

Górna granica mnogości liczb jest to liczba najmniejsza ze wszystkich liczb większych lub co najmniej równych liczbom mnogości.

Z tego określenia wynikają następujące wnioski:

W mnogości ograniczonej istnieje tylko jedna górna granica.

Jeżeli mnogość zawiera liczbę, większą od wszystkich liczb mnogości, to ta liczba jest jej górną granicą.

Jeżeli mnogość E_1 jest zawarta w mnogości E, to jej górna granica jest mniejsza lub co najwyżej równa górnej granicy mnogości E, co piszemy:

$$M(E_1) \leq M(E)$$

Jeżeli wszystkie liczby mnogości E są mniejsze lub co najwyżej równe dowolnej liczbie A, to górna granica tej mnogości musi być mniejsza lub co najwyżej równa tej liczbie A.

$$M(E) \leq A$$

Dowolna liczba λ jest górną granicą mnogości wszystkich liczb, mniejszych od λ.

In the modern language, the author proves that every bounded set on a real line possesses its lowest upper bound.

Dolna granica m ma własności analogiczne:

1. Każda liczba mnogości (E) jest większa lub co najmniej równa m (E).

2. Jeżeli weźmiemy pod uwagę dowolną liczbę

$$\lambda > m,$$

to w mnogości istnieje jeszcze liczba, która jest mniejsza od λ.

Powiemy zatem:

Dolna granica mnogości jest to liczba największa ze wszystkich liczb mniejszych lub co najwyżej równych liczbom mnogości.

Z tego określenia wynikają następujące wnioski:

W mnogości ograniczonej istnieje tylko jedna dolna granica.

Jeżeli mnogość zawiera liczbę, mniejszą od wszystkich liczb mnogości, to ta liczba jest jej dolną granicą.

Jeżeli mnogość E_1 składa się z liczb mnogości E, to dolna granica mnogości E_1 jest większa lub co najmniej równa dolnej granicy mnogości E, co piszemy

$$m(E_1) \geq m(E)$$

Jeżeli wszystkie liczby mnogości E są większe lub co najmniej równe dowolnej liczbie A, to dolna granica tej mnogości musi być większa lub co najmniej równa tej liczbie A:

$$m(E) \geq A.$$

Dowolna liczba λ jest dolną granicą wszystkich liczb większych od λ.

Mnogość ograniczoną z góry i z dołu t. zn. posiadającą górną i dolną granicę nazywamy krótko *mnogością ograniczoną*. Biorąc pod uwagę powyższe określenia, dowiedziemy teraz następujących twierdzeń:

Twierdzenie I. Jeżeli z dwu mnogości A i B, mnogość A ma tę własność, że każda jej liczba jest mniejsza od każdej liczby mnogości B, to górna granica mnogości A jest mniejsza lub co najwyżej równa dolnej granicy mnogości B, co piszemy:

$$M(A) \leq m(B)$$

Według założenia dowolna liczba mnogości B jest większa od dowolnej liczby mnogości A, czyli

$$b > a$$

a więc także $\quad b \geq M(A)$

Ponieważ według określenia dolnej granicy jest ona liczbą

An analogical arguments can be applied to the greatest lower bound.

największą ze wszystkich liczb mniejszych od liczb mnogości czyli

$$m(B) \geqq b$$

zatem także $m(B) \geqq M(A)$ c. b. d. u.

Twierdzenie II. Jeżeli mamy dwie mnogości A i B takie, że każda liczba mnogości A jest mniejsza od każdej liczby mnogości B i między liczbami mnogości A istnieją liczby nieskończenie się zbliżające do liczb mnogości B, to znaczy istnieje, przy dowolnie małej dodatniej liczbie ε, nierówność

$$b - a < \varepsilon$$

to wtedy górna granica mnogości A musi być równa dolnej granicy mnogości B, co piszemy:

$$M(A) = m(B)$$

Twierdzenie I. daje nam i w tym razie nierówność

$$M(A) \leqq m(B),$$

jeżeli zatem pierwsza część tej nierówności t. zn.

$$M(A) < m(B)$$

jest niemożliwa, nasze twierdzenie jest słuszne. W rzeczy samej z określenia granic wynika

$$a \leqq M(A) < m(B) \leqq b$$

stąd, biorąc pod uwagę mnogość liczb $b - a$, otrzymamy

$$b - a \geqq m(B) - M(A)$$

Możemy tu obrać dowolnie małą liczbę dodatnią ε taką, że

$$\varepsilon < m(B) - M(A)$$

a więc z ostatniej nierówności otrzymalibyśmy

$$b - a > \varepsilon$$

co się sprzeciwia założeniu, w którem

$$b - a < \varepsilon$$

Do tej sprzeczności doszliśmy, zakładając, że

$$M(A) < m(B)$$

przeto biorąc pod uwagę twierdzenie I., musi być

$M(A) = m(B)$ c. b. d. u.

If every element of a set A of reals is less than every element of a set B of reals, then the least upper bound of A is less than or equal to the greatest lower bound B.

Ludwik Hordyński (1882–1920), *Z podstaw rachunku wektorowego* [Elementary vectorial calculus], IV Management Reports of II c.k. Gymnasium in Rzeszów for 1907/08 school year.

Teodor Hrycak (ur. 1881), *Kwestye dydaktyki matematyki w szkole realnej* [Problems of didactics of mathematics in real school], Management Reports of c.k. Higher Real School in Stanisławów for 1910/11 school year.

The author discusses the concepts of function, continuity of functions, limit, derivative and differential. He gave the basic patterns and rules of differentiation and their application, solving of equations by the graphical method.

Roman Jamrógiewicz (ur. 1877), *O powierzchni Kummera* [About Kummer's surface], Management Reports of c.k. Gymnasium in Sanok for 1899/1900 school year.

Jan Jączek (ur. 1881), *O teoryach przewodzenia ciepła, matematyczna teorya przewodzenia ciepła* [Theories of heat conduction, mathematical theory of heat conduction], Management Reports of c.k. Gymnasium in Rzeszów for 1909/10 school year.

Stefan Juński (born 1877), *Miejsce geometryczne pewnego kompleksu kół* [Geometrical place of a certain complex of circles], Management Reports of c.k. Higher Real School in Tarnopol for 1909/10 school year.

The author talked over the chosen properties of cycloids i hipocycloids.

Augustyn Klimaszewski (ur. 1879), *Kilka zagadnień z teoryi liczb z uwzględnieniem Hoene – Wrońskiego* [Several problems in numbers theory including Hoene-Wroński], School Reports of Gymnasium SS. Urszulanek in Stanisławów for 1912/13 school year.

Augustyn Klimaszewski (ur. 1879), *Rozwiązywanie kongruencyi* [The solving of congruence], Management Reports of c.k. Gymnasium in Jarosław for 1913/14 school year.

Włodzimierz Kowalski *O powierzchniach stożkowych rzędu trzeciego* [On conical surfaces of the third order], Management Reports of Sprawozdania Dyrekcyi c.k. Wyższej Szkoły Realnej w Jarosławiu za r. sz. 1907/08.

Anton[i] Krygowski (1824–1904), *Konstruktion der trigonometrischen Tafeln*, Jahres-Bericht kaiserl. Königl. Ober-Gymnasiums, Tarnopol 1862

Antoni Krygowski (1824–1904), *Rozprawa o promieniu koła wykreślonego w trójkącie* [Dissertation about radius of the circle marked inscribed in the triangle], Jahres-Bericht des k.k. zweiten Ober-Gymnasiums in Lemberg für das Schul-Jahr 1872/73.

The author solves the exercise: find the equation of the circle inscribed in the triangle if it is given the side $AB = c$ and perimeter $c + x + y = 2S$.

Antoni Krygowski (1824–1904), *Zastosowanie twierdzenia Newtona do oznaczenia logarytmu jakiejś lizcby* [The use of Newton's theorem to determine the logarithm of some number], ibidem., 1874/75 school year.

Zdzisław Krygowski, (1872–1955), *O pewnym zastosowaniu funkcji theta* [On certain application of the theta function], Management Reports of c.k. Gymnasium I in Przemyśl, for 1899/90 school year.

O pewnem zastosowaniu funkcyi theta.

Wśród rozlicznych sposobów przekształcania jednych szeregów na drugie, metody wprowadzone przez Riemanna w teoryi funkcyi theta, wydoskonalone przez Pryma, Webera, Frobeniusa, Weierstrassa i innych, opierają się z jednej strony na stosowaniu tak zwanej zasady Hermite'a, z drugiej zaś strony na badaniu własności charakterystyk, z którymi istotne własności funkcyi theta są jak najściślej związane. Przekształcenia te stosować można i do szeregów pochodnych n. p. do takich, jakie można otrzymać przy pomocy metody Riemanna-Appella [1]); w tym też kierunku można uzyskać pewne uogólnienia i modyfikacye, które dotyczą pewnej szczególnej grupy charakterystyk

$$\begin{bmatrix} \nu_1, & \nu_2, \ldots, & \nu_r \\ 0, & 0, \ldots, & 0 \end{bmatrix},$$

tworzącej według terminu wprowadzonego przez Frobeniusa [2]) tak zwaną grupę Goepela.

1. Funkcya ogólna theta r zmiennych $\upsilon_1\ \upsilon_2 \ldots, \upsilon_r$, o dowolnej charakterystyce

$$A = \begin{bmatrix} \nu_a \\ \mu_a \end{bmatrix}, \quad (a = 1, 2, \ldots, r)$$

określona szeregiem

$$\Theta\left[A\right](\upsilon) = \sum_{(n_1, n_2, \ldots, n_r) = -\infty}^{+\infty} e^{2\pi i \sum\limits_{a=1}^{r} (\upsilon_a + {}^1/_2\mu_a)(n_a + {}^1/_2\nu_a) + \pi i \sum\limits_{(a,b)-1}^{r} \tau_{ab} (n_a + {}^1/_2\nu_a)(n_b + {}^1/_2\nu_b)}$$

spełnia związki *):

$$\Theta[ABB](\upsilon) = \begin{pmatrix} A \\ B \end{pmatrix} \Theta[A](\upsilon)$$

[1]) Por: Appell: Développements en séries trigonométriques de certaines fonctions périodiques vérifiant l'équation $\Delta F = 0$ (Journal de C. Jordan, 1887), oraz

Krygowski: O pewnej klasie funkcyj przestępnych etc., Prace matematyczno-fiz., t. V., Warszawa, 1895.

[2]) Frobenius: Ueber das Additionstheorem der Thetafunctionen mehrerer Variabeln (Crelle, t. 89, str. 185) i Baker: Abels Theorem and the allied theory etc., Cambridge, 1897, str. 486.

*) Frobenius, l. c, str. 192.

The first page of Z. Krygowski's article *On certain application of the theta function*. The article contains some applications of the theta-functions to the series.

Zdzislaw Krygowski (1872–1955), *O rozwijaniu funkcyi hypereliptycznych pierwszego rzędu na szeregi Fouriera* [On expansion of hyperelliptic functions of the first kind into Fourier series], Reports of Management of II imperial Real School in Lvov 1904/05 school year.

The author, later the professor of the University in Poznań, represented normal integrals of the first kind and their periods, bilinear combinations and integrals of the second kind. He presented relationships between normal integrals and functions theta. He introduced also manners of defining a function on sections of Riemann's surface.

O rozwijaniu funkcyi hypereliptycznych pierwszego rzędu na szeregi Fouriera.

Wiadomo, iż zasadniczych funkcyi hypereliptycznych pierwszego rzędu jest piętnaście. Są to funkcye $\frac{\vartheta^2_{\mu}(v_1,v_2)}{\vartheta_5^2(v_1,v_2)}$, $(\mu=o, 1,\ldots,4)$, oraz funkcye $\frac{\vartheta^2_{\mu\nu}(v_1,v_2)}{\vartheta_5^2(v_1,v_2)}$, $(\mu,\nu=o, 1,\ldots, 4;\ \mu<\nu)$; pierwszej kategoryi jest funkcyi pięć, drugiej dziesięć. Funkcye pierwszej kategoryi*) w układzie Rosenhaina rozwinął Appell *(por. Appell: Sur les intégrales de fonctions à multiplicateurs et leur application au développement des fonctions abéliennes en séries trigonométriques, Acta Mathematica, t. XIII, str. 122)* na szeregi Fouriera, funkcyi drugiej kategoryi nie można z powodu obecności czynnika $(x_1 - x_2)^2$ w wyrażeniach tychże rozwijać w ten sposób (por. Appell l. c. str. 139), nie można bowiem całki podwójnej przedstawiającej spółczynnik szeregu Fouriera w tym razie rozłożyć na iloczyn całek pojedynczych, jak to właśnie ma miejsce we wszystkich przypadkach rozważanych przez Appella. Istotnie, mając układ zasadniczy równań

$$\int_{a_1}^{x_1} d\omega_1 + \int_{a_3}^{x_2} d\omega_1 = v_1\,;\ \int_{a_1}^{x_1} d\omega_2 + \int_{a_3}^{x_2} d\omega_2 = v_2\,,$$

*) Wyrażają się one wzorem $\frac{\vartheta^2_{\mu}(v_1,v_2)}{\vartheta_5^2(v_1,v_2)} = c^2_{\mu}(a_{\mu}-x_1)(a_{\mu}-x_2)$, $(\mu = o,1,\ldots,4)$, gdzie c_{μ} są pewnymi stałymi.

The first page of the article *On expansion of hyperelliptic functions of the first kind into Fourier series*.

Leon Lemoch (1845–1906), *O rozwiązywaniu równań jakiegokolwiek stopnia podług metody Fouriera* [On solving equations of any order according to Fourier's method], Management Reports of c.k. Higher Real School in Stryj for 1883/84 schoool year.

Tadeusz Łopuszański (1874–1955), *Z podstaw teoryi funkcyji,* [From the basic function theory], Management Reports of c.k. Higher Gymnasium in Rzeszów for 1902/03 school year.

Michał Maryniak, *O równaniach algebraicznych. Według broszury „Studien to them Gebiete der numerischen Gleichungen von L. Żmurko"* [On algebraic equations. According to the brochure „Studien to them Gebiete der numerischen Gleichungen von L. Żmurko"], Reports of Management of imperial Higher Real School in Lvov behind 1874/75 school year.

Roman Moskwa, *O sześciokącie Pascala i sześcioboku Brianchona* [On Brianchon's and Pascal's hexagon], Management Reports of c.k. Higher Real Gymnasium named after Francis Joseph on Drohobycz for 1892/93 school year.

Franciszek Nowosielski, *Niektóre własności układu dwóch i więcej kół* [Some properties of a system of two and more disks], Management Reports of c.k. Higher Gymnasium in Sambor for 1881/82 school year.

Franciszek Nowotny (1847–1888), *O rozwiązywaniu równań różniczkowych* [About solving of differential equations], Management Reports of c.k. Gymnasium in Jasło for 1881/82 school year.
The author focuses on a way of solving equations with separated variables.

Józef Nussbaum, *Kolineacya na płaszczyźnie w zastosowaniu do szkół średnich* [Colineation on the plane adapted for secondary schools], Reports of Management of imperial Higher Real School in Śniatyn in school year 1911/12.

Stanisław Piątkiewicz, *Algebra w logice* [Algebra in logic], Reports of Management of the IV imperial Gimnasium in Lvov during 1887/88 school year.

The author proposes "introduction" of logic in an algebraic way. This manner is supposed to contribute to the better understanding of the range of formal logic. In dissertation the work of E. Schröder was quoted (1841–1902), who was one of creators of formalized language *Vorlesungen über die the Algebra der the Logician*.

O rozwiązywaniu zrównań różniczkowych.

Sposobów do rozwiązywania zrównań różniczkowych jest kilka. Każde zrównanie różniczkowe da się zprowadzić do zrównania całkowego, jeżeli tylko zmienne zachodzące w zrównaniu dadzą się rozdzielić tak, że na jednej stronie znajduje się tylko jedna, a na drugiej tylko druga zmienna — jeżeli więc zrównanie może przyjąć kształty następujące:

$$\int X dx + \int Y dy = C$$

$$X\mathfrak{Y} dy + \mathfrak{X}, Y dx = 0$$

$$\frac{\mathfrak{X} dx}{X} + \frac{\mathfrak{Y} dy}{Y} = 0$$

$$\int \frac{\mathfrak{X} dx}{X} + \int \frac{\mathfrak{Y} dy}{Y} = C$$

Przykłady:

1) $\frac{y dx}{dy} = 2x$ z tego $\frac{dx}{x} - 2\frac{dy}{y} = 0$

$$\int \frac{dx}{x} - 2 \int \frac{dy}{y} = C$$

$$\log x + \log h = 2 \log y$$

$$xh = y^2 \quad a \quad y = \sqrt{hx}$$

2) $\frac{y dx}{dy} = \varphi(x)$ albo $\frac{dx}{\varphi(x)} = \frac{dy}{y}$

jeżeli więc w ogóle $\int \frac{dx}{\varphi(x)} = \psi(x)$ to będzie

$$\psi(x) = \log y + \log h = \log(yh)$$

a zatem $e^{\psi(x)} = hy$ czyli

$$y = \frac{1}{h} e^{\psi(x)}$$

Podobnych przykładów dostarczają zagadnienia o podstycznych i podsiecznych.

The first page of the article concerns the method of separate variables. One can see from it that the author used material from textbooks.

Jan Ralski, *Niezmienniki dwóch powierzchni drugiego rzędu i ich znaczenie geometryczne* [Invariants of two surfaces of the second degree and their geometrical meaning], Reports of Management of c.k. High Gymnasium in Tarnopol in 1894/95 school year.

Jan Ralski, Zasady *rachunku różniczkowego i całkowego dla użytku szkół średnich* [Rules of differential and integral calculus for the use at secondary schools], Reports of Management of the imperial High Real School in Jarosław in 1909/10 school year, the continuation in 1910/11 school year.
Extensive 48 pages work, consists of 22 chapters on the following titles: 1. *General remarks*. 2.*Concept of the function. 3. Elementary functions. 4. Image of the function. 5. The limit of function. 6. The continuity of the function. 7. The implicit function. 8.* Image *of the implicit function. 9. The continuity of the implicit function. 10. The infinite series. 11. Two characters of the convergence of a sequence. 12. Applications of characters of the convergence of a series. 13. The notion of the derivative of a function. 14. Derivatives of some elementary functions. 15. Derivatives of* complicated *functions. 16. The derivative of the function of function. 17. The notion of the differential. 18. Relation between the differential function and the derivative. 19. Examples expressing differentials of the function. 20. The differential of the function of two variables. 21. The differential of the implicit function. 22. The meaning of the differential function.* The continuation was more extensive and consisted of 64 pages.

Michał Rembacz (1854–1931), *Krótko zebrana historya geometryi wykreślnej (Cz. 1.)* [Short collected history of descriptive geometry, part I], Management Reports of High Real School in Stanisławów for 1889/90 school year, II part – 1890/91.

The author introduced the development of geometry in Poland.

Michał Rembacz (1854–1931), *Nowy sposób wykreślania kąta nachylenia dwu płaszczyzn w rzutach prostokątnych* [A new metod of drawing the angle of inclination of two planes in rectangular projections], ibidem, 1886/87 school year.

Michał Rembacz (1854–1931), *Przyczynek do Apolloniuszowych zagadnień styczności* [A contribution to Appolonius' adjacency issues], ibidem, 1886/87.

Michał Rembacz (1854–1931), *O biegunowem przekształcaniu krzywych 2-go rzędu na koła i o zastosowaniu tego przekształcenia do rozwiązania niektórych zagadnień odnoszących się do tych krzywych* [On polar transformation of second degree curves into circles and on application of this transformation to solving problems concerning these curves], ibidem, 1883/84 school year.

Michał Rembacz, *Obliczanie planów umarzania pożyczek, spłaconych za pomocą anuitetów* [Computing the plans of mortification of loans paid off by annuities], ibidem, 1894/95.

Czesław Rodecki, *Rysunki geometryczne, zastosowane do rozwiązywania zadań algebraicznych i arytmetycznych w szkołach realnych* [Geometrical pictures, used to solving algebraic and arithmetical exercises in real schools], Reports of Management of the imperial Higher Real School in Lvov in 1886/87 school year.

Stanisław Rudnicki (1841–1888) *Kilka uwag dotyczących nauki trygonometryi w szkołach średnich* [Several remarks about teaching trigonometry in high schools], Management Reports of c.k. Gymnasium in Kołomyja for 1883/84school year.

Stanisław Ruxer (ur. 1875), *O transformacyach punktów i ich grupach na podstawie teoryi Liego* [On transformations of points and groups of points according to Lie's theories] Management Reports of c.k. High Real School in Stanisławów for 1903/4 school year.

Nicefor Sadowski, *Nauka o funkcyach w matematyce gimnazyalnej* [On functions in gymnasium mathematics], Management Reports of c.k. Gymnasium II in Tarnopol with Polish language of teaching for 1909/10 school year.

Bazyli Sanat, *O przestawianiu liczb układu dziesiątkowego* [Rearranging numbers in the decimal system], Management Reports of c.k. High Real Gymnasium named after Francis Joseph in Drohobycz for 1881/82 school year, the rest in 1882/83 school year.

Bazyli Sanat, *O własnościach współczynników dwumianowych* [On properties of binomial coefficients], ibidem, 1887/88 school year.

Bazyli Sanat, *O sumowaniu n-tych potęg z p początkowych liczb całkowitych* [Adding n^{th} powers to initial p of integral numbers], Management Reports of c.k. High Gymnasium in Brzeżany for 1889/90 school year.

Jan Sitnicki (born 1881), *Elementarne pojęcia rachunku różniczkowego i całkowego* [Basic notions of differential and integral calculus] Management Reports of High Real School in Tarnopol for 1909/10 school year.

Michał Służewski, *Wykład równań w zakresie VII klasy szkół realnych* [Exposition on equations in VII class of real schools], Management Reports of Sprawozdania High Real School in Lvov for 1878/79 school year.

Kazimierz Strutyński (born 1878), *Krytyka podstaw geometrii elementarnej* [Criticism of the foundations of elementary geometry], Management Reports of High Real School in Kołomyja for 1906/7 school year.

Ignacy Tychowicz (1849–1889), *Elementarne uzasadnienie twierdzenia Eulera* [An elementary verification of Euler's Theorem], Management Reports of c.k. Gymnasium in Przemyśl for 1884/85 school year.
The author presents an argument which is a modification of the proof of Euler's theorem on polyhedron introduced by Cauchy.

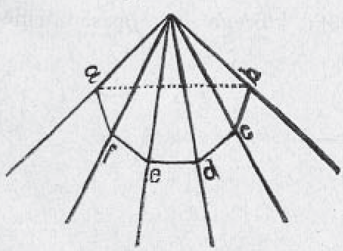
Michał Urysz, *O niektórych foremnych bryłach geometrycznych, wynikających z poszukiwania analitycznego* [Some regular solids resulting from analytic search] Management Reports of c.k. Gymnasium in Stanisławów for 1885/86 school year.
This work concerns the theory of invariants in three dimensional space.

Elementarne uzasadnienie twierdzenia Eulera.

Wykładając po raz pierwszy naukę geometryi w VI. klasie c k. gimnazyum brzeżańskiego w r. 1879. posługiwałem się podręcznikiem przepisanym pod tytułem „Geometrya dlas wyższych gimnazyalnych napisana przez Dr. Franciszka Mocnika. Z ósmego wydania niemieckiego przełożył i uzupełnił Dr. Tomasz Stanecki. Lwów 1868 ‟

Przyszedłszy w toku nauki do uzasadnienia twierdzenia Eulera ze stereometryi, nie podobna było zgodzić się na sposób udowodniania tego twierdzenia, podany w rzeczonym podręczniku na str. 165 i 166, gdzie w toku dowodu zakłada się, że ilość ścian wielościanu może być jednostką t. j. F = 1 Takie założenie jest absolutnie nie możebne, gdyż potrzeba najmniej czterech ścian do ograniczenia wielościanu Nie mogąc więc posługiwać się dowodem podanym w rzeczonej książce uzasadniłem twierdzenie Eulera w sposób następujący:

Niech figura umieszczona przedstawia naroże n ścienne dowolnego wielościanu, który ma S ścian N naroży i K krawędzi. Po ścięciu tego naroża płaszczyzną a b c d e f powstaje nowy wielościan, krórego



ilość ścian niech wyraża S'
„ naroży „ „ N
„ krawędzi „ „ K'

— 22 —

Łatwo tedy wyrozumieć następujące zrównanie:

$$S' = S + 1 \quad (1)$$

$$N' = N - 1 + n \quad (2)$$

$$K' = K + n \quad (3)$$

Odjąwszy od sumy z (1) i (2) zrównanie (3) otrzymamy:

$$S' + N' - K' = S + N - K \quad (4)$$

Odcinając w podobny sposób dowolne naroże a nawet i kilka z nich równocześnie i tworząc zrównania podobne jak w (1) (2) (3) otrzymamy, że S + N — K jest ilością stałą dla wielościanów o dowolnej ilości ścian t. j.

$$S + N - K = C \quad (5)$$

W celu oznaczenia stałej ilości C należy utworzyć wyrażenie (5) dla dowolnej bryły n p. dla sześcianu

$$S = 6$$

$$N = 8$$

$$K = 12$$

$$S + N - K = 6 + 8 - 12 = 2$$

zatem $C = 2$ (6)

Podstawiwszy wartość z (6) w (5) na wiejscu C otrzymujemy

$$S + N - K = 2$$ a z tego

$$S + N = K + 2 \quad (7)$$

Zrównanie (7) wyraża twierdzenie Eulera, które opiewa: suma ze ścian i naroży w dowolnym wielościanie (NB o wypukłych narożach) równa się ilości krawędzi zwiększonej o 2.

Powyższy sposób uzasadnienia przyjęli niektórzy pp. koledzy odemnie i posługują się nim, jak mi sami oświadczyli, z dobrym skutkiem. Ta okoliczność była powodem, dla którego to uzasadnienie w niniejszém sprawozdaniu umieściłem.

In the paper ”An elementary verification of Euler’s Theorem” the author, a gymnasium teacher, modified and enhanced the proof of Euler’s theorem given in the famous Močnik’s textbook of geometry.

Edmund W. Wierzbicki (born 1874), *O liczbach André’go i ich związku z liczbami Bernoulli’ego i Eulera. Część I.* [On André’s numbers and their relations to Bernoulli’s and Euler’s numbers. Part I], Management Reports of c.k. II Higher Real School in Lvov for 1907/1908 school year.

Stanislaw Zabielski (ur. 1880), *Arytmetyka w klasie trzeciej* [Arithmetics in the third grade], Management Reports of c.k. Gymnasium in Stryj for 1909/10 school year.
Władysław Zbierzchowski, *Przyczynek do rachunku całkowego dra Wawrzyńca Żmurki* [Contribution to dr. Wawrzyniec Żmurko's integral calculus], Management Reports of c.k. High Real School in Lvov for 1881/82 school year.

Władysław Zbierzchowski, *O liczbie kierunkowej w nauce matematyki w szkole średniej* [Directional numbers in teaching mathematics in high school], Management Reports of Higher Real School and c.k. Gymnasium in Jarosław for 1886/87 school year.