3 Charakteristika publikací

[1] *О колеблющихся интегралах дифференциальных линейных уравнений 2-ого порядка.* Czech. Math. J. **3** (**78**) (1953), 199–255. (Russian. French summary)

MR **15**,706:

Let $Q(x)$ be continuous and negative for all real x and such that all the non-trivial solutions of the equation (a) $y'' = Q(x)y$ are oscillatory, i.e. have infinitely many zeros with no finite limit point. Let $\cdots < \alpha_{-1} < \alpha_0 < \alpha_1 < \dots$ be the ordered zeros of an integral $y_0(x)$ of (a). Since $y_0(x)$ is determined, except for a constant factor, by any one of its zeros, $\alpha_n (n = \pm1, \pm2, \dots)$ is uniquely determined by α_0: $\alpha_n = \varphi_n(\alpha_0)$. The author calls $\varphi_n(x)$ the central dispersion (of the first kind) of index n; $\varphi_n(x)$ is monotone increasing and belongs to C_3. The $\varphi_n(x)$ form a cyclic group $\mathfrak{C}$ in the sense that $\varphi_n(\varphi_m(x)) = \varphi_{n+m}(x)$, $\varphi_1(x)$ being the generator, and $\varphi_0(x) = x$ the unit element. The $\varphi_{2n}(x)$ $(n = 0, \pm1, \dots)$ form an invariant subgroup $\mathfrak{S}$ of $\mathfrak{C}$. Let $U(x)$, $V(x)$ and $u(x)$, $v(x)$ be two fundamental systems of solutions of (a). Then a relation **p** is set up between the integrals $Y(x) = aU(x) + bV(x)$ and $y(x) = au(x) + bv(x)$ (a, b any real constants): $y =$**p**Y. This leads to a relation $\alpha = \zeta(A)$ between the zeros A of Y and the zeros α (properly chosen) of y, which the author calls proper dispersion. A proper dispersion $\zeta(x)$ is in C_3 and either monotone increasing („direct") or decreasing („indirect"). The $\zeta(x)$ form a 3-dimensional group $\mathfrak{G}$ with $\zeta(x) = x$ as identity. The elements of $\mathfrak{G}$ are all the the solutions of the equation of third order (b) $T''/T + \zeta'^2 Q(\zeta) = Q(x)$, $T = |\zeta'|^{-1/2}$. The direct proper dispersions form an invariant subgroup $\mathfrak{P}$ of $\mathfrak{G}$ whose center is $\mathfrak{C}$. The group $\mathfrak{G}/\mathfrak{S}$ is isomorphic to the group of real unimodular matrices of order 2. Also considered are dispersions of the second, third and fourth kind referring to the zeros and the extrema of integrals of equation (a).

M. Golomb (Lafayette, Ind.)

Zbl 053.05805

A great number of elementary properties of the oscillatory solutions of an equation $y'' = Q(x)y$ are discussed, such as the distribution of zeros and extreme values.

J. L. Massera

[2] *Замечания к рецензии М. И. Ельшина моей статьи „О колеблющихся интегралах дифференциальных линейных уравнений 2-ого порядка".* Czech. Math. J. **6** (**81**) (1956), 431–433. (Russian. French summary)

MR **20** #4053

Reply to the review in RŽ Mat **1956** #406, of the article in same J. **3 (78)** (1953), 199–255 [MR **15**, 706].

Zbl 075.26901

Verf. gibt einige Bemerkungen und Berichtigungen zu dem im Titel genannten Referat von M. I. El'šin (R. Ž. Mat. 1956, Nr. 406) über die Arbeit des Verf. [Czechosl. math. J. **3** (**78**), 199–255 (1953, dies. Zbl. **53**, 58)].

[3] *Sur la transformation des intégrales des équations différentielles linéaires ordinaires du second ordre*. Ann. Mat. Pura Appl., (4) **41** (1956), 325–342. (French)

MR **20** #1814:

Given a function $X(t)$ with a non-vanishing derivative X', define $\{X,t\} = \frac{1}{2}\frac{X'''}{X'} - \frac{3}{4}\frac{X''^2}{X'^2}$; similarly for functions of T, the derivatives being indicated by dots. Let q, Q be two continuous functions and consider equations (b): $-\{X,t\} + Q(X)X'^2 = q(t)$, (B): $-\{x,T\} + q(x)\dot{x}^2 = Q(T)$, (b'): $-\{X,t\} + q(X)X'^2 = q(t)$, (B'): $-\{x,T\} + Q(x)\dot{x}^2 = Q(T)$. The following results are typical: 1. If X is an integral of (b) its inverse function is an integral of (B); 2. If X, y, $\overline{X}$, $\overline{y}$ are integrals of (b), (B), (b'), (B'), respectively, the composite functions $X\overline{X}$, $\overline{y}X$, $y\overline{y}$, $\overline{X}y$, yX, Xy are solutions of (b), (b), (B), (B), (b'), (B'), respectively; 3. Let X be an integral of (b) and U an integral of (A): $Y'' = Q(T)Y$; then (13): $u = U(X)X'^{-1/2}$ is an integral of (a): $\ddot{y} = q(t)y$; and conversely (with certain restrictions which we do not reproduce explicitly) if u, U are integrals of (a), (A), there is an integral X of (b) such that (13) holds.

J. L. Massera (Zbl **72**, 89)

Zbl 072.08902

Stejné jako v MR.

[4] *Théorie analytique et constructive des transformations différentielles linéaires du second ordre.* Bull. Math. Soc. Sci. Math. Phys. R. P. Roumaine **1** (**49**) (1957), 125–130. (French)

MR **21** #3608

Es handelt sich um folgendes noch von Kummer herrührendes Problem: Wenn zwei Differentialgleichungen von sog. Jacobischem Typus (1) $y'' = q(t)y$; (2) $Y'' = Q(T)Y$ und ein Integral $U(T)$ der Gleichung (2) gegeben sind, wobei q und Q kontinuierliche Funktionen sind, zwei Funktionen $w(t)$ und $X(t)$ derart zu finden, dass $u(t) = w(t) \cdot U[X(t)]$ ein Integral der Gleichung (2) wird. Zu dem Zweck entwickelt der Verf. eine Theorie der linearen Differentialtransformationen von zweiter Ordnung, die aus einem analytischen und einem sog. konstruktiven Teil besteht. In der vorliegenden Arbeit wird der analytische Teil ganz kurz gestreift, da er ausführlich vom Verf. früher veröffentlicht wurde [Ann. Mat. Pura Appl. (4) **41** (1956), 325–342; MR **20** #1814], während der konstruktive Teil etwas vollständiger behandelt wird.

T. P. Anđelić (Belgrade)

Zbl 082.07501

Let (a): $y'' = q(t)y$, (A): $Y'' = Q(T)Y$, (b): $-\{X,t\} + Q(X)X'^2 = q(t)$, where q, Q are continuous functions in open intervals j, J and $\{X,t\} = X'''(2X')^{-1} - 3X''^2(2X')^{-2}$ is the Schwartzian derivative; assume that the integrals of (a), (A) are oscillatory (i.e., have infinitely many roots) at both ends of both intervals j, J. Given $t_0 \in j$, $X_0 \in J$ and two integrals y, Y of (a), (A) which are both zero or both different from zero at t_0, X_0, respectively, a direct (indirect) correspondence between the roots of y, Y is established by associated roots with equal ordinal numbers counted from t_0, X_0 in the same (opposite) direction. Let σ, Σ be the families of all solutions of (a), (A) and $p : \sigma \to \Sigma$ any isomorphism; if u, $v \in \sigma$ are linearly independent, the characteristic of p is the sign of the quotient of the two Wronskian determinants of u, v and pu, pv; p is regular if $y \in \sigma$, $y(t_0) = 0$ implies $(py)(X_0) = 0$. Let p be a regular isomorphism, $t \in j$, $y \in \sigma$, $y(t) = 0$; the direct (indirect) dispersions $D(\overline{D})$ are defined as functions of t by: $D(t)$ $(\overline{D}(t))$ is equal to the root of py associated to t in the direct (indirect) correspondence. The following theorem is stated: the solutions of (b) exist in

j and they are the direct dispersions corresponding to the different regular isomorphisms with positive characteristic and the indirect dispersions of the regular isomorphisms with negative characteristic; the former represent all the increasing, the latter all the decreasing solutions. Indications of other related results are also given.

J. L. Massera

[5] *Sur les transformations différentielles linéaires complètes du second ordre.* Ann. Mat. Pura Appl., (4) **49** (1960), 229–251. (French)

MR **22** #5771 The results of a previous paper [same Ann. (4) **41** (1956), 325–342; MR **20** #1814] are only of a local character, i.e., the solutions of (b) exist (and hence the transformation (13) applies) only in intervals which are smaller than the intervals j, J of definition of (a), (A). A solution of (b) is called complete if it is defined on j and its values cover J; the corresponding transformation is also called complete. The present work is devoted to the investigation of the existence of complete solutions. The decisive condition is that (a), (A) have the same type m (supposed to be finite and $\geq$ 2), the type being the maximum number of zeros of the solutions in the interval of definition, and are both simultaneously special or non-special, an equation (a) being special if inf $\{t \in j; t$ has a conjugate in j which is $< t\}$ is conjugate to sup $\{t \in j; t$ has a conjugate in j which is $> t\}$. A detailed description of the results would be too lengthy to be reproduced here.

J. L. Massera (Montevideo)

Zbl 095.28603

Stejné jako v MR.

[6] *Transformations des équations différentielles linéaires du deuxième ordre. – Décompositions dans les ensembles et théories des groupoïdes*. Algèbre et Théorie des Nombres. Sém. P. Dubreil, M.-L. Dubreil-Jacotin et C. Pisot 14 (1960/61), Nr. 22, 18 et 17p. (1963).

Zbl 121.07103

Verf. gibt einen Überblick über die Hauptdefinitionen und Hauptergebnisse der Theorie der Transformation der Lösungen der Differentialgleichung (a) $y'' = q(t)y$, $t \in j$, in die Lösungen der Gleichung (b) $Y'' = Q(T)Y$, $T \in J$. Diese Transformation ist durch die Formel $y(t) = (|X'(t)|)^{-1/2}Y(X(t))$ gegeben. Dabei ist $X(t)$ eine Lösung der nichtlinearen Differentialgleichung (c) $-\{X, t\} + Q(x)X'^2 = q(t)$, wo $\{X, t\}$ die sogenannte Schwarzsche Ableitung bedeutet. Es zeigte sich daß der Begriff des Typus einer Differentialgleichung in dieser Transformation eine wichtige Rolle spielt. Die Gleichung (a) ist vom Typus m, wenn es eine Lösung von (a) gibt, welche auf j m Nullstellen hat, aber keine Lösung von (a) mehr als m Nullstellen besitzt. Hat irgendeine Lösung von (a) auf j unendlich viele Nullstellen, so ist (a) vom unendlichen Typus. Hinreichende und notwendige Bedingungen wurden dafür abgeleitet, daß (a) vom Typus m ist. Weitere Grundbegriffe, wie eine Basis von (a), d. h. ein geordnetes Paar (u, v) von linear unabhängigen Lösungen von (a), die Phase α und die Amplitude ϱ, die durch die Formeln: $\operatorname{tg} \alpha = u(t)/v(t)$, $\varrho = \sqrt{u^2(t) + v^2(t)}$ definiert sind, haben sich als sehr fruchtbar in der Theorie der Transformation erwiesen. Mit Hilfe dieser wurde zum Beispiel die Frage der Existenz und Eindeutigkeit der Lösung von (c) gelöst. Es wurden weiter die entsprechenden Intervalle $i \subset j$, $I \subset J$ abgeleitet, auf welchen sich die Transformation abspielt. Die Transformation heißt komplett, wenn $i = j$, $I = J$ ist. Notwendige und hinreichende

Bedingungen für eine solche komplette Transformation werden angegeben. Verf. beschäftigt sich weiter mit den speziellen Lösungen von (c), den sogenannten Zentraldispersionen. Es sei t eine beliebige Zahl aus j und es sei $u(v)$ die Lösung von (a), welche in t (deren Ableitung in t) eine Nullstelle hat. Dann ist $\varphi_n(t)$ ($\psi_n(t)$, $\chi_n(t)$, $\omega_n(t)$) die n-te nach t liegende Nullstelle von u (v', u', v). Die Funktion $\varphi_n(t)$ ($\psi_n(t)$, $\chi_n(t)$, $\omega_n(t)$) heißt die n-te Zentraldispersion erster (zweiter, dritter, vierter) Gattung. Die Zentraldispersionen sind die Lösungen von (c). Man kann eine ausführliche Analysis der Zentraldispersionen durchführen. Einige ihrer Eigenschaften sind hier angegeben. Zum Schluß führt Verf. einige Probleme an, die mittels der Theorie der Transformation schon gelöst wurden.

M. Švec

[7] *Sur la structure de l'ensemble des transformations différentielles linéaires complètes du second ordre*. Ann. Mat. Pura Appl., (4) **58** (1962), 317–333. (French)

MR **26** #3981

Further results on the subject studied in previous papers by the author [same Ann. (4) **41** (1956), 325–342; MR **20** #1814; ibid. **49** (1960), 229–251; MR **22** #5771]. It is shown, for instance, that the complete solutions in the nonspecial case may be split into two families, each of which admits an ordering which makes them order-isomorphic to the set of real numbers. In the special case the situation is more involved since each one of the two families depends on two parameters. Other properties of these families are too complicated to be summarized here.

J. L. Massera (Montevideo)

Zbl 111.28001

Dans un Mémoire antérieur (ce Zbl. **95**, 286) l'A. a étudié l'existence et la généralité de ces transformations. Dans celui-ci il introduit la théorie de façon plus directe et il étudie les propriétés de ces transformations moyennant les solutions d'une certaine équation différentielle non-linéaire de troisième ordre.

A. de Castro

[8] *Über einige Ergebnisse aus der Theorie der linearen Differentialtransformationen 2. Ordnung*. Heft 13 der Schriftenreihe der Institute für Mathematik. Bericht von der Dirichlet-Tagung. Akademie-Verlag, Berlin, 1963, 51–57. (German)

MR **31** #429

An expository lecture presented in 1959.

Zbl 114.28803

Bericht über einige Ergebnisse aus der Transformationstheorie der gewöhnlichen linearen Differentialgleichung 2. Ordnung [s. a. Verf., dies. Zbl. **72**, 89; **82**, 75; **95**, 286; sowie M. Laitoch, Czechosl. Math. J. **6** (**81**), 265–380 (1956); J. Chrastina, Časopis Mat. **87**, 188–197 (1962)].

[9] *Sur l'ensemble des équations différentielles linéaires ordinaires du deuxième ordre qui ont la même dispersion fondamentale.* Bul. Inst. Politehn. Iaşi, 9 (**13**) (1963), no. 3–4, 11–20. (French. Russian, Romanian summary)

MR **31** #3655

Consider an equation (1) $y'' = q(t)y$ which is oscillatory for $t \to \pm\infty$, and for any real t, let $\varphi(t)$ be the first right conjugate point of t. The function φ is called the fundamental dispersion of the equation (1). It is shown that the set of all equations with the same fundamental dispersion has the power of the continuum.

W. A. Coppel (Canberra)

Zbl 138.32403

Let (q): $y'' = q(t)y$ be a given linear differential equation of second order where $q \in C_0(-\infty, \infty)$. Let each solution of (q) have infinitely many zeros both to the left and to the right of an arbitrary number. The basic central dispersion $\varphi(t)$ of (q) is defined as follows: Let $u(t)$ be a non-trivial solution of (q) which vanishes at t_0; then $\varphi(t_0)$ is the first zero of $u(t)$ lying on the right of t_0. The following result is proved: The power of the set of all equations (q) with the same basic central dispersion $\varphi(t)$ does not depend on $\varphi(t)$ and it is equal to the power of the continuum. To this aim the theory of so called phases of (q) was developed. Let u, v be two independent solutions of (q). Then a phase of (q) is a continuous solution of $\operatorname{tg}\alpha(t) = u(t)/v(t)$, $v(t) \neq 0$, for $t \in (-\infty, \infty)$. For example, there is proved: The set of all phases of all (q) forms a group and the set of so-called elementary phases [i. e. phases $\alpha(t)$ satisfying the relation $\alpha(t+\pi) = \alpha(t) + \pi \operatorname{sign}\alpha'$] is its subgroup. The formula is also derived establishing all (q) with the same given basic central dispersion:

$$q = q_\alpha + (f''\alpha + 2f'\alpha \operatorname{cotg}\alpha)\alpha'^2,$$

where $f \in C_2$ is a periodic function with period π and such that $f(0) = f'(0) = 0$, $\int_0^\pi \frac{e^{-2f(\sigma)}}{\sin^2\sigma} d\sigma = 0$, α is given phase and q_α is the coefficient of (q) with the phase α.

M. Greguš

[10] *Transformation of ordinary second-order linear differential equations*. Differential Equations and their Applications (Proc. Conf. Equadiff I, Prague 1962). Publ. House Czechoslovak Acad. Sci., Prague; Academic Press, New York, 1963, 27–38. (English)

MR **30** #295

This paper is concerned with conditions under which the equations $y'' + q(t)y = 0$, $\ddot{Y} + Q(T)Y = 0$ can be transformed into one another by a change of variables $y = w(t)Y$, $T = X(t)$. The problem was solved formally by Kummer in the last century. The author outlines a rigorous treatment for the real domain and refers to previous papers for applications.

W. A. Coppel (Canberra)

Zbl 138.32402

Der Verf. behandelt das Kummersche Problem: Von zwei linearen Differentialgleichungen II. Ordnung $y'' + q(t)y = 0$, $\ddot{Y} + Q(T)Y = 0$ ist die Lösung einer Gleichung bekannt. Wie kann die Lösung der einen Differentialgleichung durch die der anderen ausgedrückt werden? Dieses Problem führt auf die Schwarzsche Differentialgleichung, einer nichtlinearen Differentialgleichung III. Ordnung. Die Existenz und Eindeutigkeit ihrer Lösungen wird mit topologischen Methoden untersucht, die auch qualitative Aussagen über den Zusammenhang zwischen Differentialgleichung und ihren Lösungen ermöglichen.

H.-J. Bangen

[11] *Über die algebraische Struktur der Phasenmenge der linearen oszillatorischen Differentialgleichungen 2. Ordnung*. Bericht von der Tagung über geordnete Mengen, Brno, November 1963. Publ. Fac. Sci. Univ. J. E. P., Brno, n°457, 1964, 461–462.

Tato práce nebyla recenzována ani v MR ani v Zbl. Jedná se o přednášku o algebraické struktuře množiny fází oscilatorických lineárních diferenciálních rovnic 2. řádu, kterou O. Borůvka proslovil na konferenci o uspořádaných množinách, jež se konala 4. – 7. prosince 1963 v Brně.

[12] *Sur quelques applications des dispersions centrales dans la théorie des équations différentielles linéaires du deuxième ordre.* Arch. Math. (Brno), **1** (1965), 1–20. (French)

MR **33** #5984

The second-order equation $y'' = q(t)y$ is said to be oscillatory on the interval (a, b) if its solutions have an infinite sequence of zeros as $t \to a$ and as $t \to b$. If $y(t)$ is a solution and x is not a zero of $y(t)$, then $\overline{y}(t) = y(t) \int_x^t [y^2(\sigma)]^{-1} d\sigma$ is also a solution on an interval about x not containing any zeros of $y(t)$. The author shows how to extend this solution to the whole interval (a, b). He also studies the properties of equations with the same fundamental dispersion (i.e., whose solutions have the same zeros) as the given equation.

F. Brauer (Madison, Wis.)

Zbl 151.10804

The author considers the following differential equation $y'' = q(t)y$, where $q(t)$ is a continuous function in the interval (a, b), which may be infinite. Two independent solutions of this differential equation denoted by u and v are supposed to posses an infinite number of zeroes. In this paper are considered the phases $\operatorname{tg} \alpha(t) = u(t)/v(t)$. The author regards the solution $y(t) \int_x^t \frac{d\sigma}{y^2(\sigma)}$ in the interval $(-\infty, \infty)$ and gives some properties concerning the asymptotic behaviour of the solutions and zeroes. The importance of the central dispersion in the theory of the above mentioned differential equation is stressed and considered in details.

T. Tietz

[13] *Über die allgemeinen Dispersionen der linearen Differentialgleichungen 2. Ordnung*. Ann. Şti. Univ. „Al. I. Cuza“ Iaşi, **11B** (1965), 217–238. (German. Romanian, Russian summary)

MR **34** #1595

Another exposition of the transformation theory of second-order linear differential equations and the author's theory of dispersions [cf. the author, *Differential equations and their applications* (Proc. Conf., Prague, 1962), pp. 27–38, Publ. House Czechoslovak Acad. Sci., Prague, 1963; MR **30** #295].

W. A. Coppel (Canberra)

Zbl 173.34003

Consider oscillatoric differential equations (q): $y'' = q(t)y$ and (Q): $\ddot{Y} = Q(T)Y$, where $q(t)$, $Q(T)$ are continuous functions on $(-\infty, \infty)$. Let (u, v) and (U, V) be two linearly independent solutions of (q) and (Q), resp. A phase $\alpha(t)$ and $A(T)$ with respect to (u, v) and (U, V) is defined as a continuous function on $(-\infty, \infty)$ satisfying $\operatorname{tg} \alpha(t) = u(t)/v(t)$ and $\operatorname{tg} A(t) = U(T)/V(T)$, resp. Let p be a mapping of the set of all solutions of (q) into the set of all solutions of (Q) defined in the following way: if $y = \lambda u + \mu v$ then $p(y) = \lambda U + \mu V$. The characteristic χp of p is number $(uv' - u'v)/(U\dot{V} - \dot{U}V)$. Let t_0, T_0 be arbitrary numbers, (u, v) be a pair of independent solutions of (q). There exist λ, μ such that $y(t) = \lambda u + \mu v$ has a zero at t_0. Choose (U, V) such that $Y(T) = \lambda U + \mu V$ has a zero at T_0. A mapping p defined by means of these pairs (u, v) and (U, V) is called normed mapping with respect to t_0, T_0. If, moreover, phases α and A are chosen (which is always possible) such that $\alpha(t_0) = A(T_0) = 0$, then α, A are called canonical with respect to t_0, T_0 and p. Let $\cdots < t_{-1} < t_0 < t_1 < \ldots$ and $\cdots < T_{-1} < T_0 < T_1 < \ldots$ be all zeros of a solution y of (q) and of a solution Y of (Q), resp. Let p be a normed mapping with respect to t_0, T_0. Now, let $t^\star \in (-\infty, \infty)$ be a number and y be such a solution of (q) that $y(t^\star) = 0$. Let $t^\star \in< t_\nu, t_{\nu+1})$ and $T^\star$ be the zero of $Y = p(y)$ lying in $< T_\nu, T_{\nu+1})$ if $\chi p > 0$ or in $(T_{-\nu-1}, T_{-\nu} >$ if $\chi p < 0$. Then $T^\star = X(t^\star)$ is a general dispersion of (q), (Q) (in this order) with respect to t_0, T_0 and p. – Some results: „Let $X(t)$ be a general dispersion with respect to t_0, T_0 and p. If α, A are canonical with respect to t_0, T_0 and p, then $\alpha(t) = A(X(t))$ on $(-\infty, \infty)$“. Further, all solutions of Kummer's equation (Q, q): $-\{X, t\} + Q(X)X'^2 = q(t)$, where $\{X, t\}$ is Schwarz's derivative $\frac{1}{2}X'''/X' - \frac{3}{4}X''^2/X'^2$, are constructed: „All solutions of (Q, q) defined on $(-\infty, \infty)$ are exactly all general dispersions of (q), (Q).“ Algebraic structure of general dispersions is deeply studied as well.

F. Neuman

[14] *Sur une application géométrique des dispersions centrales des équations différentielles linéaires du deuxième ordre*. Ann. Mat. Pura Appl., (4) **71** (1966), 165–187. (French)

MR **34** #6647

Given a collection of straight lines, the author studies the plane curves such that (i) each straight line of the collection cuts the curve in at least two points, and (ii) the tangents to the curves at each intersection are parallel. He characterizes such curves, using global properties of second-order linear differential equations.

F. Brauer (Madison, Wis.)

Zbl 148.06001

„Sont étudiées les courbes planes caractérisées par la propriété d'être coupées par toute droite d'un faisceau de droites en au moins deux points et de telle façon que les tangentes de la courbe, dans les différents points d'intersection, sont mutuellement paralléles. L'étude est basée sur les notions empruntées de la théorie des équations différentielles linéaires ordinaires du deuxième ordre. Il s'agit des matières dans le domaine réel et de caractère global.“ (Authors's summary.)

A. M. Krall

[15] *Neuere Ergebnisse in der Transformationstheorie der gewöhnlichen linearen Differentialgleichungen 2. Ordnung*. Vorträge der 3. Tagung über Probleme und Methoden der mathematischen Physik. Technische Hochschule Karl-Marx-Stadt, 1966, Heft 1, 13–27.

Zbl 161.05804

This paper is a review one. The author states some results of his own and some of his disciples for the equation $(\star)$ $y'' = q(t)y$ with a continuous function $q(t)$ in an open interval. Firstly results are given concerning oscillatory equations $(\star)$ with the same zeroes of solutions in $(-\infty, \infty)$. These results concern the power of the set of such equations (it is equal to C), connection between functions $q(t)$ of these equations and properties of their solutions. Then the theory of transformation of $(\star)$ equations and its physical application is considered.

E. J. Grudo

[16] *Lineare Differentialtransformationen 2. Ordnung*. Hochschulbücher für Mathematik, Band 67. VEB Deutscher Verlag der Wissenschaften, Berlin, 1967, xiv+218. (German)

MR **38** #4743

This book gives a connected account of work of the past twenty years by the author and other Czechoslovak mathematicians on the transformation theory of second-order linear differential equations.

Contents: (I) Grundlagen der Theorie: (A) Allgemeine Eigenschaften der gewöhnlichen linearen homogenen Differentialgleichungen 2. Ordnung; (B) Phasentheorie der gewöhnlichen linearen homogenen Differentialgleichungen 2. Ordnung; (II) Dispersionstheorie: (A) Theorie der Zentraldispersionen; (B) Spezielle Probleme über Zentraldispersion; (C) Theorie der allgemeinen dispersionen; (III) Allgemeine Transformationstheorie: (A) Allgemeine Transformationen; (B) Vollständige Transformationen.

Zbl 153.11201

Es handelt sich um eine Transformationstheorie für gewöhnliche lineare homogene Differentialgleichungen 2. Ordnung im Reellen, bei der man untersucht, wie sich Variablentransformationen und damit zusammenhängende Vorgänge auf Lösungen auswirken, also um Fragen, die zuerst von E. E. Kummer (1834) und dann später von Laguerre, Brioschi, Halphen, Forsyth, Lie und anderen behandelt wurden. Die vorliegende Gestalt verdankt die Theorie neueren Arbeiten von E. Barvínek, Verf., M. Greguš, Z. Hustý, M. Laitoch, F. Neuman, M. Ráb, V. Šeda und anderen. Diese Theorie ist qualitativ und global und stützt sich wesentlich auf neue Begriffe. Sie hat 2 Teile: 1. Die „Dispersionstheorie" betrifft oszillatorische Differentialgleichungen. Sie beruht auf dem Begriff der Zentraldispersion und umfaßt eine konstruktive Integrationstheorie der Kummerschen Differentialgleichungen. 2. Die „allgemeine Transformationstheorie" untersucht Eigenschaften von Lösungen der Kummerschen Differentialgleichung im Zusammenhang mit Transformationen bei linearen Differentialgleichungen 2. Ordnung. Demgemäß gliedert sich das Buch, wie folgt: I. Grundlagen: A. Allgemeine Eigenschaften gewöhnlicher homogener linearer Differentialgleichen 2. Ordnung (u. a. Eigenschaften von Integralen, konjugierte Zahlen, zentroaffine Eigenschaften ebener Kurven). B. Phasentheorie der genannten Gleichungen (Polarkoordinaten der Basen, Polarfunktionen, lokale und Randeigenschaften der Phasen, algebraische Struktur der Phasenmenge oszillatorischer Differentialgleichungen usw., also die methodische Grundlage der zu entwickelnden Transformationstheorie). II. Dispersionstheorie: A. Zentraldispersionen (Z. D.) B. Spezielle Probleme (z. B. Differentialgleichungen mit denselben Z. D. 1. Art, mit zusammenfallenden Z. D. k-ter und $(k+1)$-ter Art, usw.). C. Allgemeine Dispersionen (unter Zugrundelegung zweier oszillatorischer Differentialgleichungen (q) $y'' = q(t)y$, $a < t < b$, und (Q) $\ddot{Y} = Q(T)Y$, $A < T < B$. III. Allgemeine Transformationstheorie: A. Allgemeine Transformationen (Transformationseigenschaften der Lösungen der von Kummer angegebenen nichtlinearen Differentialgleichung (Qq) dritter Ordnung für die Transformierende der Differentialgleichungen (q), (Q)m, sowie Existenz- und Eindeutigkeitsfragen bei (Qq), physikalische Anwendungen auf geradlinige und harmonische Bewegungen). B. Vollständige Transformationen (Existenz und Allgemeinheit der vollständigen Transformationen, Struktur der Menge vollständiger Lösungen von (Qq)). Die Darstellung ist klar und leicht lesbar und erfordert keine besonderen Vorkenntnisse. Auf historische Zusammenhänge und geometrische Motivierungen wird Wert gelegt. Alles in allem hat man damit eine abgerundete und wohlausgewogene Neuerscheinung, die sehr begrüßenswert ist.

E. Kreyszig

[17] *L'état actuel de la théorie des transformations des équations différentielles linéaires du deuxième ordre*. Colloque sur la théorie de l'approximation des fonctions. Cluj, 15. – 20. Septembre, 1967, 1–14.

Tato práce nebyla recenzována v MR ani v Zbl. Jedná se o stručný přehled nejdůležitějších výsledků celé transformační teorie: Zavedení první a druhé fáze, Kummerův transformační problém, teorie centrálních dispersí (zavedení, základní vlastnosti, derivace dispersí, souvislost transformační teorie a centrálních dispersí), obecné disperse a algebraická struktura fází. Je zde uvedeno, že jde o hlavní výsledky zpracované v monografii [16], která byla v té době v tisku.

[18] *Théorie des transformations des équations différentielles linéaires du deuxième ordre.* Rend. Mat. e Appl., (5) **26** (1967), 187–246. (French)

MR **37** #5453

This is an expository paper containing the text of four invited lectures delivered at the University of Rome in April, 1967, concerning the author's theory of transformations of differential equations of the form $y'' + p(x)y' + q(x)y = 0$. This material has been reviewed previously [the author, Bull. Math. Soc. Sci. Math. Phys. R. P. Roumaine **1** (**49**) (1957), 125–130; MR **21** #3608; Ann. Mat. Pura Appl. (4) **49** (1960), 229–251; MR **22** #5771; ibid. (4) **58** (1962), 317–333; MR **26** #3981; *Differential equations and their applications* (Proc. Conf., Prague, 1962), pp. 27–38, Publ. House Czechoslovak Akad. Sci., Prague, 1963; MR **30** #295; Bul. Inst. Politehn. Iaşi (N. S.) **9 (13)** (1963), no. 3–4, 11–20; MR **31** #3655; Arch. Math. (Brno) **1** (1965), 1–20; MR **33** #5984; An. Şti. Univ. „Al. I. Cuza" Iaşi Secţ. I a Mat. (N. S.) **11B** (1965), 217–238; MR **34** #1595; Ann. Mat. Pura Appl. (4) **71** (1966), 165–187; MR **34** #6647].

C. A. Swanson (Vancouver, B.C.)

Zbl 165.10002

Die vorliegende Arbeit enthält den Wortlaut von vier Vorträgen, die der Verf. über seine Transformationstheorie der gewöhnlichen linearen Differentialgleichungen 2. Ordnung im Seminar des Herrn Prof. G. Fichera an der Universität in Rom gehalten hatte (April 1967). Diese Theorie ist inzwischen in ausführlicher monographischer Bearbeitung in Buchform erschienen [vgl. Verf., Lineare Differentialtransformationen 2. Ordnung (1967; dies. Zbl. **153**, 112)]. Die Arbeit bringt eine Übersicht über die Struktur und den Inhalt der erwähnten Theorie, wobei namentlich die neuartigen methodisch und sachlich wichtigsten Elemente dieser letzteren hervorgehoben werden. Dies betrifft insbesondere den Begriff von verschiedenen Arten von Dispersionen, die sogenannten vollständigen Transformationen, sowie die algebraischen auf gruppentheoretische Sätze gestützten Methoden, die bei Untersuchungen der Transformationsprozesse im oszillatorischen Fall tiefliegende Resultate ergeben. Ferner findet man in der Arbeit in kurzgefaßter Form die Lösung von einigen Problemen analytischer und geometrischer Natur, die die Tragweite der erwähnten Transformationstheorie beleuchten.

Autorreferat.

[19] *Éléments géométriques dans la théorie transformations des équations différentielles linéaires et ordinaires du deuxièmes ordre.* Atti Convegno internaz. Geom. diff. Ist. Geom. Univ. Bologna 1967, 97–108 (1970).

Zbl 243.34050

[This article was published in the book announced in this Zbl. 226.00013.] The author gives a short geometric introduction to the theory developed in his book „Lineare Differentialtransformationen 2. Ordnung" (1967; this Zbl. **153**, 112). The theory concerns conjugate and focal points (the latter both in the sense of the calculus of variations and in the sense adopted by most ungeometric writers in the theory of ODE) of linear second order differential equations. Major tools discussed are (a) the Kummer transform and (b) centro-affine differential geometry of curves $x(t)$ that satisfy the equation, in particular Radon curves. The analyst should be aware of the following dictionary: Central dispersion of first kind = conjugate point, of second kind = conjugate point of the derivative of a solution of the DE, of third kind = focal point in the unhistoric sense of the word, of fourth kind = focal point in the sense of M. Morse.

H. Guggenheimer

[20] *Über eine Charakterisierung der allgemeinen Dispersionen linearer Differentialgleichungen 2. Ordnung*. Math. Nachr. **38** (1968), H 5/6, 261–266. (German)

MR **39** #5854

The equations under discussion are (q) $y'' - q(t)y = 0$, (Q) $Y'' - Q(t)Y = 0$ and (Qq) $-\{X,t\}+Q(X)X'^2 = q(t)$, where $\{X,t\}$ denotes the Schwarzian derivative. (q) and (Q) are related by $y(t) = |X'(t)|^{-1/2}Y[X(t)]$. With equation (q) one can associate a phase function $\alpha(t)$, defined by tan $\alpha(t) = y_1/y_2$, where y_1 and y_2 are linearly independent solutions of (q). If (q) is of oscillatory type, then $\alpha(t)$ increases monotonically from $-\infty$ to $+\infty$. The set of all phase functions forms a group under the operation of composition of functions. The identity $\alpha(t) = t$ is associated with the equation (-1). One can define a linear mapping between the solution spaces of (q) and (Q). To each such mapping p one can assign a function $X(t)$, namely, the general dispersion. If y satisfies (q) and vanishes at t, then py satisfies (Q) and vanishes at $X(t)$, and $X(t)$ satisfies (Qq). (For more details on these concepts, see the author's book [*Lineare Differentialtransformationen 2. Ordnung*, VEB Deutsch. Verlag Wissensch., Berlin, 1967; MR **38** #4743].) The set of all general dispersions associated with (Qq) is denoted by $I(Qq)$. The author proves, using substantially grouptheoretic arguments, that a phase-function ξ is a general dispersion of (Qq) if and only if $\xi^{-1}I(QQ)\xi = I(qq)$.

H. Hochstadt (Brooklyn, N. Y.)

Zbl 193.04301

In the theory of transformations of linear differential equations of the 2nd order (Q) $Y'' = Q(T)Y$, (q) $y'' = q(t)y$, the central place is assumed by Kummer's differential equation (Qq) $-\{X,t\} + Q(X)X'^2 = q(t)$, $\{X,t\} = (1/2)(X'''/X') - (3/4)(X''^2/X'^2)$,whose solutions X transform every integral Y of the equation (Q) into a certain integral y of the equation (q) in the sense of the formula: $Y[X(t)]/\sqrt{|X'(t)|} = y(t)$. In case of the definition interval of the equations (Q), (q) being $(-\infty,\infty)$ and these equations being oscillatory, the set $I(Q,q)$ of all solutions of the equation (Qq) is just formed of the so-called general dispersion of this equation, which may be constructively described. In addition to it, $I(Q,q)$ is known to be a subset in the group of phases $\mathfrak{G}$ of linear differential equations of the 2nd order and is given by the formula $I(Q,q) = A^{-1}\mathfrak{C}\alpha$, where A, α denote arbitrarily chosen (first) phases of the equations (Q), (q) respectively and $\mathfrak{C}$ denotes the fundamental subgroup in $\mathfrak{G}$, i. e. $\mathfrak{C}= I(-1,q)$. For details, see author: Lineare Differentialtransformationen 2. Ordnung (1967; this Zbl. **153**, 112). In the paper an algebraic characterization of the set $I(Q,q)$ is studied. The article consists of three parts: In the first part a certain theorem on conjugated subgroups of an abstract group is proved: Let G be an (abstract) group, E $(\subset G)$ its subgroup and N the normalizator of this subgroup in G. Further let A, $a \in G$ be arbitrary elements and $g_a = a^{-1}Ea$, $g_A = A^{-1}EA$ conjugated subgroups with E with respect to a, A. Finally let $G/_l\, g_a$ and $G/_r\, g_A$ denote the left and the right decompositions of the group G with respect to the subgroup g_a and g_A, respectively. Then N coincides with E if and only if the set $A^{-1}Ea$ is the only common element of both the decompositions $G/_l\, g_a$, $G/_r\, g_A$. In the second part is proved that the supposition of this theorem is fulfilled in the case of the group of phases $\mathfrak{G}$ and its fundamental subgroup $\mathfrak{C}{:}\mathfrak{N}{=}\mathfrak{C}$, where $\mathfrak{N}$ is the normalizator of $\mathfrak{C}$ in $\mathfrak{G}$. In the third part the above mentioned theorem is realized by the general dispersions of the equation (Qq). The result is the following characterization of the general dispersions of the equation (Qq): $\xi \in I(Q,q) \Leftrightarrow \xi^{-1}I(Q,Q)\xi = I(q,q)$.

F. Neuman

[21] *Sur les solutions simultanées de deux équations différentielles de Kummer.* IVème Congrès des mathématiciens d'expression latine et Commémoration de Elie Cartan, Bucuresti-Brasov, 1969. Résumés, 3–4.

Tato práce nebyla recenzována v MR ani v Zbl. Jedná se o resumé přednášky proslovené na 4. kongresu matematiků, jenž se konal ve dnech 17. – 24. září 1969.

Přednáška byla věnována otázce, za jakých podmínek mají dvě různé Kummerovy rovnice oscilatorického typu stejné řešení. K řešení této problematiky je využit algebraický přístup k teorii transformací.

[22] *Algebraic elements in the transformation theory of 2nd order linear oscillatory differential equations*. Acta F. R. N. Univ. Comenianae, Mathematica **17** (1967) (Proc. Conf. Equadiff II, Bratislava 1966), 27–36 (1969).

Zbl 218.34005

A survey article dealing with the algebraic aspects of the theory developed in the author's book [Lineare Differentialtransformationen 2. Ordnung (1967; this Zbl. **153**, 112)].

H. Guggenheimer

[23] *Geometric elements in the theory of transformations of ordinary second-order linear differential equations.* Symposium on Differential Equations and Dynamical Systems. Mathematics Institute, University of Warwick, 1968–69, 19–22.

Tato práce nebyla recenzována v MR ani v Zbl. Jedná se o sborník příspěvků proslovených v seminářích, které se konaly v Matematickém Institutu University Warwick v období od 1. září 1968 do 30. června 1969. O. Borůvka zde proslovil přednášku s výše uvedeným názvem. Zavedl pojmy fáze, centrální disperse, transformace a naznačil geometrický význam centrálních dispersí. Odkazuje se přitom na monografii [16] a na práci H. Guggenheimer, *Some geometric remarks about dispersions* (Arch. Math. (Brno), 4 (1968), 193–199).

[24] *Sur quelques propriétés de structure du groupe des phases des équations différentielles linéaires du deuxième ordre*. Rev. Roumaine Math. Pures Appl., **15** (1970), 1345–1356. (French)

MR **43** #6502

In this paper the author investigates some properties of certain groups that arise in the qualitive study of oscillatory differential equations of the form (q) $y'' + qy = 0$, $q \in C(-\infty,\infty)$. The first part of the paper is devoted to the proof of theorems and statement of terminology necessary for the second part. In particular there are results pertaining to the transformation of one subgroup Z_a of a group G into another subgroup Z_b of G. In the second part the following terminology is introduced. By a (first) phase of equation (q) one understands all functions $\alpha(t) \in C(-\infty,\infty)$, satisfying (with the exception of the zeros of the function v) the relation $\tan\alpha(t) = u(t)/v(t)$, where u, v are a linearly independent set of solutions of (q). The basic group of concern to the author is the group of phases G, defined as the group of all function phases that are equivalent to (first) phases and defined as all functions $\alpha(t) \in C(-\infty,\infty)$ with $\alpha'(t) \neq 0$, $\lim_{t\to\pm\infty}\alpha(t) = \pm\infty(\operatorname{sgn}\alpha')$ with group multiplication given by composition of functions. The identity of the group is t. Then there are ten results given, involving this group, its subgroups, and associated group theoretic concepts. A typical result is following: if $c(t) = (t+\pi)c_\nu$ ($\nu = 0, \pm 1, \dots$) is the composition of c with itself ν times, $Z = (\dots, c_{-2}, c_{-1}, t, c_1, c_2, \dots)$ and

$$Z_m = (\dots, c_{-2m}, c_{-m}, t, c_m, c_{2m}, \dots)$$

then all function phases that transform the subgroup Z_m to Z_n are precisely the phases of equation (q) with q defined by

$$q(t) = -\frac{1}{2}\frac{G_n'''(t)}{(\varepsilon + G_n'(t))} + \frac{3}{4}\frac{G_n''^2(t)}{(\varepsilon + G_n'(t))^2} - \frac{m^2}{n^2}(\varepsilon + G_n'(t))^2,$$

ε being ± 1, and $G_n(t)$ a function $\in C^3(-\infty,\infty)$ that is periodic of period $n\pi$ and such that $\operatorname{sgn}(\varepsilon + G_n'(t)) = \varepsilon$. The author's book *Lineare Differential-Transformationen 2. Ordnung*, VEB Deutsch. Verlag der Wissensch., Berlin, 1967 [MR **38** #4743], will be of help as a reference during a first reading of the paper.

H. C. Howard (Lexington, Ky.)

Zbl 216.10901

„Dans le groupe des phases des équations différentielles linéaires du deuxième ordre, G, le centre du sous-groupe formé par les phases-éléments du sous-groupe fondamental qui sont croissantes est un groupe monogène. On étudie les propriétés de structure du groupe G en relation avec le centre en question.“ (Résumé de l'A.)

A. de Castro

[25] *Linear differential transformations of the second order*. Translated from the German by F. M. Arscott. The English Universities Press, Ltd., London, 1971, xvi+254. (English)

MR **57** #3484

The original has been reviewed [Deutsch. Verlag Wissensch., Berlin, 1967; MR **38** #4743]. The author has written two additional chapters for this translation, dealing with (i) an abstract algebraic model for the transformation theory of Jacobian oscillatory differential equations, and (ii) a survey of recent results in transformation theory.

The translator has appended a glossary of English equivalents for certain German technical terms.

Zbl 222.34002

Vgl. die Besprechung des deutschen Originals in diesem Zbl. **153**, 112.

[26] *Sur la périodicité de la distance des zéros des intégrales de l'équation différentielle* $Y'' = Q(T)Y$. Commemoration volumes for Prof. Dr. Akitsugu Kawaguchi's seventieth birthday, Vol. III. Tensor (N.S.) **26** (1972), 121–128. (French)

MR **49** #5438

The author considers the ordinary differential equation (q) $y'' = q(t)y$, $q \in C^0(j)$, $j = (-\infty, \infty)$, and assumes that it is oscillatory. Under these conditions it is possible to define in the interval j a countable system of functions: $\ldots, \varphi_{-2}, \varphi_{-1}, \varphi_0, \varphi_1, \varphi_2, \ldots$, such that, for any integral y of the equation (q) vanishing at $t = \bar{t}$, the values $\varphi_n(\bar{t})$, $\varphi_{-n}(\bar{t})$ represent the nth zero of this integral y, to the right and to the left of $\bar{t}$, respectively. The author then defines the distance function $d(t)$ for the equation (q) by $d(t) = \varphi_1(t) - t$, and studies the equations (q) that admit such a distance function that is periodic with π, i.e., $d(t + \pi) = d(t) + \pi$. He proves several propositions, which are too complicated to be reported here.

{For more complete bibliographic information about the collection in which this article appears, including the table of contents, see MR **48** #15.}

A. Averna (Perugia)

Zbl. 237.34051 (předběžný autoreferát)[20]

Es sei (q) $y'' = q(t)y$, $q \in C^0_{j=(-\infty,\infty)}$, eine oszillatorische Differentialgleichung und φ die Fundamentaldispersion erster Art von (q) ($\varphi(t)$ ist also die erste rechts von t liegende und mit t konjugierte Zahl erster Art). Die Abstandsfunktion d von (q) wird so definiert: $d(t) = \varphi(t) - t$. In der vorliegenden Arbeit werden Differentialgleichungen (q) mit π-periodischen Abstandsfunktionen untersucht: $d(t + \pi) = d(t)$. Dabei kommt insbesondere ein neuer Begriff, u.zw. der von inversen Differentialgleichungen, wesentlich zur Geltung. Die Differentialgleichungen (q), $(\overline{q})$ heißen (zueinander) invers, wenn sie inverse Phasen α, $\overline{\alpha}$ zulassen: $\overline{\alpha}(t) = \alpha^{-1}(t)$, $t \in j$. Einige Resultate: 1. Die Abstandsfunktion von (q) ist dann und nur dann π-periodisch, wenn die Fundamentaldispersion von (q) elementar ist: $\varphi(t + \pi) = \varphi(t) + \pi$. 2. Die Abstandsfunktion von (q) ist dann und nur dann π-periodisch, wenn die Differentialgleichungen mit den Trägern $q(t)$, $q(t + \pi)$ dieselbe Fundamentaldispersion haben. 3. Ist die Abstandsfunktion von (q) π-periodisch, so ist auch die von jeder inversen Differentialgleichung $(\overline{q})$ π-periodisch. 4. Die Differentialgleichungen (q) mit π-periodischen Trägern haben π-periodische Abstandsfunktionen. 5. Der Träger von (q) ist dann und nur dann π-periodisch, wenn die Fundamentaldispersion jeder zu (q) inversen Differentialgleichung eine Phase von $y'' = -y$ ist.

Autorreferat.

Zbl 254.34037

Vgl. das Autorreferat (Voranzeige) in diesem Zbl. 237.34051.

[20] Předběžný autoreferát psal autor v okamžiku, když byl článek přijat k publikaci (viz Zbl 237.34051). Ve chvíli skutečného vydání článku jsou v Zbl (viz Zbl 254.34037) uvedeny pouze přesné bibliografické údaje a odkaz na předešlý autoreferát.

[27] *On central dispersions of the differential equation $y'' = q(t)y$ with periodic coefficients*. Ordinary and partial differential equations (Proc. Conf., Univ. Dundee, Dundee, 1974), Lecture Notes in Math., Vol. 415. Springer – Verlag, Berlin, 1974, 47–61. (English)

MR **56** #8984

The paper continues the author's long history of work on the central dispersion theory for the equation $y'' = q(t)y$. A comprehensive review of this subject may be found in O. Borůvka's article [Differencial′nye Uravnenija **12** (1976), no. 8, 1347–1383; MR **55** #13003.]

{For the entire collection see MR **50** #10391.}

T. L. Sherman (Tempe, Ariz.)

Zbl 313.34008

[This article was published in the book announced in this Zbl. 284.00008.] In this lecture the author shows how his theory of dispersions of linear differential equations of the second order (q): $y'' = q(t)y$, enriches the classical Floquet theory in deep consequences. Let (q) be a both side oscillatory equation defined on $\mathbb{R} = (-\infty, \infty)$. A phase of (q) is any function $\alpha : \mathbb{R} \to \mathbb{R}$ continuous on $\mathbb{R}$ and satisfying $\tan \alpha(t) = u(t)/v(t)$ on $\mathbb{R} - \{t \in \mathbb{R}; v(t) = 0\}$, where u, v are linearly independent solutions of (q). α is called a dispersion phase if $\alpha'(t) > 0$ and either $\alpha(t) > t$ or $\alpha(t) < t$ on $\mathbb{R}$. α is elementary if $\alpha(t + \pi) = \alpha(t) + \pi \cdot \operatorname{sgn} \alpha'$ on $\mathbb{R}$. All phases of the equation $y'' = y$ on $\mathbb{R}$ with the composition rule form the so called fundamental group $\mathfrak{C}$. For each integer n, the central dispersion φ_n of (q) is defined as follows: $\varphi_n(t)$ is the $|n|$-th conjugate number with the number t, greater or smaller than t according as $n > 0$ or $n < 0$; $\varphi_0(t) = t$. An equation $(\overline{q})$ is called inverse of (q) if $(\overline{q})$ has a phase $\overline{\alpha}$ which is inverse function of some phase α of (q), $\overline{\alpha} = \alpha^{-1}$. For more details see author [Linear differential transformations of the second order (London 1971, German original 1967; Zbl **153**, 112)]. The set of all equations (q) with $q(t + \pi) = q(t)$ on $\mathbb{R}$ is denoted as A_p. The set A is formed by all (q), whose φ_1 satisfy $\varphi_1(t + \pi) = \varphi_1(t) + \pi$ on $\mathbb{R}$. Some of results introduced in the lecture: „$(q) \in A_p$ iff each phase α of (q) satisfies $\alpha(t + \pi) = \varepsilon\alpha(t)$, where $\varepsilon \in \mathfrak{C}$ is a dispersion phase“, or „$(q) \in A_p$ iff all the central dispersions of each inverse equation of (q) lie in the fundamental group $\mathfrak{C}$“, or „ $(q) \in A$ iff each phase α of (q) satisfies $\alpha(t + \pi) = h\alpha(t)$ on $\mathbb{R}$ for an elementary dispersion phase h“, or

$$"(\frac{m}{M})^{\frac{\sqrt{M}}{2}} \leq |s_{1,2}| \leq (\frac{M}{m})^{\frac{\sqrt{M}}{2}},$$

where $s_{1,2}$ are real periodicity factors of $(q) \in A_p$ according to the Floquet theory, $q(t) > 0$ on $\mathbb{R}$ and $m := \min q(t)$ and $M = \max q(t)$ for $t \in \mathbb{R}$". It is very important that the whole theory of differential equations (q) with periodic coefficients can be expressed in a purely algebraic way and that these equations can be studied in the range of the abstract algebraic theory of oscillatory equations (q) given axiomatically.

F. Neuman

[28] *Sur la structure algébrique de la théorie des transformations différentielles linéaires du deuxième ordre.* Acta F. R. N. Univ. Comenianae, Mathematica **31** (1975), 59–71. (French. Czech, Russian summary)[21]

MR **52** #11169

This paper is closely connected with the author's book [*Lineare Differentialtransformationen 2. Ordnung*, VEB Deutscher Verlag Wissensch., Berlin, 1967; MR **38** #4743; English translation, English Universities Press, London, 1971] and uses the terminology and notation introduced there. The author considers oscillatory Sturm-Liouville equations (q) $y'' = q(t)y$ in the interval $j = (-\infty, \infty)$ $(q \in C_j^0)$. A phase $\alpha(t)$ is defined by $\operatorname{tg} \alpha(t) = u(t)/v(t)$, $\{u, v\}$ being a fundamental system of solutions. A bijective transformation (X): $T = X(t)$, $y = c/\sqrt{|X'(t)|}Y$ of the (T, Y)-plane on the (t, y)-plane induces a transformation of bases and phase functions. The phase functions form a group with respect to superposition. The author gives a more detailed study of these groups and an abstract characterization of these groups and of the transformation theory of oscillatory Sturm-Liouville equations in general.

G. Eisenreich (Leipzig)

Zbl 332.34009

The first part of this lecture (given at the „Czechoslovak Conference on Differential Equations and their Applications" in Brno, 1972) contains a survey of the theory of transformations of oscillatory Jacobian differential equations (1) $y'' = -q(t)y$ [cf. author, Lineare Differentialtransformationen 2. Ordnung (1967; Zbl **153**, 112), English translation by F. M. Arscott (London 1971)]. In the main second part the algebraic structure of this theory is described by a model essentially composed of the following ingredients. Given a group G (set of phase-functions), a subgroup G_0 (set of increasing phase-functions) of index 2 and a subgroup E (phases of $y'' = -y$), which have the properties: The normalizer of E equals E. The center Z of $E_0 := E \cap G_0$ is an infinite cyclic group corresponding to the set of phases $\{t + k\pi | k \in \mathbb{Z}\}$. Furthermore, there is given a homomorphism H of E onto the group $U := \{A \in GL(2, \mathbb{R}) | \det A = \pm 1\}$ such that $H(E_0) = \{A \in U | \det A = 1\}$ and the kernel of H is the subgroup of Z of index 2. The decomposition of G into right cosets of E corresponds to the phases of the different equations (1). By associating to each coset $\overline{a} := Ea$ a 2-dimensional real vector space $L\overline{a}$ (the space of solutions of the corresponding equation (1)) and by defining a quasinorm for bases of $L\overline{a}$, it is finally possible to describe the Kummer transformations in terms of the given algebraic objects. A list of open questions concludes the paper.

J. Hainzl

[21] Jedná se o plenární přednášku, kterou O. Borůvka proslovil na konferenci Equadiff III v Brně dne 28. 8. 1972. Tato přednáška nebyla otisknuta v Proceedings - Equadiff 3, Brno, 1972 (Vydala UJEP Brno, 1973). Důvodem nezahrnutí této přednášky do Proceedings byla politicky podmíněná skutečnost, že možnosti O. Borůvky při publikování příspěvků byly v té době poněkud omezené zejména v publikacích brněnské univerzity. Laskavostí bratislavských matematiků proto bylo zahrnutí této přednášky do časopisu Acta F. R. N. Univ. Comenianae.

Citujme z dopisu O. Borůvky redakci Acta F. R. N. Univ. Comenianae z 26. 3. 1973:

... po dohodě s prof. M. Gregušem, který byl předsedou mezinárodní konference EQUADIFF III, která se konala v Brně ve dnech 28. 8. – 1. 9. 1972, žádám Vás o laskavé uveřejnění přiloženého rukopisu, který nemohl vyjít ve Sborníku konference. Byl bych Vám vděčen, kdybyste můj rukopis mohli zařadit k uveřejnění co nejdříve, aby nenastalo příliš velké zpoždění za Sborníkem, který vyjde v nejbližší době...

[29] *Sur quelques compléments à la théorie de Floquet pour les équations différentielles du deuxième ordre*. Ann. Mat. Pura Appl., (4) **102** (1975), 71–77. (French)

MR **51** #10732

The author considers the ordinary differential equation (q) $y'' = q(t)y$, $q \in C^0(J)$, on the interval $J = (-\infty, \infty)$, where $q(t) < 0$ and $q(t+\pi) = q(t)$ for $t \in J$. He shows that the characteristic roots of the given equation and the function q are closely related. We state Theorem 2: If the characteristic roots s_1 and s_2 of the equation (q) are real then

$$(\frac{m}{M})^{\frac{\sqrt{M}}{2}} \leq |s_i| \leq (\frac{M}{m})^{\frac{\sqrt{M}}{2}} \qquad (i = 1, 2),$$

where $m = \min_{t\in j}|q(t)|$ and $M = \max_{t\in j}|q(t)|$.

A. Averna (Perugia)

Zbl 311.34012

In this article the author demonstrates connections between Floquet theory and his „Theory of dispersions" of linear oscillatory differential equations (1): $y'' = q(t)y$, $t \in \mathbb{R}$, and derives results that enriche both the theories in deep and interesting consequences. A function φ is (basic central) dispersion (of the 1st kind) of (1), if $\varphi(t_0)$ is the 1st zero on the right of t_0 of any nontrivial solution y of (1) that vanishes at t_0: $y(t_0) = 0$. Then φ_n denotes the n-th iterate of φ, and $d_n(t) := \varphi_n(t) - t$ is called the distance function of the index n. For more complete details and results see author [Linear diffeential transformations of the second order (London 1971, German original 1967; Zbl. **153**, 112)]. For $q(t) < 0$, $q(t+\pi) = q(t)$, let $m := \min\{|q(t)|;\, t \in \mathbb{R}\}$, $M := \max\{|q(t)|;\, t \in \mathbb{R}\}$, and s_1, s_2 denote the roots of the characteristic equation corresponding to (1) according to Floquet theory. Then s_1 and s_2 are real and positive (negative) iff there exist $x \in \mathbb{R}$ and positive even (odd) integer n such that $\sqrt{m} \leq n \leq \sqrt{M}$ and $d_n(x) = \pi$. In such a situation, $s_1 = (-1)^n \cdot (\varphi_n'(x))^{-1/2}$ and $s_2 = (-1)^n \cdot (\varphi_n'(x))^{1/2}$. Moreover, $s_1 \neq s_2$ iff $d_n'(x) \neq 0$. If s_i, $i = 1, 2$, are real, then

$$(\frac{m}{M})^{\frac{\sqrt{M}}{2}} \leq |s_i| \leq (\frac{M}{m})^{\frac{\sqrt{M}}{2}}.$$

Especially for Mathieu equation $y'' + (\lambda - 2h^2\cos 2t)y = 0$, $h \in \mathbb{R}$, $\lambda > 2h^2$, we get

$$\left(\frac{1-2h^2/\lambda}{1+2h^2/\lambda}\right)^{\frac{1}{2}\sqrt{\lambda+2h^2}} \leq |s_i| \leq \left(\frac{1+2h^2/\lambda}{1-2h^2/\lambda}\right)^{\frac{1}{2}\sqrt{\lambda+2h^2}}, \qquad i = 1, 2.$$

F. Neuman

[30] *Sur les blocs des équations différentielles $Y'' = Q(T)Y$ aux coefficients périodiques*. Rend. Mat., (6) **8** (1975), 519–532. (French. Italian summary)

MR **52** #849

The author continues the algebraic study of differential equations begun in his book [*Lineare Differentialtransformationen 2. Ordnung*, VEB Deutsch. Verlag Wissensch., Berlin, 1967; MR **38** #4743; English translation, English Universities Press, London, 1971]. The book not only explores an area of linear differential equations little noted in the U. S. A. (though the basic formulas are due to W. Leighton [Trans. Amer. Math. Soc. **68** (1949), 253–274; MR **11**, 603]) and aspects of affine and projective differential geometry going back to E. Kummer and H. A.

Schwarz and rediscovered by more recent authors [cf. H. Flanders, J. Differential Geometry **4** (1970), 515–519; MR **43** #2619] but also has an interesting approach to groups of monotone functions defined on an interval and hence should be of interest to students of simple groups. The setting of the present paper is the following. Let $\mathfrak{G}$ be the group of strictly monotone C^2 functions on $[0,\infty)$ and $\mathfrak{F}_0$ the isomorphic image in $\mathfrak{G}$ of the double covering of SL(2, $\mathbb{R}$) in $\mathfrak{G}$ given by $t \mapsto \tan^{-1}(a\tan t + b)/(c\tan t + d)$, $ad - bc = \pm 1$, $\tan^{-1} 0 = 0$. Let $\mathfrak{F}$ be the group generated by $\cup\mathfrak{F}_n$, where $\mathfrak{F}_n$ is defined as $\mathfrak{F}_0$ but with $\tan^{-1} 0 = n\pi$. The groups that play a role in the theory of differential equations are the groups $\mathfrak{H}$ that contain $\mathfrak{F}$: $\mathfrak{F} \subset \mathfrak{H} \subset \mathfrak{G}$. A coset $\mathfrak{F}g$ for $g \in\mathfrak{G}$ is a differential equation $y'' + q(t)y = 0$, i.e., all elements of $\mathfrak{F}g$ are phase functions α, $\tan\alpha = y_2/y_1$ for some basis of the solution space of the equation for $q(t) = \frac{1}{2}\{\tan\alpha, t\}$ and all phase functions belonging to one equation are in the coset. The cosets $g\mathfrak{F}$ contain one phase function each of a „block" of differential equations. Theorem: Let $\mathfrak{H}$ be the group of phases of Hill equations $(q(t+\pi) = q(t))$. Then the Floquet multipliers [and the Ljapunov discriminant] are constant on the blocks of $\mathfrak{H}/\mathfrak{F}$ [are functions of the homogeneous space $\mathfrak{H}/\mathfrak{F}$]. {The author would help the general acceptance of his approach by changing the name of his „central dispersions" to the accepted „conjugate points".}

{For the entire collection see MR **51** #9998.}

H. W. Guggenheimer (Brooklyn, N. Y.)

Zbl 326.34007

Let (1): $y'' = q(t)y$ be oscillatory both for $t \to -\infty$ and for $t \to \infty$. In his transformation theory of those equations the author introduced a (1st) phase of (1) as a continuous function α satisfying $\tan\alpha(t) = u(t)/v(t)$ for an independent pair u, v of solutions of (1). All phases for all equations (1) form a group G (with the superposition law) and all phases of the equation $y'' = -y$ on $(-\infty,\infty)$ form the subgroup F. Elements of the right decomposition G with respect to F are in 1 – 1 correspondence with the equations (1). Elements of the least common covering of the right and left decompositions G with respect to F are called „blocks". The author shows how this algebraic approach enriches the classical Floquet theory of periodic equations (1) – „The law of inertia of characteristic multipliers": All equations (1) with π-periodic coefficients q corresponding to the same block have the same characteristic multipliers.

F. Neuman

[31] *Ueber die Differentialgleichungen $y'' = q(t)y$ mit periodischen Abständen der Nullstellen ihrer Integrale*. 5. Tagung über Probleme und Methoden der Mathematischen Physik, Technische Hochschule Karl-Marx-Stadt, 1975. Wiss. Schr. Techn. Hochsch. Karl-Marx-Stadt, Heft 2, 1975, 239–255. (English)

MR **55** #3416

The author investigates a class of differential equations $y'' = q(t)y$ by the aid of transformation theory and his own theory of dispersions. Although the important ideas are explained, the author's earlier book [*Lineare Differentialtransformationen 2. Ordnung*, Deutsch. Verlag Wissensch., Berlin, 1967; MR **38** #4743] will be of help during the reading of this lecture.

{For the entire collection see MR **54** #12428.}

L. Pintér (Szeged)

Zbl 398.34031

[This article was published in the book announced in this Zbl. 373.00008.] The aim of this paper is to investigate oscillatory properties of solutions of second order differential equation

(q) $y'' = q(t)y$, with $q : \mathbb{R} \to \mathbb{R}$ continuous, and satisfying further conditions. For each integer n, let $\rho_n(t)$ be defined as follows: $\rho_0(t) = t$, $t \in \mathbb{R}$; for $|n| \geq 1$, $\rho_n(t)$ is the $|n|$-th conjugate point to the point t, $t \in \mathbb{R}$, situated at right with respect to t when $n > 0$, and at left with respect to when $n < 0$. The function $\rho_n(t)$ is called the central dispersion with index n. The main attention is paid to the case when the functions $d_n(t) = \rho_n(t) - t$ are periodic of period π. The equations (q) for which this property holds true are said to belong to the class A. Connections with groups theory are emphasized. The paper is very much in the vein of the author's book „Linear differential transformations of the second order" (1971; Zbl. 222.34002).

C. Corduneanu

[32] *Contribution à la théorie algébrique des équations $Y'' = Q(T)Y$.* Boll. Un. Mat. Ital., B (5) **13** (1976), 896–915. (French. Italian summary)

MR **58** #22789

Linear second-order differential equations of the stated type are assumed to have periodic solutions on the real line for each function Q. The set of all such equations is divided into (disjoint) equivalence classes via the phases or phase functions of the equation $y'' = -y$; two phases are equivalent if there is an invertible bilinear transformation linking them. Thus the phases form a group under (function) composition and the properties of the phases are translated into algebraic terms: periodicity, evenness, monotonicity are made equivalent to the properties of various subgroups. Explicit formulae are given for all transformations and properties, and an example is worked out for the Mathieu equation. The author does not make clear why one is studying the phases of differential equations, nor to what use these may be put; the article relies somewhat on the author's book referred to therein [*Lineare Differentialtransformationen 2. Ordnung*, Deutsch. Verlag Wissensch., Berlin, 1967; MR **38** #4743].

J. J. Cross (Zbl **364** #34002)

Zbl 364.34002 (stejné jako v MR)

Linear second-order differential equations of the stated type are assumed to have periodic solutions on the real line for each function Q. The set of all such equations is divided into (disjoint) equivalence classes via the phases or phase functions of the equation $y'' = -y$; two phases are equivalent if there is an invertible bilinear transformation linking them. Thus the phases form a group under (function) composition and the properties of the phases are translated into algebraic terms: periodicity, evenness, monotonicity are made equivalent to the properties of various subgroups. Explicit formulae are given for all transformations and properties, and an example is worked out for the Mathieu equation. The author does not make clear why one is studying the phases of differential equations, nor to what use these may be put; the article relies somewhat on the author's book referred to therein [Lineare Differentialtransformationen 2. Ordnung (1967; **153**, 122)].

J. J. Cross

[33] *Diferenciální rovnice $Y'' = Q(T)Y$ s periodickými koeficienty v souvislosti s teorií dispersí.* Knižnice odborných a vědeckých spisů VUT v Brně, B-67, 1976, 31–42.

Zbl 416.34034

In the article, the relations are described between the characteristic roots of the Hill differential equation and some elements of the dispersion theory, especially phases and central dispersions. Special attention is devoted to the so-called blocks of oscillatoric equations $y'' = q(t)y$ ($q(t) \in C_j^0$, $j = (-\infty, \infty)$). These are characterized by that all equations of the same block originate from one of them by transformations of the independent variable by phases $\varepsilon(t)$ of the equation $y'' = -y : t \to \varepsilon(t)$. Simultaneously, all the equations of the same block have or have not periodic carriers, e. g. with π. For the blocks the so called theorem of inertia of characteristic roots holds: Equations of the same block with π-periodic carriers have identical characteristic roots and, at the same time, they all have or have not all their integrals π-semiperiodic or π-periodic.

Summary.

[34] *Теория глобальных свойств обыкновенных линейных дифференциальных уравнений второго порядка.* Дифференциальные уравнения, Минск, **12** (1976), no. 8, 1347–1383, 1523. (Russian)

Theory of the global properties of ordinary second order linear differential equations. Differential Equations **12** (1976), no. 8, 949–975 (1977).

MR **55** #13003

The author gives a self-contained survey of the global theory of Sturm-Liouville equations developed in his book [*Lineare Differentialtransformationen 2. Ordnung*, Deutsch. Verlag Wissensch., Berlin, 1967; MR **38** #4743; English translation, *Linear differential transformations of the second order*, English Univ. Press, London, 1971] and in some other papers (especially theory of phases, dispersion theory, Kummer transformations, algebraic theory of oscillatory equations).

{English translation: Differential Equations **12** (1976), no. 8, 949–975 (1977).}

G. Eisenreich (Leipzig)

Zbl 348.34007

The article is a very nice survey of the global structure of linear homogeneous differential equations of the second order in the real case and give s brief but comprehensive view on the results developed and published by O. Borůvka [Lineare Differentialtransformationen 2. Ordnung (1967; Zbl. **153**, 112) (English translation with supplement: Linear differential transformations of the second order, The English Universities Press, London 1971)] and in several papers of the author and his pupils in the last 25 years. On the contrary to the investigations of Kummer, Laguerre, Brioschi, Forsyth, and others, started in the middle of the last century and having been of local character, here the original investigations are global, and besides many new results concerning mainly the algebraic structure of both side oscillatory differential equations of the form $y'' = q(t)y$ on $\mathbb{R}$, they cover also classical subjects, like differential equations with periodic coefficients and Floquet Theory, studying them in this modern and original setting. In the last chapter there is a short account of the latest results concerning the global theory of linear differential equations of the n-th order, $n \geq 2$. The article should not be left without attention of those who are interested in the global theory of linear differential equations since it presents an original and modern approach to the area in brief however exact form and gives a perfect orientation in the field without requirements on supplementary sources.

F. Neuman

Zbl 375.34012 (anglický překlad)

Translation from Differencial'nye Uravnenija **12**, 1347–1383 (1976; Zbl 348.34007)

[35] *Algebraic methods in the theory of global properties of the oscillatory equations* $Y'' = Q(t)Y$. Lecture Notes in Mathematics 703 (Proc. Conf. Equadiff IV, Prague 1977). Springer – Verlag, Berlin, 1979, 35–45. (English)

MR **80** #34037

The basic statements of the algebraic method, developed by the author and his scientific school, for studying global properties of oscillatory equations $y'' = Q(t)y$ are presented.
{For the entire collection see MR 80c:34002.}

I. Kiguradze (Tbilisi)

Zbl 405.34009

[Dieser Artikel erschien in dem in diesem Zbl. 393.00004 angezeigten Sammelwerk.] Die Theorie der linearen Differentialgleichungen (Diffgen) 2. Ordnung im reellen Gebiet, (Q): $y'' = Q(t)y$, $Q \in C_j^{(0)}$, $j = (-\infty, \infty)$, in ihrem vollen Umfang, weist zahlreiche Bindungen an algebraischen und differentialgeometrischen Fragestellungen auf [vgl. O. Borůvka, Linear Differential Transformations of the Second Order (1967; Zbl. **153**, 112)]. Dies gilt insbesondere für oszillatorische Diffgen (Q). In diesem Fall werden jeder Diffg (Q) die sogen. adjungierten Gruppen zugeordnet: $\mathfrak{U}_Q \supset \mathfrak{B}_Q \supset \mathfrak{B}_Q^+ \supset \mathfrak{L}_Q$; $\mathfrak{B}_Q$ ist die Dispersionsgruppe von (Q), $\mathfrak{B}_Q^+$ die von den wachsenden Dispersionen von (Q) gebildete Untergruppe von $\mathfrak{B}_Q$, $\mathfrak{L}_Q$ ist das Zentrum von $\mathfrak{B}_Q^+$, $\mathfrak{U}_Q$ ist der Normalisator von $\mathfrak{L}_Q$ in der Phasengruppe. Zwischen den adjungierten Gruppen von zwei Diffgen (Q), (P) bestehen gewisse Beziehungen, die in den sogen. Inklusionssätzen beschrieben werden, wobei diese zum Teil von dualem Charakter sind. Wird (Q) mittels des Transformators X in (P) (global) transformiert, so gehen die mit X transformierten adjungierten Gruppen von (Q) in die entsprechenden adjungierten Gruppen von (P) über: $X^{-1}\mathfrak{U}_Q X = \mathfrak{U}_P$, usw. Im Fall $Q = -1$ sind die adjungierten Gruppen explizit darstellbar. Jede Diffg (Q) kann vermöge ihrer Phasen in die Diffg (-1) transformiert werden. Es gereicht zum Vorteil, diese letztere wegen ihrer Einfachheit in den Mittelpunkt von Betrachtungen zu stellen, in dem Sinn, daß die Diffgen (Q) als Transformierte von (-1) angesehen werden. Die entsprechende Theorie wird als die spezialisierte algebraische Theorie der Diffgen (Q) bezeichnet. Weitere den Diffgen (Q) zugeordneten Objekten algebraischen Ursprungs sind die inversen Diffgen und Blöcke von Diffgen. Die Menge aller Diffgen (Q) zerfällt in Blöcke, die paarweise zueinander invers sind. Je zwei in zueinander inversen Blöcken liegende Diffgen sind zueinander invers. Untersuchungen über diese Begriffe, im Rahmen der gesamten Theorie ergeben zahlreiche neue Resultate. Im Fall von Diffgen (Q) mit periodischen Koeffizienten erhält man eine weitgehende Erweiterung und Vertiefung der Floquetschen Theorie der linearen Diffgen 2. Ordnung.

Autorreferat.

[36] *Sur une classe des groupes continus à un paramètre formés des fonctions réelles d'une variable*. Ann. Polon. Math. **42** (1983), 25–35. (French)

MR **85** #58014

The author studies the groups of continuous one-to-one mappings from $\mathbb{R}$ to $\mathbb{R}$ satisfying: for all $(t, x) \in \mathbb{R}^2$ there is exactly one $g \in \mathfrak{G}$ such that $g(t) = x$. The mappings $g \in \mathfrak{G}$ are strictly increasing; and the relation $g \preceq h$, defined as "$g(t) \leq h(t)$ for some $t \subset \mathbb{R}$", is a linear and Archimedean order on $\mathfrak{G}$. So there are isomorphisms $h\colon \mathbb{R} \to \mathfrak{G}$. If $S\colon \mathbb{R}^2 \to \mathbb{R}$ is defined by $S(t, x) = h(x)(t)$, there is a continuous and increasing function $G\colon \mathbb{R} \to \mathbb{R}$ such that $G(S(t, x)) = G(t) + G(x)$. The author shows that the function G, which is uniquely defined except for a constant multiplier, characterizes the group $\mathfrak{G}$.

François Aribaud (Paris)

Zbl 533.26003

The theory of continuous iteration groups on $\mathbb{R}$ is developed under supposition of 'planarity' ('completeness'): through every point of $\mathbb{R}^2$ goes the graph of exactly one function of the group. Further results are due to *G. Blanton* [Arch. Math., Brno 18, 121–128 (1982; Zbl. 518.26002), joint wth *J. Baker*; C. R. Math. Acad. Sci., Soc. R. Can. 5, 169–172 (1983; Zbl. 518.26003); Aequationes Math. (to appear)] on]0,1[rather than $\mathbb{R}$.

J. Aczél

[37] *Sur les transformations simultanées de deux équations différentielles linéaires du deuxième ordre dans elles-mêmes*, Applicable Analysis **15** (1983), no. 1–4, 187–200. (French)

MR **85** #34034

Consider the second-order linear, oscillatory differential equation (Q): $y'' = Q(t)y$ $(t \in \mathbb{R} = (-\infty, \infty), Q \in C^0)$. We know that there exist functions $X : \mathbb{R} \to \mathbb{R}$, called the dispersions of (Q), which transform the differential equation (Q) into itself. The dispersions of (Q) are precisely the solutions of the differential equation $-\{X, t\} + Q(X)X'(t) = Q(t)$, and they form a group $\mathfrak{B}_Q$ depending on three parameters. The increasing dispersions of (Q) form an invariant subgroup $\mathfrak{B}_Q^+$ of $\mathfrak{B}_Q$.

Let $(Q_1), (Q_2)$ be arbitrary equations of the kind considered. The intersection $\mathfrak{P}^+_{Q_1Q_2} = \mathfrak{B}^+_{Q_1} \cap \mathfrak{B}^+_{Q_2}$ is, of course, a group whose elements simultaneously transform the equations $(Q_1), (Q_2)$ into themselves. The main result of the present paper implies that the group $\mathfrak{P}^+_{Q_1Q_2}$ is *o*-isomorphic to a subgroup of the additive group of real numbers. Further properties of $\mathfrak{P}^+_{Q_1Q_2}$ depend primarily on whether the set $B = E_x = \{x \in \mathbb{R},\, Q_1(x) - Q_2(x) = 0\}$ is empty or not and also, in both cases, on other relations between the functions Q_1, Q_2. These relations cannot be given in detail here. In particular, if $B \neq 0$, then the group $\mathfrak{P}^+_{Q_1Q_2}$ is trivial $(= \{\text{identity}\})$ or is an infinite cyclic group.

O. Borůvka (Brno)

Zbl 494.34021 (předběžný autoreferát)

Eine lineare oszillatorische Differentialgleichung zweiter Ordnung (Q): $y'' = Q(t)y$ $(t \in \mathbb{R}, Q \in C^0)$ kann durch geeignete Funktionen, die Dispersionen von (Q), in sich transformiert werden. Die Dispersionen von (Q) sind genau die Integrale der Differentialgleichung $-\{X, t\} + Q(X)X'^2(t) = Q(t)$ und bilden eine dreiparametrige Gruppe $\mathfrak{L}_Q$. Die wachsenden Dispersionen von (Q) bilden einen Normalteiler $\mathfrak{L}^+_Q$ von $\mathfrak{L}_Q$. In der vorliegenden Arbeit werden gemeinsame wachsende Dispersionen von zwei Differentialgleichungen (Q_1), (Q_2) $(Q_1 \neq Q_2)$ untersucht. Dieselben bilden die Gruppe $\mathfrak{P}^+_{Q_1Q_2} = \mathfrak{L}^+_{Q_1} \cap \mathfrak{L}^+_{Q_2}$. Das Hauptergebnis der Untersuchung ist der Satz, daß die Gruppe $\mathfrak{P}^+_{Q_1Q_2}$ zu einer Untergruppe der additiven Gruppe der reellen Zahlen o-isomorph ist. Die Eigenschaften der Gruppe $\mathfrak{P}^+_{Q_1Q_2}$ hängen zunächst davon ab, ob die Menge $B = E$ $(x \in \mathbb{R}, Q_1(x) - Q_2(x) = 0)$ leer ist oder nicht, und in beiden Fällen auch noch von anderen Beziehungen der Funktionen Q_1, Q_2 zueinander. Im Falle $B \neq \emptyset$ ist die Gruppe $\mathfrak{P}^+_{Q_1Q_2}$ entweder die triviale Gruppe $\{id\}$ (z. B. dann, wenn die Menge B beschränkt ist) oder aber eine unendliche zyklische Gruppe. Im Falle $B = \emptyset$ kommen für Gruppe $\mathfrak{P}^+_{Q_1Q_2}$ neben den zwei soeben angeführten Typen auch andere Typen in Betracht. Unter gewissen Umständen ist die Gruppe $\mathfrak{P}^+_{Q_1Q_2}$ planar, d. h. so beschaffen, daß durch jeden Punkt der Ebene $\mathbb{R} \times \mathbb{R}$ genau ein Element von $\mathfrak{P}^+_{Q_1Q_2}$ hindurchgeht.

Autorreferat.

Zbl 506.34031

See the preview (Autorreferat) in Zbl. 494.34021.

[38] *Sur les sous-groupes planaires des groupes des dispersions des équations différentielles lineáires du deuxième ordre*. Proc. Roy. Soc. Edinburgh Sect. A **97** (1984), 35–41. (French. English summary)

MR **86** #34058

The paper is a continuation of previous works of the author [cf. *Linear differential transformations of the second order*, English translation, English Universities Press, London, 1971; MR **57** #3484; Ann. Polon. Math. **42** (1983), 25–35; MR 85b:58014]. Let Q be a continuous real-valued function which is defined on $\mathbb{R}$, and (E) the second-order differential equation $y'' = Q(t)y$. A C^3-diffeomorphism of $\mathbb{R}$ is called a dispersion of Q if, for each solution y of E, the function $z(t) = y(f(t))/\sqrt{f'(t)}$ is also a solution of (E). With the usual composition of mappings, the set of the dispersions of Q is a group $\mathfrak{V}_Q^+$. Dispersions are characterized as solutions of the highly nonlinear differential equation $-\frac{1}{2}S(f) + Q(f)f'^2 = Q$, where $S(f)$ denotes the Schwarzian derivative of f.

The paper is devoted to some results concerning the algebraic structure of the group $\mathfrak{V}_Q^+$. A subgroup of $\mathfrak{V}_Q^+$ is said to be planar if through each point of the plane $\mathbb{R}^2$ there passes just one element of the subgroup. The author shows: (i) the planar subgroups of a given $\mathfrak{V}_Q^+$ form a system depending on two constants, the intersection of which is exactly the center of $\mathfrak{V}_Q^+$; (ii) the intersection of those groups $\mathfrak{V}_Q^+$ which contain a given planar group $\mathfrak{S}$ is exactly $\mathfrak{S}$.

François Aribaud (Paris)

Zbl 554.34026

A group $\mathfrak{S}$ consisting of real continuous functions of one real variable on the interval $j = (-\infty, \infty)$ is called planar if through each point of the plane $j \times j$ there passes just one element $s \in \mathfrak{S}$. Every differential oscillatory equation (Q): $y'' = Q(t)y\ (t \in j = (-\infty, \infty), Q \in C^{(0)})$ admits functions, called the dispersions of (Q), that transform (Q) into itself. These dispersions are integrals of Kummer's equation (QQ): $-\{X, t\} + Q(X)X'^2(t) = Q(t)$ and form a three-parameter group $\mathfrak{B}_Q$, known as the dispersion group of (Q). The increasing dispersions of (Q) form a three-parameter group $\mathfrak{B}_Q^+(\subset \mathfrak{B}_Q)$ invariant in $\mathfrak{B}_Q$. The centre of the group $\mathfrak{B}_Q^+$ is an infinite cyclic group $\mathfrak{C}_Q$, whose elements, the central dispersions of (Q), describe the position of conjugate points of (Q).

The present paper contains new results concerning the algebraic structure of the group $\mathfrak{B}_Q^+$. It provides information on the following: (1) the existence and properties of planar subgroups of a given group $\mathfrak{B}_Q^+$ and (2) the existence and properties of the groups $\mathfrak{B}_Q^+$ containing a given planar group $\mathfrak{S}$. The results obtained are: the planar subgroups of a given group $\mathfrak{B}_Q^+$ form a system depending on two constants, $\mathcal{S}Q$, such that $\cap\mathfrak{S} = \mathfrak{C}_Q$ for all $\mathfrak{S} \in \mathcal{S}Q$. The equations (Q) whose groups $\mathfrak{B}_Q^+$ contain the given planar group $\mathfrak{S}$ form a system dependent on one constant, $\mathcal{Q}S$, such that $\cap\mathfrak{B}_Q^+ = \mathfrak{S} = \cup\mathfrak{C}_Q$ for all (Q)$\in \mathcal{Q}S$.

Autorreferat.

[39] *Sur les blocs des équations différentielles linéaires du deuxième ordre et leurs transformations*. Čas. pěst. mat. fys. 1, **111** (1986), 78–88, 90. (French)

MR **88** #34046

The author considers a class (E) of second-order linear equations $(P)\ y'' = P(t)y$, where $P: (-\infty,\infty) \to (-\infty,\infty)$ is continuous. It is assumed that each equation in the class (E) is oscillatory and that the solutions oscillate as $t \to \infty$ and as $t \to -\infty$. Let B_p be the group of transformations of (P) into itself. This paper is concerned with the decompositions of B_p.

John R. Graef (1-MSS)

Zbl 596.34022

Es sei M die Menge der linearen oszillatorischen Differentialgleichungen $P: y'' = P(t)y$ ($P \in C^0_R$, $R = (-\infty,\infty)$). Ferner sei G die Phasengruppe und α der folgende Homomorphismus von G in die Gruppe S(M) der bijektiven Abbildungen der Menge M auf sich: Das Bild von $P \in M$ in der dem Element $\omega \in G$ zugeordneten Bijektion $\alpha(\omega) = \varphi_\omega \in S(M)$ ist die Differentialgleichung ($\varphi_\omega(P) =$) $Q \in M$ mit dem Träger $Q(t) = -\{\omega, t\} + P[\omega(t)] \cdot \omega'^2(t)$ ($t \in R$). Man spricht von der ω-Transformation von P in Q. In dieser Situation stellt (M,G;α) einen (algebraischen) homogenen Raum mit dem Operatorenbereich G dar.

Es sei $P \in M$. Ausgehend von P werden gewisse Untermengen von M, die Blöcke mit der Basis P, konstruktiv definiert und ihre Eigenschaften untersucht. Insbesondere ist die Menge dieser Blöcke eine Zerlegung von M und der die Differentialgleichung $X \in M$ enthaltende Block stellt die von der Dispersionsgruppe von P erzeugte Trajektorie von X dar. Die betrachteten Zerlegungen von M sind im bezug auf ω- Transformationen ihrer Basen mit den letzteren kovariant.

Autorreferat.