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Otakar Borůvka (1899–1995)

OTAKAR BORŮVKA AND FRENCH MATHEMATICS

Eduard Fuchs

We are standing at the threshold of the 21^{st} century and in all kinds of connections we recapitulate the events of the last century. If we compare the state of mathematics today and one hundred years ago we often remain astonished how wise and far-sighted our predecessors were not only as far as the questions of science are concerned, but even referring to methods of its further development. At the same time, the aspects of the development of science which even the greatest thinkers could not foresee and anticipate, take our breath away.

It would be easy to document both these poles on one of the greatest mathematical personalities of the last break of times, on David Hilbert (1862–1943), who so foresightedly outlined the development of mathematics in 20^{th} century in his famous lecture on 23 problems, which he read at 2^{nd} World Mathematical Congress in Paris in August 1900.

In this work we want to deal with a personality who influenced outstandingly by his work the development of mathematics in 20^{th} century in both mentioned aspects – in "classical" spheres, for mathematics of 20th century "anticipated", as the differential geometry, algebra or the theory of differential equations are as well as at the same time in new theories, which dynamic development could not be foreseen at the end of 19^{th} century as e.g. the theory of graphs and discrete optimalizations are. This personality is one of the greatest Czech mathematicicans, Otakar Borůvka, whose life and scientific work are in many aspects typical for fate of intellectuals and for the development of mathematics in 20^{th} century. Moreover we shall try to demonstrate that his link with the French mathematics and French mathematicians was for his life and work fundamental and in any case it cannot be considered marginal or violently created.

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OTAKAR BORŮVKA was born at the end of 19^{th} century on 10^{th} May 1899 in a small South Moravian town Uherský Ostroh. If we wanted to express in one sentence what was characteristic for 20^{th} century in his life we could talk about his studies during World War I when, under the pressure of recruitment and command to front, he interrupted studies at the secondary school from where he went on to the Military Technical Academy in Mödling near Vienna in Austria (for us who knew him as a respectable old man the idea of a young O. Borůvka riding and fencing with a helmet not very comfortable because of strong glasses he had been then wearing was very funny); we could talk about his young days in a new born democratic Czechoslovakia, about tragic years of World War II when the Czech universities were closed, about difficult life under the communist régime when the non-communist Borůvka was a thorn in the eye of the then powerful party and whose "bourgeois origin", contacts with foreign scientists and world acceptance of his work were only "the aggravating" circumstances; it would not be easy to describe the events following the Russian occupation in 1968 when professor Borůvka, after 50 years of lecturing at the University, was fired without a word of appreciation not to say thanks. And finally we could talk about the last years of his life when he still lived to see a return of democracy in his country and although physically not completely well, mentally absolutely fit, he was highly appreciated for his lifework which was finally closed on 22^{nd} July 1995 when he died in Brno at the age of 96.

We assume that 20^{th} century European history can be observed on his life very accurately.

However, we want to follow his mathematical career. Two circumstances had a great influence on it. The first was that in 1919, after a number of contests, the second Czech University named after the first Czechoslovak president T. G. MASARYK (1850–1937) was established in Brno, the second largest town in the new Czechoslovakia. The second was the fact that MATYÁŠ LERCH (1860–1922) was appointed the first professor of mathematics at the Masaryk University. Professor Lerch brought O. Borůvka to mathematics and O. Borůvka remembered him with respect and gratitude as his first and life-long teacher.

Matyáš Lerch was the first Czech mathematician of world reputation.[1] In 1900 he received the Great prize of Paris Academy for his work *Essais sur le calcul du nombre des classes de formes quadratique*

[1] We leave out in this moment the eminent personality of BERNARD BOLZANO (1781–1848) who was born and worked in Prague but can be considered as a "Czech" scientist only in the geographical and not in the national sense.

binaires sux coefficients entiers. Czech situation at the end of 19th century was characterized by the fact that in spite of his extraordinary scientific results there was no post for him at Czech universities although they were for the more mediocre mathematicians. On recommendation of his good friend and protector Ch. Hermite (1822–1901) he becomes the professor of mathematics in the Swiss town Fribourg. Only in 1906 he returns to Czech universities and he is appointed the professor at the Czech Technical University in Brno. After World War I O. Borůvka starts his studies at the Czech Technical University and more or less by chance enrolls for the lectures of mathematics read by Lerch. Lerch's lectures were not popular among students for their difficulty.

A meeting of professor Lerch and his young student Borůvka was for Borůvka fateful. As he himself was saying with pleasure, mathematics became his life mission because he was not good at it. Beginnings with Lerch were definitely not easy. Borůvka with his typical diligence, however, took such a great interest in mathematics that when Lerch had become the first professor of mathematics at the just established Masaryk University, he had offered him the position of assistant lecturer. And thus Borůvka, still a student of the Technical University, starts his external studies at the Masaryk University and at the same time starts working as Lerch's assistant lecturer.

Lerch was engaged in various areas of mathematics, but he achieved his most remarkable results in the theory of numbers and especially in mathematical analysis. He lead O. Borůvka especially to mathematical analysis particularly to studies of qualities of special functions. And thus Borůvka started quite purposefully to devote his attention to analysis.

However, in 1922 Professor Lerch died all of a sudden and in 1923 he was succeeded by a young and ambitious Eduard Čech (1893–1960). E. Čech was then interested in differential projective geometry which he was studying in the years 1921–1922 in Turin at Guido Fubini (1879–1943) and he was world famous in it.

Čech was in many respects quite different from Lerch. Lerch was a typical representative of the ending 19$^{\text{th}}$ century, the representative of a generation knowing "classical" mathematics, generation whose view and knowledge of mathematics was extraordinary and whose abilities e.g. in mathematical analysis nowadays mathematicians could hardly imagine.[2] Nevertheless, this generation slowly abandoned its place for the coming

[2]While reading Lerch's papers from analysis one is astonished by his knowledge and logic by means of which he solves problems which in "reconstruction" seem to be mastered only with maximum difficulties.

generation; and Čech was its prominent and respectable representative.

Čech brought excitement to Brno mathematical life. According to Borůvka's recollections, (see e.g. [1]) brought fresh air by his lectures on various topics, till that time practically not used, by his organizational ideas, by his foreign relations and trips, by his new working methods. Owing to Čech's influence and requirement, Borůvka focused himself soon on projective differential geometry which he had to study from the very beginning. And here starts Borůvka's way to his "second great teacher", as he always called him with respect. Years later, Borůvka recollects [1, page 94]:

> *Within a short time Eduard Čech imposed me to study and get knowledge of methods of Parisian professor Elie Cartan in the differential geometry, mainly his method of moving reper. The task was almost above my capacity. There were no excercise-books at that time and works of Elie Cartan existed only in journals but they were hardly available. I was completely isolated in my work and not only in Brno (Eduard Čech had not known Cartan's methods at that time), but maybe on this planet as a matter of fact, because these methods were brand-new, original, deep-created and not easy to read. I remember that I knew that Cartan's treatise on projective deformation of spaces which was the main source of studies for me nearly by heart but I could not understand the roots of the method.*

Let us say that Borůvka's words about the difficulty of Cartan's methods are not an exaggeration at all. The method of a moving reper belongs even today to the most difficult methods of modern differential geometry. From the formal point of view it leads to the complicated systems of partial differential equations where their prolongation must be taken into account. This difficulty is however balanced by profound results which can be obtained.

And Borůvka, although not easily, masters this method and Eduard Čech, knowing the value, sends Borůvka to Paris for a course so that he could study ELIE CARTAN's progresssive methods. On 1st October 1926 Borůvka leaves for Paris for a year's course.[3] He attends lectures

[3] Borůvka had excellent memory. Even after more than 60 years he remembered unbelievable details from all his foreign trips. Nearly at all his foreign lectures (and there were tens of them) he remembered significant participants including such details as where they sat and how they reacted to the problems.

of mathematicians who up to now belong to the most significant ones and whose names are legends: ÉMILE PICARD (1856–1941), JACQUES HADAMARD (1865–1963), MAURICE FRÉCHET (1878–1973), EDOUARDE GOURSAT (1858–1936), above all ELIE CARTAN (1869–1951). First he visits him regularly in his study at Sorbonna, then also in his flat in Versailles and they become friends not only on scientific field but also from human point of view and the very Elie Cartan becomes his "second great teacher" and he was remembering him gratefully all his life. Borůvka, a kind man but severe and self-exacting says at the end of his life about Elie Cartan [2, p. 60]: *And thus I must say that during my frequent communication with him, a great respect to this man was growing in me and I found all features of an ideal man in his person.*[4]

Elie Cartan

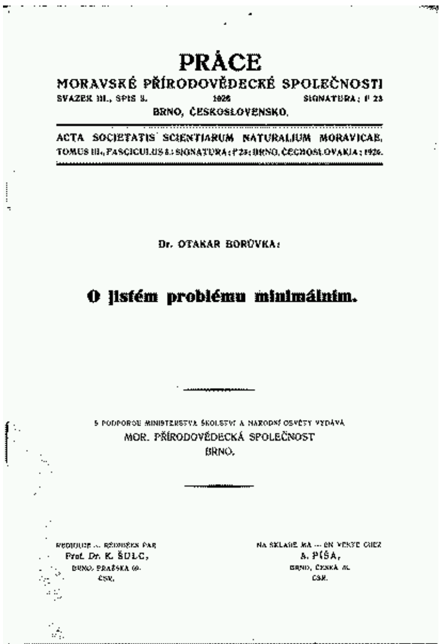
Borůvka mastered, as one of the first mathematicians, Cartan's method in differential geometry and during the following years, he achieved remarkable results in a number of publications in this field. His results were included in textbooks of geometry and the famous Bologna School followed his results from the theory of analytical correspodences. When in 1952 the international committee for complete edition of Elie Cartan's work was established, O. Borůvka was one of its members.

Let us come back once more to Borůvka's first stay in Paris. Then another interesting event took place which was directly connected with other sphere of Borůvka's work, with the result, with which he entered into the history of mathematics for ever. As usual at that first moment he himself was not aware of the extraordinary importance of this result. What was the matter?

[4]The author can confirm the relation of O. Borůvka to E. Cartan and his family by personal memory. During his second stay in Paris in 1929–1930, Borůvka took part in family holiday of Elie Cartan in the Alps where he became friends with Cartan's four children. A son Henri (*1904) (who became an excellent mathematician himself, a professor at Paris university and one of the founders of famous Bourbaki group) in June 1969, after Prof. Borůvka's several years effort, visited Brno, where he read two lectures, in which author also took part. It was impressing to see Prof. Borůvka's pleasure in indroducing him as his friend and son of one of the most eminent people he had ever met.

Borůvka remembers (see [1]):

> *Studies at technical schools brought me near to mechanical sciences and caused that I had always fully understood technical and other applications of mathematics. Soon after the First World War at the beginning of twenties the Westmoravian power plant in Brno electrificated the South-Moravian region. Because of my friendly terms with some workers I was asked to solve the problem of cost minimalization of electro-water net from the mathematical point of view. I succeeded in finding a construction …*



PRÁCE
MORAVSKÉ PŘÍRODOVĚDECKÉ SPOLEČNOSTI
SVAZEK III., SPIS 8. 1926 SIGNATURA: F 23
BRNO, ČESKOSLOVENSKO.

ACTA SOCIETATIS SCIENTIARUM NATURALIUM MORAVICAE,
TOMUS III., FASCICULUS 8.; SIGNATURA: F 23; BRNO, ČECHOSLOVAKIA; 1926.

Dr. OTAKAR BORŮVKA:

O jistém problému minimálním.

S PODPOROU MINISTERSTVA ŠKOLSTVÍ A NÁRODNÍ OSVĚTY VYDÁVÁ
MOR. PŘÍRODOVĚDECKÁ SPOLEČNOST
BRNO.

Prof. Dr. K. ŠULC,
ČSR.

A. PÍŠA,
ČSR.

Borůvka published the abovementioned result in his work *On a certain minimum problem* (in Czech: *O jistém problému minimálním* – see the figure) published in 1926 . At that time the theory of graphs did not exist and was established ca. ten years later. In all of Borůvka's work, however, the term "graph" did not appear at all. In fact Borůvka discovered algorithm for finding a minimal skeleton of final continuous graph which today plays a fundamental role in theory of graph algorithms and in the whole modern discrete mathematics.[5] As usual, the above result was "discovered" several times in the following decades but Borůvka's priority is without any doubt.

During his first stay in Paris, Borůvka also attended the seminar of Prof. COOLIDGE who was lecturing in Paris at that time. In spring 1927 Borůvka was invited by Coolidge to inform about his results. Although Borůvka more or less supposed that he would speak about his results in differential geometry, he offered Coolidge three topics, one

[5]O. Borůvka uses in this work the theory of matrixes and in a certain sense he anticipates some methods of the theory of matroids which arose at the beginning of the 1930s. However, to consider Borůvka a founder of the theory of matroids, as recently some authors mention, seems to be exaggerated.

of them concerning the "minimum problem". To Borůvka's great surprise, it was precisely this topic that was chosen, without hesitation, by Coolidge for the seminar. And thus Borůvka delivered lecture concerning a very non-traditional problem at that time in the seminar. The following event is rather piquant: although Elie Cartan[6] participated in the above-mentioned lecture, he most probably forgot the topic of the lecture because in 1938, he recommended a work by G. Choquet, in which Borůvka's algorithm is repeated without quotation, for publication in *Comptes Rendus.*

Only for completeness let us add that further "discovery" of the same algorithm was due to G. Sollin in 1961. He, however, did not publish the manuscript, though the work was quoted in a book by Berge and Ghoula-Houri, *Programming, Games and Transportation Networks.* [Wiley 1965].

In the mid-1930s, Borůvka gradually leaves differential geometry in his scientific work. He refuses an offer of Eduard Čech to co-work in his Brno topological seminar, which undoubtedly becomes world known; Borůvka is more than interested in the rapid progress of modern algebra after the publication of van der Waerden's pioneering book in 1931.

And thus for several following years he works purposefully on algebraic problems. He creates the theory of groupoids as far-reaching generalization of the theory of groups which he started to read at the faculty. He creates systematically the theory of set partitions which he uses in the theory of groupoids. Let us say that till that time the partitions were in mathematics speculated only in the language of equivalence relations, which, simultaneously with Borůvka, was discovered also by the French mathematicians Paul Dubreil (1904–1994), and M. L. Dubreil-Jacotin. (Borůvka was, however, not acquainted with their work as during the Second World War, as he had no access to foreign journals during the Nazi occupation.)

Although the theories of partitions and equivalences are completely equal each of them is convenient in a different type of considerations. Borůvka's theory of partitions enabled to derive the far-reaching generalization of the famous Jordan-Hölder-Schreier-Zassenhaus theorem on one hand and at the same time to come to beginnings of the theory of lattices which began to be created by in Garett Birkhoff (1911–1996) at that time. By the way, Borůvka is also a co-creator in this field. The Czech term "svaz" for the English term "lattice" was used for the first time by Borůvka in 1939.

[6]Personal notification of Prof. Borůvka to the author.

After the Second World War the situation was quite normal for a short period of time. Borůvka, as many times in his professional career, re-orientates himself on a totally new mathematical discipline. The last decade he focuses on the study of ordinary differential equations and he created the school of an international significance. This activity of O. Borůvka, however, is beyond the scope of this paper.

We have omitted many aspects of Borůvka's life, above all his excellent pedagogical work by which he made the history of the Czech and Slovak mathematics. Let us conclude this lecture with one of his own quotations which briefly gives a picture of his thinking and life rule:

> *Taking a critical look at my work today I believe that the method I was using in my work was correct: I studied problems from different areas of mathematics, usually starting from unanswered questions close to classical matters.That was how I achieved my overall knowledge of vast spheres of mathematics enabling me to find relations between knowledge from mutually remote mathematical areas of mathematics.*
>
> *I study thoroughly each serious problem trying to find the best solution and when I do find the solution I perform it to the best of my knowledge, belief and capacity. Being successful has no particular importance for me I take it as the natural consequence of my acting.I consider failures as the complexities of life and I try to take advice from them. But I never regret my decisions, as I always acted to the best of my belief.*

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