Jarník's note of the lecture course Punktmengen und reelle Funktionen by P. S. Aleksandrov (Göttingen 1928)

Pavel Sergeevich Aleksandrov (1896–1982)

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PAVEL SERGEEVICH ALEKSANDROV

(1896 – 1982)[1]

Childhood and studies

Pavel Sergeevich Aleksandrov was born on May 7, 1896 in Bogorodsk (now called Noginsk).[2] His father Sergei Aleksandrovich Aleksandrov (?–1920), a graduate of the Medical School at Moscow University, chose not to follow an academic career in Moscow but rather to become a general practitioner in Yaroslavskii.[3] He later obtained more senior positions in Bogorodsk's hospital.

When Pavel Sergeevich Aleksandrov was one year old, his family moved to Smolensk[4] where his father worked at the State hospital and established himself as an outstanding surgeon. From the end of the nineteenth century, Aleksandrov's family lived in Smolensk.

As were his brothers and sisters, he was initially educated at home by his mother Tsezariya Akimovna Aleksandrova (born Zdanovskaya) who herself had had a very good and extensive education. It was from her that he learnt French and German, and acquired a deep love of music and theatre; most of the family was tallented and the house was often filled with music.

He studied at the grammar school in Smolensk where his mathematics teacher Aleksander Romanovich Eiges recognised that his pupil had an exceptional talent for mathematics and science. Eiges not only influenced Aleksandrov's choice of his future career in mathematics but also had a hand in forming his taste in literature and the arts. At school Aleksandrov was not interested in solving the usual mathematical exercises or brain-teasers designed for secondary schools students but he concerned himself with the fundamentals of classic and non-Euclidean geometry. In 1913 Pavel Sergeevich Aleksandrov graduated from the grammar school in Smolensk. He was the dux of school and awarded a gold medal. Under the influence of Aleksander Romanovich Eiges he decided to become a secondary school teacher of mathematics.

First mathematical activities and results

In 1913 he entered Moscow University where studied under Vyacheslaw Vassilievich Stepanov (1889–1950),[5] also of Smolensk, who had often visited the

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[1] There are many different ways of transliterating of his name *Павел Сергеевич Александров* into the Roman alphabet. The most common ways are Pavel Sergeevich Aleksandrov or Paul (Pawel) Sergejevitsch Alexandroff.

[2] Noginsk is a city in the Moscow region cca 50 km from Moscow.

[3] Yaroslavskii is a city in central Siberia (the region of Jakutsk) on the Lena River.

[4] Smolensk is a city on the Dnieper River 420 km west of Moscow.

[5] Vyacheslaw Vassilievich Stepanov after his studies at the high school in Smolensk and at Moscow University became an assistant lecturer in 1909 there. Then he travelled abroad visiting Göttingen where he was influenced by Edmund Landau (1877–1938). Stepanov

Aleksandrov family there when Pavel Sergeevich was just a child. As a result of this early acquaintanceship Stepanov exerted a strong influence on Aleksandrov and recommended that he joins Egorov's mathematical seminar although he was only in his first year of studies at Moscow University. The following year, Aleksandrov met Nikolai Nikolaevich Luzin (1883–1950)[6] and their relationship played a continuing role in Aleksandrov's mathematical interests, his future professional choices as well as his working and teaching methods.

Aleksandrov's first important result, namely that every uncountable Borel set contains a perfect subset, was published in 1916. The method which was created by Aleksandrov played a very important role in the future development of descriptive set theory. Following the publication of this theory

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attended lectures by David Hilbert and Edmund Landau. In 1915 he returned to Moscow University and continued lecturing on mathematics. In 1921 he was involved in training young scientists at the Research Institute of Mathematics and Mechanics which had been founded in that year. In 1928 he became a professor of mathematics at Moscow University and in 1939 he was appointed the director of the Research Institute of Mathematics and Mechanics continuing to hold this post until his death. Following his studies, he collaborated mainly with N. N. Luzin and D. F. Egorov. He influenced many of his pupils – future distinguished Russian mathematicians (his most famous student was Aleksander Osipovich Gel'fond (1906–1968)). He was one of the founders of the Soviet school of differential equations and real analysis.

[6] Nikolai Nikolaevich Luzin graduated from Moscow University; in 1905–1906 he had a scholarship to study in Paris at Emil Borel (1871–1956). After returning to Russia, Luzin studied medicine and theology as well as mathematics. Not until 1909 did he decide for a mathematical career. In 1910 he was appointed as an assistant lecturer in pure mathematics at Moscow University. From 1910 to 1913 he studied in Göttingen where he was influenced by Edmund Landau. In 1917 he became a professor of pure mathematics at Moscow University. From 1927 he was a member of the USSR Academy of Sciences, two years later he became a full member of the Department of Philosophy (then the Department of Pure Mathematics). He also worked in the Steklov Institute in Moscow where he became a head of the Department of the Theory of Function of Real Variables (1935). He was interested in the theory of functions, topology and measure theory, set theory, differential equations, differential geometry, probability theory, control theory, foundations of mathematics and the history of mathematics. He influenced the development of modern mathematics, not only in the USSR. In 1936, he became a victim of a fanatical political campaign organized by the Soviet authorities and the newspaper "Pravda". He was accused of anti-Soviet propaganda and of sabotaging the development of Soviet sciences based on the evidence that of all his important and influential results were published abroad in foreign languages and because of his close international contacts. The main aim of the Luzin affair was to get rid of him as a representative of the old pre-Soviet Moscow mathematical school. The role of Aleksandrov in this affair is described in [Lo]. The most visible consequence was that, from this difficult moment, Soviet mathematicians began to publish almost exclusively in Russian in Soviet journals and they lost their international contacts for some years. For more information about Luzin's life and work and new archival materials see Russian articles in Историко-математические исследования [Istoriko-Matematicheskie Issledovanija] 25(1980), pp. 335–361; 28(1985), pp. 278–287; 31(1989), pp. 116–124, 191–203, 203–272; 34(1993), pp. 246–255; 36(1995), No. 1, pp. 19–24; 37(1997), pp. 33–43, 43–66, 133–152; 38(1999), pp. 92–99, 100–118, 119–127; 39(1999), pp. 156–171, 171–185; 40(2000), pp. 119–142 etc. See also [ZD], S. S. Demidov, Ch. E. Ford: *N. N. Luzin and the affair of the "National Fascist Center"*, pp. 137–148, in J. Dauben (ed.): *History of mathematics*, States of Arts, New York, 1990, and A. P. Youschkevitch, P. Dugac: *L'affaire de l'académicien Luzin de 1936*, La Gazette des mathématiciens 3(1988), pp. 31–35.

Luzin, recognizing that Aleksandrov was one of the most talented young mathematicians in Russia, urged him to try to solve the continuum hypothesis – the famous open problem in set theory.[7]

Aleksandrov failed to solve this problem and disappointed, believed himself unable to go on with his mathematical career. He left his university studies, moved to Novgorod-Severskii[8] and became a theatre producer. He then went to Chernikov[9] where became the director of the theatre company, part of the Regional Educational Committee, and lectured on Russian and foreign literature. He prepared a cycle of lectures on F. M. Dostojevski, N. V. Gogol and J. W. Goethe which enjoyed considerable popularity. Because of his musical and artistic interests and talent he found many friends among poets, artists and musicians (for example L. V. Sobinov). After a short time spent in prison (1919),[10] he returned to Moscow in 1920. At that time, N. N. Luzin and Dmitri Fedorovich Egorov (1869–1931)[11] had started to put together a large research group of mathematicians at Moscow University called "Luzitania" by its members and students.[12] They brought together a pool of talented students and young researchers and managed to create a very friendly working atmosphere despite the many difficulties occurring in the first years after the October Revolution. Aleksandrov's former teachers and colleagues welcomed Aleksandrov's return.

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[7] Now, thanks Paul Joseph Cohen's work from the 1960's we know that the continuum hypothesis can neither be proved nor disproved.

[8] Novgorod-Severskii is a very old and famous city in Russia on the Volhov River 250 km south-east of St. Peterburg.

[9] Chernikov (Chernigov) is a city in the Ukraine on the Desna River 150 km north of Kyjiv.

[10] His jailing was a consequence of difficulties connected with the time of the Russian revolution.

[11] Dmitri Fedorovich Egorov studied mathematics and physics at Moscow University. He lectured there from 1894. After spending a year abroad, he returned to Moscow and he became an ordinary professor of mathematics in 1903. In 1923 he was named director of the Institute for Mathematics and Mechanics at Moscow University. Because of his deep religious orientation, in 1929 he was dismissed as director although the Moscow Mathematical Society supported him and refused to expel him. He was arrested as a religious sectarian. Egorov went on a hunger strike in the prison and he was taken to the prison hospital in Kazan where he died. He was interested in differential geometry and its applications, integral equations and theory of functions of real variables. He was one of the founders of Moscow school of theory of functions. Many important Russian mathematicians were among his pupils (N. N. Luzin, V. V. Stepanov, I. I. Privalov, V. V. Golubev, I. G. Petrovskii, L. N. Sretenskii etc.). For more information about Egorov's life and work see Russian articles in Историко-математические исследования [Istoriko-Matematicheskie Issledovanija] 35(1994), pp. 324–336; 36(1996), No. 2, pp. 146–165; 39(1999), pp. 123–156; 45(2005), pp. 13–19; Успехи математических наук [Uspekhi Matematicheskikh Nauk] 26(1971), No. 5, pp. 169–210 etc. See also [ZD], Ch. E. Ford: *Dmitrii Egorov: Mathematics and religion in Moscow*, The Mathematical Intelligencer 13(1991), No. 2, pp. 24–30.

[12] During 1920's V. V. Stepanov, N. N. Luzin, D. F. Egorov, P. S. Aleksandrov, V. I. Veniaminov, P. S. Urysohn, N. K. Bari, U. A. Royanskaya, V. I. Glivenko, N. A. Selivanov, L. G. Schnirelman, A. N. Kolmogorov, M. A. Lavrentiev, L. V. Keldysh, E. A. Leontovich, P. S. Novikov, I. N. Khlodovskii, G. A. Seliverstov, I. I. Privalov, D. E. Menshov and A. Ya. Khinchin were active members of "Luzitania". For more information see [ZD].

However, he was not allowed to stay in Moscow and spent 1920–1921 in Smolensk where he taught mathematics at the university. Despite this, he managed to keep in touch with mathematicians in Moscow, and so could continue his research and prepare himself for the state examinations. After successfully taking them in 1921, he was appointed lecturer at Moscow University and started giving lectures on several interesting topics (functions of real variable, topology, Galois theory etc.).

During this time, Aleksandrov became a friend with Pavel Samuilovich Urysohn (1898–1924)[13] who was a member of "Luzitania". Their friendship soon developed into a major and useful mathematical collaboration. In the summer of 1922, they went with other young Moscow mathematicians to the village at Burkov near the town Bolshev (a holiday center on the banks of the river Kalyazma) where they began to study topology; inspired by Hausdorff's famous book, *Grundzüge der Mengenlehre* published in 1914,[14] they made essential contributions to the theory of topological and metric spaces.[15] Here they had the opportunity to work, think and discuss their ideas in congenial surroundings and to find new inspirations. Aleksandrov and Urysohn worked on the general definition of dimension in topology; they applied their new

[13] In 1915 after studies at a private grammar school in Moscow, Pavel Samuilovich Urysohn entered Moscow University to study physics but his interest after attending lectures by Luzin and Egorov began to concentrate on mathematics. In 1919, after his graduation, he became an assistant professor at Moscow University. Two years later, he was appointed a private docent at the Institute of Mathematics and Physics at the First Moscow University and in 1923 he became a professor at the Second Moscow University. He was interested in topology, namely in topological and metric spaces, theory of integral equations, theory of functions of complex variables etc. On August 17, 1924, he tragically drowned while swimming in the Atlantic Ocean near Batz-sur-Mer.

[14] The substantially revised edition from 1914 appeared in 1927 and 1935. The 1914 edition was reprinted in 1949 and 1965 by Chelsea, the 1927 edition was published in Russian in 1937, the 1935 edition was translated into English and published in 1957. See *Felix Hausdorff – gesammelte Werke. Band II. Grundzüge der Mengenlehre*, edited and with commentary by E. Brieskorn, S. D. Chatterji, M. Epple, U. Felgner, H. Herrlich, M. Hušek, V. Kanovei, P. Koepke, G. Preuß, W. Purkert and E. Scholz, Springer-Verlag, Berlin, 2002, and *Felix Hausdorff – gesammelte Werke. Band III. Mengenlehre (1927, 1935): deskriptive Mengenlehre und Topologie*, edited by U. Felgner, H. Herrlich, M. Hušek, V. Kanovei, P. Koepke, G. Preuß, W. Purkert and E. Scholz, Springer-Verlag, Berlin, 2008.

[15] Felix Hausdorff (1868–1942) was one of the most important and inspirational German mathematician. From secondary school he was attracted to literature and music, he wanted to pursue a career in music or literature but under the influence of his parents he turned towards mathematics. He studied at Leipzig University, graduated in 1891 with a doctorate in applications of mathematics to astronomy, four years later he obtained *Habilitation* based on his research in astronomy and optics. In 1902, he was appointed an extraordinary professor of mathematics at Leipzig University and he turned down the offer of a similar post in Göttingen. From 1910 to 1913 he taught mathematics at the University in Bonn, from 1913 to 1921 at the University in Greifswald. In 1921 he returned to Bonn and worked there until 1935 when he was forced to retire by the Nazi regime. Unfortunately, he had made no attempt to emigrate while it was possible, and the position of Jews continued to deteriorate. Together with his wife and his wife's sister, he committed suicide on January 26, 1942. He is an author of many influential results on set theory, topology, measure theory, functional analysis, group theory, number theory etc.

theory and its consequences on countable compact spaces and they obtained some results of fundamental importance, namely in the theory of compact spaces and locally compact spaces, which immediately attracted the interest of European mathematicians. In the 1920's, Aleksandrov formulated general axioms of topological space.

Studies and stays abroad

After the signing of the Rapallo Pact in 1922,[16] the Soviet state sent many young and talented scientists to Germany to broaden their knowledge and to come into contact with the best mathematical centers of Western Europe, as well as to possibly publish their results there. In May 1923, Aleksandrov, Urysohn and Kovner (1896–1962)[17] arrived at Göttingen with Luzin's letter of recommendation.[18] They started to study at one of the most important centers of European mathematics. In June 1923, Aleksandrov and Urysohn took part in mathematical lectures, seminars, informal meetings and discussions with Emmy Amalie Noether (1882–1935),[19] Richard Courant (1888–1972),[20] David Hilbert

[16] In 1922, Germany and the USSR signed the pact in the Italian seaside Rapallo. The bilateral demands on the war compensation were annulled; diplomatic relations, cultural contacts and economic collaboration were renewed.

[17] S. V. Kovner had no important mathematical results.

[18] For more information see [To1] and Tobies R.: *Zu den Beziehungen deutscher und sowjetischer Mathematiker während der Zeit der Weimarer Republik*, Mitteilungen der Mathematischen Gesellschaft der DDR 1(1985), pp. 66–80.

[19] Emmy Amalie Noether was a daughter of Max Noether, professor of mathematics in Erlangen. After her studies at the "Höhere Töchter Schule in Erlangen" (1889–1897) she took the examinations of the State of Bavaria and became a certificated teacher of English and French at girls schools (from 1900). She never accepted this position as she decided to study mathematics at Erlangen University (1900–1902). She then continued in Nürnberg (1903) and finally completed her studies at Göttingen University (1903–1904) where she attended lectures by O. Blumenthal, D. Hilbert, F. Klein and H. Minkowski. In 1907, she obtained a doctorate in Erlangen under P. Gordan. From 1907 to 1915 she helped her father with his lectures at Erlangen University but was not named an assistant; for a woman it was then impossible. In 1915 she moved to Göttigen where she lectured thanks to the support of Hilbert. After a long battle with the university authorities, she was appointed professor of mathematics (1922) and she taught there up to 1933. During the school year 1928/1929 she gave some special courses on abstract algebra at Moscow University and she organised a research seminar on algebraic geometry which took place at the Academy in Moscow. In 1933, she had to emigrate to the USA; Nazi laws made her academic career no longer possible. She obtained a position at Bryn Mawr College in Pennsylvania and she also lectured at the Institute for Advanced Study in Princeton. Noether was incredibly influential for modern abstract algebra. From 1907 up to 1919 she was interested in solving Jordan's and Hilbert's problems, from 1920 up 1926 she worked on ideal theory and from 1927 she studied and solved many problems on non-commutative algebra. She opened new and modern directions in abstract algebra which influenced the development of mathematical thinking. Her fundamental results were extended, generalized and popularized by her pupils and co-workers (for instance, B. L. van der Waerden).

[20] Richard Courant after his studies at the König Wilhelm Gymnasium in Breslau attended classes in mathematics and physics at the University of Breslau but found them lacking in excitement and interest. In the spring of 1907 he left Breslau and spent one semester in Zurich. Then he moved to Göttingen which he found to be full of outstanding

(1862–1943),[21] their collaborators and pupils. Aleksandrov's collaboration with Noether is briefly described in [Te]:

In 1923, P. S. Alexandroff, a prominent Russian mathematician who could both speak and write excellent German, came to deliver a series of lectures at Göttingen. Noether, who had been fascinated for years with events in Russia, was enchanted with what she perceived as the Bolshevik idealistic view of society and socialism's potential as more humane organizing force in society. She even joined the Social Democratic party, which may have been a contributing factor in her problems a few years later, when she was labeled as a left-leaning radical.

Noether was impressed with the mathematician Alexandroff. His work in topology complemented her abstract algebra in exciting new ways, and she relished their interactions. He, in turn, recognized that she was a great mathematician with whom he could work productively. Both Noether and Alexandroff could see how Noether's abstract algebra could contribute to Alexandroff's topology. He was impressed with her intellectual enthusiasm, with her commitment to the importance of her remarkable new ideas, and with the simplicity and warmth of her interactions with her students. To him, she was a new kind of scholar, committed to both her mathematics and her budding students. He looked forward to future collaborations with her.[22]

The mathematicians at Göttingen University as well as in the Göttingen Mathematical Society[23] were impressed by their results on topology, particularly on topological and metric spaces, in particular, by their definition of

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mathematicians. There he attended courses by Hilbert and Minkowski and their joint seminars. In 1908 he became Hilbert's assistant and under his supervision he obtained his doctorate in 1910. Following this, he collaborated with Hilbert and became a mathematics lecturer at Göttingen University where he taught until the start of World War I. After the war, which interrupted his career, he returned to Göttingen and was appointed a professor of mathematics there. In 1922 he founded the University's Mathematics Institute and taught there up 1933 when he had to emigrate because of the Nazi regime. He then spent a short time in England before than he moved to New York. After some difficulties, he was appointed a professor of mathematics at the University in New York (1936) where he built up an applied mathematics research center based on the Göttingen model. Thanks to his excellent reputation he helped many mathematicians who were forced to leave Germany to obtain positions in the USA. His outstanding mathematical results are connected with the Dirichlet problem, theory of conformal mapping, mathematical physics, partial differential equations.

[21] David Hilbert after graduating from the gymnasium entered the University of Königsberg where he studied from 1880 to 1883. In 1885 he received the doctorate and from 1885 to 1895 he taught there. In 1895 he was appointed to the chair of mathematics at the University in Göttingen and lectured there till 1933. His research and results had an enormous impact on many mathematicians and turned Göttingen into the world center of mathematics in the first third of the 20th century. His interests were incredibly vast, he was one of the last "universal" mathematicians. In his active life, there were eight fundamental periods: invariant theory (1885–1898), algebraic numbers fields (1893–1898), the fundamentals of geometry (1898–1902), the Dirichlet problem, theory of differential equations (1900–1906), theory of integral equations (1900–1910), solution of Waring's problems (1908–1909), mathematical physics (1909–1922) and the logical foundations of mathematics (1922–1939). In all these topics he obtained important, influential and complex results.

[22] [Te], pp. 116–117.

[23] The Göttingen Mathematical Society was founded in 1892 by Felix Klein and Heinrich

dimension. In July 1923, Aleksandrov and Urysohn prepared their lectures based on articles which were accepted for publication in Mathematische Annalen.[24] In the same year they also published their new topological results in Mathematische Zeitschrift.[25]

In September 1923, they participated in the Annual Meeting of the German Mathematical Society which took place in Marburg. Aleksandrov lectured on the theory of point sets and Urysohn on the theory of general Cantor's curves. During that time they were in the touch with Hellmuth Kneser[26] who helped them with some administrative problems connected with their new stay in Göttingen in 1924.[27]

In the summer of 1924, Aleksandrov and Urysohn arrived to Göttingen again. In July they lectured at the meeting of the Göttingen Mathematical Society and their lectures were published in Mathematische Annalen.[28]

Weber (1842–1913). At the beginning of the 20th century it became one of the most famous and influential world mathematical centers. For more information see http://www-groups.dcs.st-andrews.ac.uk/∼history/Societies/Gottingen.html.

[24] P. S. Alexandroff and P. S. Urysohn: *Zur Theorie der topologischen Räume*, Mathematische Annalen 92(1924), pp. 258–266 (the article is dated Göttingen, den 26. Juni 1923 [Eingegangen am 1. 8. 1923]); P. S. Alexandroff: *Über die Strukture der bikompakten topologische Räume*, Mathematische Annalen 92(1924), pp. 267–274 (the article is dated Göttingen, den 3. Juli 1923 [Eingegangen am 1. 8. 1923]), *Über die Metrisation der im kleinen kompakten topologischen Räume*, ibid. 92(1924), pp. 294–301 (the article is dated Göttingen, den 10. Juli 1923 [Eingegangen am 1. 8. 1923]); P. S. Urysohn: *Über die Metrisation der bikompakten topologischen Räume*, Mathematische Annalen 92(1924), pp. 275–293 (the article is dated Göttingen, den 15. VII. 1923 [Eingegangen am 1. 8. 1923]), *Der Hilbertsche Raum als Urbild der metrischen Räume*, ibid. 92(1924), pp. 302–304 (the article is dated Göttingen, den 22. Juli 1923 [Eingegangen am 1. 8. 1923]). These articles contain important investigations on normal spaces, metrization theorems, existence theorem concerning an imbedding into a Hilbert space.

[25] P. S. Alexandroff: *Über die Äquivalenz des Perronschen und des Denjoyschen Integralbegriffes*, Mathematische Zeitschrift 20(1924), pp. 213–222 (the article is dated Göttingen, den 14. Juni 1923 [Eingegangen am 31. Juli 1923]); P. S. Urysohn: *Ein Beitrag zur Theorie der ebenen Gebiete unendlich hohen Zusammenhanges*, Mathematische Zeitschrift 21(1924), pp. 133–150 (the article is dated Moskau, den 30. Dezember 1923 [Eingegangen am 25. Februar 1924]), *Zur ersten Randwertaufgabe der Potentialtheorie. Ein Fall der Unlösbarkeit*, ibid. 23(1925), pp. 155–158 (the article is dated Eingegangen am 2. October 1923).

[26] Hellmuth Kneser (1898–1973) after studies at the secondary school in Breslau attended classes in mathematics at Göttingen University (1916–1921) where he obtained his doctorate under Hilbert's supervision and he became his assistant. In 1922 he was named private docent at the university and taught there until 1925 when he moved to Greifswald as an ordinary professor of mathematics. From 1937 he lectured at Tübingen University. He was one of the founders of the German mathematical journal *Archiv der Mathematik* which was firstly published in 1952. He was interested in topology, function theory, differential geometry and algebra.

[27] Aleksandrov's recollections on his stay in Göttingen were published in his articles *Математическая жизнь в СССР*, Успехи математических наук [Mathematical life in the USSR, Uspekhi Matematicheskikh Nauk] 34(1979), pp. 219–249, 35(1980), pp. 241–278, and *Erinnerungen an Göttingen*, in Deutschland–Sowjetunion. Aus fünf Jahrzehnten kultureller Zusammenarbeit, Berlin, 1966, pp. 437–440.

[28] For example, P. S. Alexandroff: *Zur Begründung der n-dimensionalen mengentheo-*

The future development of algebra and algebraic topology was connected particularly with Hausdorff's works and namely with Noether's mathematical group in which Aleksandrov and Urysohn took part. Its members studied general problems of ideal theory, and also commutative and non-commutative algebra. Aleksandrov and Urysohn developed the theory of dimension under Noether's influence. During their stay they traveled to Bonn to visit Hausdorff and discuss the major new directions that both independently had taken in topology; Hausdorff was fascinated with their results. The evenings spent there were a mixture of discussions on topology and music. In the mornings the two young men rose early to swim in the Rhine, a dangerous sport, which horrified Hausdorff.

Before their holiday, they had decided to go to the Netherlands and France. In the Netherlands, they visited Luitzen Egbertus Jan Brouwer (1881–1966),[29] then spent a short time in Paris. In the August they took their holiday in the fishing village of Batz-sur-Mer where they continued to do mathematics, rest and swim in the Atlantic Ocean, where Urysohn tragically drowned. After his death Aleksandrov spent some parts of 1925 and 1926 in the Netherlands and he worked with Brouwer on finishing Urysohn's last paper for publication.[30] Aleksandrov also continued with his own work on the solutions of topological problems and wrote five articles.[31] His stay in Amsterdam was supported by the International Education Board (sponsored by the Rockefeller Foundation)

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retischen Topologie, Mathematische Annalen 94(1925), pp. 296–308 (the article is dated Le Batz (Loire-Inférieure), August 1924 [Eingegangen am 23. 8. 1924]); P. S. Urysohn: *Über die Mächtigkeit der zusammenhängenden Mengen. Meinem Freunde Paul Alexandroff gewidnet*, Mathematische Annalen 94(1925), pp. 262–295 (the article is dated Le Batz (Loire-Inférieure), den 14. 8. 1924 [Eingegangen am 23. 8. 1924]).

[29] Luitzen Egbertus Jan Brouwer was a Dutch mathematician. He attended the famous high school studies in Hoorn (a town on the Zuiderzee north of Amsterdam). He completed his secondary school by the age of fourteen. He then spent the next two years studying Greek and Latin and in 1897 he passed the entrance examinations for the University of Amsterdam where he obtained his master's degree (1904) and finished his doctoral dissertation (1907). In 1909 he was appointed a private docent and in 1912 a professor of set theory, function theory and axiomatic theory at the University in Amsterdam. He hold the post until his retirement in 1951. Then he lectured in South Africa, the United States and Canada till his death. During 1911–1913 he obtained almost all his fundamental results on topology and he was considered by many mathematicians to be its founder. Later he was interested in group theory, set theory, functional analysis, conceptual problems of modern mathematics and logical foundations and philosophy of mathematics. He was the first European mathematician who appreciated Urysohn's and Aleksandrov's results in topology.

[30] After Urysohn's death, his articles prepared from his notes by Aleksandrov were published – P. S. Urysohn: *Zum Metrisationproblem*, Mathematische Annalen 94(1925), pp. 309–315 (dated Eingegangen am 28. 9. 1924), *Über im kleinen zusammenhängende Kontinua. Aus dem Nachlasse von P. Urysohn † herausgegeben von Paul Alexandroff in Moskau*, ibid. 98(1928), pp. 296–308 (dated Eingegangen am 19. 6. 1926); P. S. Alexandroff, P. S. Urysohn: *Über nulldimensionale Puntkmengen*, ibid. 98(1928), pp. 89–106 (dated Eingegangen am 8. 5. 1926).

[31] P. S. Alexandroff: *Simpliziale Approximationen in der allgemeinen Topologie*, Mathematische Annalen 96(1927), pp. 489–511 (the article is dated Le Batz (Loire Inférieure), August, 1925 [Eingegangen am 10. 9. 1925]), *Über kombinatorische Eigenschaften allgemeiner Kurven*, ibid. 96(1927), pp. 512–554 (the article is dated Collioure (Pyrénées Orientales),

which granted fellowships for young scientists on an international level and global grants for building new world scientific centers and institutions.[32]

After Urysohn's death, Aleksandrov went to Göttingen regularly every summer from 1925 until 1932 where the working atmosphere remained open and friendly. William Henry Young (1863–1942)[33] described Göttingen meetings:

The German professors have instituted at Göttingen and elsewhere a Mathematical Society of their own, meeting one evening in the week, to which the professors, Privatdozents and a few advanced students have access. The current mathematical literature is, as far as possible, divided among the members for perusal, and subsequently to report to the Society as to the contents. Free criticism and suggestion is allowed, and in particular any references to other writers, ancient or modern, in which the subjects treated of in the Society occur, are welcomed.[34]

The informal discussions of the Göttingen mathematical circle are depicted as followed:

The summer of 1925 was a charmed time at the Mathematical Institute at Göttingen. There were frequent algebraic-topological walks led by Noether, particularly when the brilliant Alexandroff was in residence. The assembled mathematicians spent many afternoons and evenings together, sometimes boating at the Courants' place on the Leine River or swimming at the Klie swimming pool. Although the pool was theoretically for men only, the Noether/Courant group didn't follow that rule. As they swam in the pool or rowed on the river or walked along its banks, the major focus was always mathematics.

The musical evenings were a setting for even more mathematics. With Courant at the piano and several people on a variety of other musical instruments, the tempo was quick and spirits were high. While it is true that Courant hit only about 75 percent of the notes that appeared on a page, he didn't worry about the other 25 percent, and he played with such gusto that his guests hardly noticed. Noether did not feel compelled to demonstrate her

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Oktober 1925 [Eingegangen am 1. 11. 1925]), *Über stetige Abbildungen kompakter Räume*, ibid. 96(1927), pp. 555–571 (the article is dated Blaricum bei Amsterdam, November 1925 [Eingegangen am 11. 12. 1925]), *Darstellung der Grundzüge der Urysohnschen Dimensionstheorie*, ibid. 98(1928), pp. 31–63 (the article is dated Eingegangen am 3. 4. 1926), *Über den allgemeinen Dimensionsbegriff und seine Beziehungen zur elementaren geometrischen Anschauung. Paul Alexandroff in Moskau. Hernn L. E. J. Brouwer gewidmet*, ibid 98(1928), pp. 617–635 (the article is dated Le Batz (Loire Inférieure), August, 1926 [Eingegangen am 10. 10. 1926]).

[32] For more information about the main aims and activities of the International Education Board and its role in the development of international mathematical collaboration see [SS1]. Aleksandrov obtained a scholarship for twelve months to work with Brouwer on topology. He was supported and recommended by Egorov (see [SS1], pp. 15–16, 288).

[33] William Henry Young, an American mathematician, was in Göttingen from 1899 to 1908.

[34] See http://www-groups.dcs.st-andrews.ac.uk/∼history/Societies/Gottingen.html.

expertise on the piano – "he Happy Farmer" would not have fit in very well – and, besides, she had mathematics to discuss.[35]

In 1926 Aleksandrov became a close friend with Heinrich Hopf (1894–1971).[36] They started working together and held a topological seminar at the University in Göttingen, where they participated as lecturers and collaborators in Noether's famous seminar.[37]

In 1926, they spent some time in the south of France, where they travelled and worked with Otto Neugebauer (1899–1990).[38]

During the first half of the 1920's Aleksandrov acquired the reputation as

[35] [Te], p. 118.

[36] Heinrich Hopf was a German and later a Swiss mathematician. After studies at secondary schools in Breslau (Dr. Karl Mittelhaus'school and König Wilhelm Gymnasium), in 1913 he entered the Silesian Friedrich Wilhelms University in Breslau. His studies were interrupted by World War I when he had to fight on the Western front as a lieutenant. After the war he returned to the university in Breslau and after a year he went to the University in Heidelberg where he took courses in mathematics, philosophy and psychology. In 1920 he moved to the University in Berlin to prepare for his doctorate. He received it in 1925 on the basis of a thesis on topology of manifolds supervised by Erhard Schmidt (1876–1959). In the same year he went to Göttingen where he met Emmy Amalie Noether. Her influence played an important role in Hopf's mathematical development. During his stay in Göttingen he worked on his *Habilitation* which was completed in 1926. During the school year 1930 he gave some special courses on topology at Moscow University. In 1931 he became a professor of mathematics at the Polytechnic in Zurich. During the World War II he was able to help German friends who had to flee Germany under the Nazis as he had previously obtained Swiss citizenship. His outstanding works are connected with algebraic topology and its applications in differential geometry, homology and cohomology of groups.

[37] Aleksandrov's recollections on his collaboration and friendship with Hopf was published in his German article. See P. S. Alexandroff: *Heinz Hopf zum Gedenken*, Jahresbericht der Deutschen Mathematiker-Vereinigung 78(1976), pp. 113–146.

[38] Otto Neugebauer was a German mathematician, historian of mathematics and astronomy. After studies at the Gymnasium in Graz, he joined the Austrian army as an artillery lieutenant and spent the end of war as a prisoner of the Italians. From 1919 up to 1921 he studied electrical engineering and physics at the University of Graz, then he moved to the University of Munich where took courses in mathematics and physics. In 1922 he settled in Göttingen where he began to study mathematics seriously. In this time, he became friends with R. Courant, Harald August Bohr (1887–1951) and P. S. Aleksandrov. In 1926 he finished his studies in Göttingen under Klein's supervision and he received his doctorate for a dissertation on the history of Egyptian unit fractions. From 1927 up 1933 he lectured on the history of ancient mathematics in Göttingen. From 1929 until 1932 Neugebauer and Courant jointly directed the newly founded Mathematical Institute in Göttingen which was built with support from the Rockefeller Foundation. Neugebauer persuaded Springer-Verlag to publish a journal reviewing all mathematical publications; the first issue of *Zentralblatt für Mathematik* edited by him appeared in 1931. When the Nazis came to power in Germany, Neugebauer's career completely changed. He had to leave Germany, from 1934 up 1938 he taught at the University in Copenhagen in Denmark and he took the editorial office of Zentralblatt für Mathematik there and continued its publication. In 1939 he had to emigrate to the USA and he was appointed a professor at Brown University in Providence. In a short time Neugebauer founded the new review journal *Mathematical Reviews* supported by the American Mathematical Society and worked as its editor until 1945. In 1947 he was appointed an ordinary professor of history of mathematics at Brown University. Neugebauer's works are particularly devoted to the history of ancient mathematics and astronomy.

one of the leading experts in topology, and the German Mathematical Society asked him to prepare a detailed survey on the newest topological results. In 1926, at the Annual Meeting of the German Mathematical Society, Aleksandrov lectured on some new methods and problems of general topology. At the special mathematical colloquium which took place at the Berlin University on May 10, 1927, Aleksandrov lectured on relations between combinatorical topology and set theoretic topology. From 1923 to 1931, Aleksandrov officially lectured six times at meetings of the Göttingen Mathematical Society.[39]

Aleksandrov and Hopf spent a large part of the academic year 1927/1928 at Princeton in the USA where they collaborated with Solomon Lefschetz (1884–1972),[40] Oswald Veblen (1880–1960)[41] and James Waddell Alexander (1888–1971).[42] Their successful and productive stay was supported by the Interna-

[39] From 1920 to 1931, 17 lectures by Soviet mathematicians were given at the meetings of Göttingen Mathematical Society (for example S. N. Bernstein (1925), G. F. Pfeifer (1928), D. M. Sincov (1927), O. Yu. Schmidt (1927), A. N. Kolmogorov (1930, 1931), L. G. Schnirelman (1931)). L. S. Pontryagin, L. A. Lyusternik and A. O. Gel'fond, younger Soviet mathematicians, spent their long study stays in Göttingen at the end of 1920's. It can be mentioned that many Soviet mathematicians published their articles in German scientific journals. For example, in the Mathematische Annalen 81(1920)–119(1937), 497 articles written by non-German mathematicians were published, from which 103 were written by Soviet ones (from 1923 up 1932 only). Aleksandrov published twelve important articles there from 1924 to 1932. Fore more information see [To1].

[40] Solomon Lefschetz was born in Moscow, he studied engineering at the Ecole Central in Paris (1902–1905) and he attended lectures by Emil Picard and Paul Appell. Not being a French citizen, he had some difficulties in obtaining an academic post in France and so went to the USA. He worked at the Baldwin Locomotive works and for Westinghouse Electric Company in Pittsburgh. During a laboratory accident in 1907, he lost both his hands. This tragedy pushed him towards mathematics. He decided to devote himself to teaching mathematics and received his doctor's degree at Clark University in Worcester (Massachusetts) with a thesis on algebraic geometry (1911). In the same year, he became an instructor in mathematics at the University of Nebraska in Lincoln. Two years later he was appointed to the University of Kansas in Lawrence. Then he was an assistant (1916), an associate professor (1919) and a full professor of mathematics (1923). During this short time, he wrote a series of important papers on topology. In 1924 he went to Princeton as a visiting professor; the following year he got a permanent position and taught there to 1953. During the 1920's and 1930's he visited many European countries, in particular France, Italy and the USSR. In 1940's he visited the National University of Mexico and as a result of these visits he helped to build a flourishing school there. Lefschetz worked in a variety of topics including algebraic geometry, theory of subvarieties of an algebraic variety, theory of algebraic representations, applications of topological methods in geometry, theory of nonlinear ordinary differential equations, theory of nonlinear oscillations.

[41] Oswald Veblen after studies at the University of Iowa (1894–1898) and at the University of Chicago (1898–1900) was awarded his doctorate in Chicago in 1903 and became an assistant of mathematics there. From 1905 up to 1932 he taught at the University in Princeton where he was promoted to professor of mathematics in 1910. In the academic year 1928–1929 he taught at Oxford University; in 1932 he lectured in Berlin, Hamburg and Göttingen. In 1932 he became the director of the famous mathematical center at the Institute for Advanced Study in Princeton which became one of the leading centers in the world for topology research. Veblen was a great American specialist in topology, projective and differential geometry and their applications in nuclear physics and relativity theory. He was one of the founders of American topological school.

[42] James Waddell Alexander studied in Princeton where he obtained his degree under

tional Education Board[43] and, it was important not only for Aleksandrov, but also for the future development of topology. During this time, Aleksandrov and Hopf planned a joint multi-volume work on topology. Its first volume was published in 1935.[44] World War II, however, prevented further collaboration on the next two volumes.[45]

On February 16, 1928, R. Courant, D. Hilbert and G. Herglotz[46] proposed to the Göttingen Scientific Society that Aleksandrov be elected as its corresponding member and their recommendation was respected.[47] From the academic

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Veblen's supervision (1910, 1911), from 1911 up to 1912 he served as an instructor at the mathematics department there. In 1912 he went to Paris and Bologna to continue his studies. In 1915, after his return to Princeton, he was awarded his Ph.D. and was appointed as a lecturer there. During World War I he served as a lieutenant in the U.S. Army. Leaving military service, he returned to Princeton where he taught as an assistant (1920), an associate professor (1926) and a full professor (1928). From 1933 until his retirement in 1951 he was a member of the Institute for Advanced Studies in Princeton. During World War II he worked as a civilian specialist for the U.S. Army Air Force. In the 1950's the McCarthy era caused strong feelings against communism in the USA and Alexander, who held left-wing political views, was under suspicion and he had to leave public life. Alexander's main results are connected with topology, particularly with topology of manifolds, theory of homology and cohomology, topological invariants, algebraic geometry and theory of algebraic surfaces, Cremona transformations, function theory and knot theory. His topological works were developed by Soviet mathematicians as, for example, P. S. Aleksandrov, A. N. Kolmogorov, L. S. Pontryagin.

[43] They were invited and recommended by Lefschetz to spend one academic year in Princeton and to study modern topology here. Lefschetz's non-standard and informal method was criticized by A. T. Trowbridge (one of the important IEB officers for European candidates). Trowbridge pointed out that Aleksandrov had had a fellowship in 1925/1926 to collaborate with Brouwer in Amsterdam. After long discussions, Aleksandrov received his second scholarship sponsored by IEB which was not common. He was again supported by Egorov; Hopf was supported by Schmidt. Aleksandrov and Hopf obtained scholarships for eight months. For more information see [SS1], pp. 88, 137–138, 275, 288, 292.

[44] It was published thanks to Courant in the famous edition *Die Grundlagen der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berücksichtigung der Anwendungsgebiete* as its 44th volume. Its full name is *Topologie. Bd. I: Grundbegriffe der mengentheoretischen Topologie. Topologie der Komplexe. Topologische Invarianzsätze und anschließende Begriffsbildungen. Verschlingungen im n-dimensionalen euklidischen Raum. Stetige Abbildungen von Polyedern*, J. Springer, Berlin, xiv + 636 pages, 39 pictures.

[45] Aleksandrov fruitfully collaborated with Hopf in Göttingen up 1931. They met again in Paris (autumn 1932) and in Moscow during the international topological conference (1935). Then they lost contact for many war years.

[46] Gustav Herglotz (1881–1953) studied physics at the University in Wien (L. Boltzmann) and at the Munich University. The academic year 1903–1904 he spent in Göttingen where he finished his doctorate thesis under Klein's supervision. In 1904 he was named a private docent of astronomy and mathematics there. Then he lectured as a professor of mathematics at the universities in Göttingen (1907), Wien (1908), Leipzig (1909–1925) and again in Göttingen (1925–1947). He was interested in mathematics and its applications in physics and astronomy.

[47] It is interesting that Aleksandrov signed the article named *Über die Dualität zwischen den Zusammenhangszahlen einer abgeschlossenen Menge und des zu ihr komplementären Raumes* which was published in 1927 with the Courant's foreword, as the corresponding member of Göttingen Scientific Society (see Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 1927, pp. 323–329; the article is dated Norderney, Pfingsten 1927, Princeton (New Jersey), Anfang Okober 1927, Vorgelegt von

year 1928–1929 up to the winter semester of the academic year 1930–1931 Aleksandrov regularly lectured in Göttingen.[48] However, after this period, he was forced, for political reasons, to discontinue his work in Germany and returned to Moscow.[49]

Professor in Moscow

From 1921, Aleksandrov, when he was not studying or giving lectures in western Europe, regularly taught mathematics at Moscow University. He founded and organised the first Russian special seminar on topology (since 1924). In 1929 he was appointed an ordinary professor at the Moscow University and his Russian teaching and pedagogical career started. His lectures were described by B. A. Rosenfeld:

I took some of Pavel Sergeevich's courses in topology. In his lectures I felt like I was at the "leading edge" of the world of mathematics, an impression that was strengthened by the distinctive timbre of his voice. Perhaps this was because, it was said, that at the time of the Civil War Aleksandrov was an artist and even a major theatrical producer.[50]

In 1929 Aleksandrov began a friendship with Andrey Nikolaevich Kolmogorov (1903–1987).[51] They spent three weeks together traveling through Russia. After the start in Yaroslavl, they went by boat down the Volga River,

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R. Courant in der Sitzung vom 25. November 1927). Later he published two articles there (*Zum allgemeinen Dimesionsproblem* (ibid., 1928, pp. 25–44; the article is dated Göttingen, den 5. Juli 1928; Vorgelegt in der Sitzung an 6. Juli 1928) and *Über geschlossene Cantorsche Mannigfaltigkeiten* (ibid., 1930, pp. 211–218; the article is undersigned Vorgelegt in der Sitzung an 18. Juli 1930)). It should be mentioned that twelve articles written by Soviet scientists were published in the Göttinger Nachrichten with the help of Aleksandrov's reputation, assistance and influence.

[48] During these years, he published two articles in Mathematische Annalen – *Bemerkung zu meiner Arbeit "Simpliziale Approximationen in der allgemeinen Topologie"*, 101(1929), pp. 452–456 (the article is dated Eingegangen am 4. 9. 1928), and *Dimensionstheorie. Ein Beitrag zur Geometrie der abgeschlossenen Mengen*, 106(1932), pp. 161–238 (the article is dated Göttingen, Mathematisches Institut, Winter 1930–31).

[49] Aleksandrov left Göttingen Scientific Society in 1938 perhaps as a consequence of the discrimination of his Jewish friends and colleagues in Germany. He renewed his contacts with German mathematicians in 1957 when he visited both German countries. In 1958 he obtained the "Gauss professorship" which was established by the Göttingen Academy of Science in 1954 in C. F. Gauss' memory and came with an award of 15 000 marks. Aleksandrov was chosen unanimously in the summer 1957; he received an invitation on August 22, 1957 and he answered on October 8, 1957. He agreed to lecture in Göttingen again but because of many teaching and administrative duties he asked to transfer his stay to the summer semester 1958. His cycle of lectures on topology started at the Göttingen University on May 1, 1958. For more information see [To1], [Ko], Aleksandrov's memories *Математическая жизнь в СССР*, Успехи математических наук [Mathematical life in the USSR, Uspekhi Matematicheskikh Nauk] 34(1979), pp. 219–249, 35(1980), pp. 241–278, and *Erinnerungen an Göttingen*, in Deutschland–Sowjetunion. Aus fünf Jahrzehnten kultureller Zusammenarbeit, Berlin, 1966, pp. 437–440.

[50] [ZD], p. 80.

[51] Andrey Nikolaevich Kolmogorov after finishing secondary school worked for a short time as a conductor on the railway and privately studied Newton's laws of mechanics. In 1920 he

then across the Caucasus mountains to Lake Sevan in Armenia. During the trip Aleksandrov worked on his famous book on topology which he co-authored with Hopf, and Kolmogorov worked on Markov processes. Their friendship was as important and useful for both them as it was for Russian mathematics. In the summer of 1931, they were off on another long trip through Europe, where they visited Berlin, Göttingen, Munich and Paris and where they spent many hours in discussions with outstanding European mathematicians such as E. A. Noether, H. Hopf, P. Lévy, M. Fréchet etc.

In 1935, Aleksandrov and Kolmogorov bought a house in Komarovka (a small village outside Moscow) which became the Russian summer mathematical center. The best undergraduate and graduate students from Moscow University (B. V. Gnedenko, A. A. Mal'cev, I. M. Gel'fand) and their supervisors as well as many famous European mathematicians (for example, J. Hadamard, M. Fréchet, S. Banach, H. Hopf, K. Kuratowski) visited their house where they spent their summer holiday. Aleksandrov and Kolmogorov prepared walks and trips; dinners and evenings were full of mathematical ideas and of discussions about current mathematical problems and their applications not yet available in books or papers. They also discussed culture, painting, architecture, music and literature.[52] For more than fifty years this house continued as an informal center of mathematics for several generations of Soviet mathematicians and their guests from abroad.

In the academic year 1938/1939 Aleksandrov as well as many leading mathematicians from Moscow University joined the Steklov Mathematical Institute of the USSR Academy of Science while retaining their positions at Moscow University. This situation was very comfortable and motivating for ongoing scientific work and the progress of their students.[53] In 1954 Aleksandrov opened

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entered Moscow University but he was interested in a number of subjects (Russian history and culture, metallurgy, physics etc.) which were far from mathematics. Stepanov, Luzin, Egorov and their famous research group "Luzitania" including Suslin, Urysohn and Aleksandrov highly evaluated his abilities and he decided for a mathematical career. In 1925 he finished his studies at Moscow University and began research under Luzin's supervision. His published results attracted of international attention, although he was only an undergraduate. In 1931 he was appointed a professor at the Moscow University, after seven years he joined the Steklov Mathematical Institute of the USSR Academy of Sciences. Kolmogorov made numerous major contributions in a whole range of different mathematical areas. His most important works dealt with probability theory, foundations of theory of Markov random processes, set theory, stochastic calculus, theory of dynamical systems and topology.

[52] More information on the development of Moscow University, its mathematical school, mathematicians and their scientific, pedagogical as well as political activities can be found in [ZD] and in many Russian articles published in Историко-математические исследования [Istoriko-Matematicheskie Issledovanija] 1(1948), pp. 9–42; 8(1955), pp. 9–54; 27(1983), pp. 312–333; 34(1993), pp. 163–184; 36(1995), No. 1, pp. 114–151; 38(1999), pp. 74–92; 41(2001), pp. 213–231; Успехи математических наук [Uspekhi Matematicheskikh Nauk] 12(1957), pp. 9–46; 22(1967), No. 1, pp. 137–161, No. 2, pp. 195–253, No. 4, pp. 147–185; 25(1970), No. 4, pp. 188–196; 34(1979), pp. 219–249; 35(1980), pp. 241–278; 41(1986), pp. 187–203 etc.

[53] Among his students we can find A. V. Arkhangel'skii, A. N. Czerkasov, V. V. Fedorczuk, O. V. Lokucievskii, A. A. Mal'cev, V. V. Nemyckiy, V. I. Ponomarev, L. S. Pontryagin,

a new seminar devoted to modern topics of topology which was aimed at first year students in order to show them some further aspects of their research and projects.

Aleksandrov died on November 16, 1982 in Moscow. In memory of his contribution to Russian topology, his work for the Moscow Mathematical Society and his influence on the Moscow mathematical school there is an annual topological Symposium Aleksandrov Proceedings which takes place every May.

Mathematical and scientific achievements

During his long career Aleksandrov wrote about 300 scientific works. His first outstanding results are connected with topology and theory of functions of real variables. He started with descriptive set theory and theory of real functions. In the 1916 and the years following he proved that every uncountable Borel set contains a perfect subset and some theorems on Borel sets. He then became interested in the foundations of topology. In 1924 he introduced the concept of a locally finite covering and he used it as a basis for his criteria for the metrizability of topological spaces. In many papers from the 1920's and 1930's he developed the basis of topology, homology and cohomology theory, theory of dimension, theory of bicompact spaces. His methods were based on arguments of combinatorial and algebraic topology, set theory and their applications. Some topological theorems bear his name (Aleksandrov set, Aleksandrov bicompact expand, Aleksandrov-Hausdorff theorem on the power of A-sets, Aleksandrov-Čech homology etc.). In the 1940's he discovered the ingredients of an exact sequence of the kernel of a homomorphism, and later he worked on the theory of continuous mappings of topological spaces. Some of his works are also connected with geometry, functional analysis, mathematical logic, foundations and history of mathematics. His results were developed by A. V. Arkhangel'skii, E. Čech, H. Hopf, M. Katětov, V. I. Kuz'minov, V. I. Ponomarev, A. N. Tikhonov.

Thanks to his reputation within the mathematical community, he was editor and a member, at various times, of the editorial board of several international mathematical journals. For many years he also edited the famous Soviet journal *Uspekhi Matematicheskikh Nauk*.

During his life, Aleksandrov obtained many honours for his outstanding mathematical contributions and results. He was the president of the Moscow Mathematical Society (1932–1964) and the Vice-President of the International Congress of Mathematicians (1958–1962). In 1929 he was elected a corresponding member of the USSR Academy of Science and he was appointed its full member in 1953. Many learned societies elected him to membership – the Göttingen Academy of Sciences, the Austrian Academy of Sciences, the Leopoldina Academy in Halle, the Polish Academy of Sciences, the National

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K. A. Sitnikov, Yu. M. Smirnov, A. N. Tikhonov, L. A. Tumarkin, N. B. Vedenissov, V. I. Zajcev.

Academy of Sciences of the United States, the London Mathematical Society, the American Philosophical Society and the Dutch Mathematical Society.

Among his many awards were the Stalin Prize (1943), six Orders of Lenin, and Lobatchevskii International Medal (1972).

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