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FRANC HOČEVAR AND HIS SCIENTIFIC WORK

Marko Razpet

Abstract: During the time of the Austrian–Hungarian Empire, some mathematicians born in the territory of what is now the Republic of Slovenia made important contributions to the fields of education and science. Having produced numerous published articles and textbooks within their professional and scholastic activities, these scholars introduced original ideas to their fields. Within this article, we will discuss two Slovenian mathematicians who had a great influence on academic literature and the study of mathematics in Central Europe: Franc Močnik (1814–1892) and Franc Hočevar (1853–1919).

1 Franc Mocnik

1.1 Life and work

Franc Močnik (1814–1892) was born in Cerkno – Kirchheim (Ger.) – Circhina (Ita.) in the western part of the present day Slovenia. After attending primary school in Idrija (also the site of a well-known mercury mine that first opened in 1500), he studied at the grammar school and lyceum in Ljubljana – Laibach (Ger.), and thereafter at the Roman-Catholic seminary in Gorica – Görz (Ger.) – Gorizia (Ita.) – Gurize (Fur.).[1]

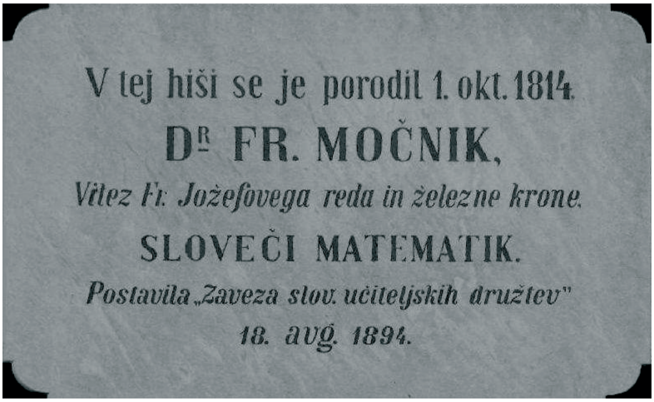

Figure 1: Memorial tablet to Franc Močnik in Cerkno.

After completing his seminary education, Močnik was unable to become a priest due to his young age, so he taught at a normal school in Gorica while simultaneously preparing his doctorate at the University of Graz in Styria – Steiermark (Ger.). After completing his doctorate, Močnik worked as a professor of mathematics in Lviv (Ukr.) – Lwów (Pol.) – Lemberg (Ger.), and in Olomouc (Cze.) – Olmütz (Ger.).

[1]The geographic names have di erent forms in di erent national languages. Here we use abbreviations: Ger. – German, Cze. – Czech, Ukr. – Ukrainian, Pol. – Polish, Ita. – Italian, Fur. – Furlanian.

During his work as a teacher, Močnik recognized severe educational deficiencies at the primary and secondary schools within the Monarchy that required significant improvements. Therefore, he submitted a proposal for educational reform to the Monarchy. Soon after, he was appointed to the position of school counselor and school inspector in Carniola and Styria (more about Močnik can be found in [3, 4, 6]).

During his career, Močnik wrote many mathematical textbooks in the German language that were later translated into other languages of the Monarchy. These books underwent several editions and were still in use after World War I. Only two years after Močnik's death, a memorial tablet was unveiled at his birth house in Cerkno.

During Močnik's time as a student at the lyceum in Ljubljana, he was encouraged to pursue mathematics by a teacher known as L. K. Schulz von Strassnitzki (1803–1852). During the course of his lifetime, Strassnitzki wrote several books on geometry, arithmetic and astronomy, and became well known after inventing a new formula for calculating the number π, namely

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{8}.$$

It contains very easy numbers (1/2, 1/5, 1/8) for calculation and the series expansion of the arctangent function can give us many digits of π very quickly. At that time there was also in Europe a man of great calculating abilities by the name of Zacharias Dase (also spelled Dahse) (1824–1861), who at 30 years old, correctly calculated 200 correct decimals of π. These results were published in Crelle's Journal in 1844. More about Strassnitzki and mathematicians in Vienna can be found in [1].

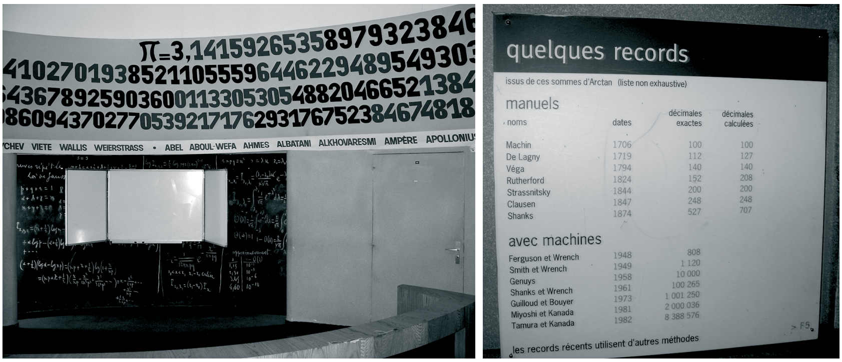

Figure 2: Number π in the science museum Palais de la Découverte, Paris

Of additional mathematical importance was the residence of the French royal family from the Bourbon dynasty in Gorica during their exile in 1830. Among them was Henry V (1820–1883), (known also as Count de Chambord or the Duke of Bordeaux) who was the grandson of the last king of France, Charles X (1757–1836). He was tutored in science at that time by the famous French mathematician A.-L. Cauchy (1789–1857) who was also in exile.

In Gorica, Franc Močnik met Cauchy just prior to his doctorate and as a result he wrote his only scientific work entitled: "Theorie der numerischen Gleichungen mit einer Unbekannten. Mit besonderer Rücksicht auf die neueste von Cauchy erfundene Auflösungsmethode."

This translates to, "Theory of Numerical Equations in One Unknown. With Special Respect to the Newest Method Invented by Cauchy." This work was published in Vienna in 1839 and served as a bridge for Močnik into the scientific world.

For his great work, Močnik was awarded twice: first by the Order of Franz Joseph, and second, by the Order of the Iron Crown Third Class where he was also Knighted "Franz Ritter von Močnik" and received his own coat of arms.

In the past few years, Močnik has been honored once again through memorial sculptures, new books and articles, and through the reprinting of some of his textbooks. In Cerkno, his coat of arms were discovered, and in Gorica a memorial tablet to Cauchy was also unveiled.

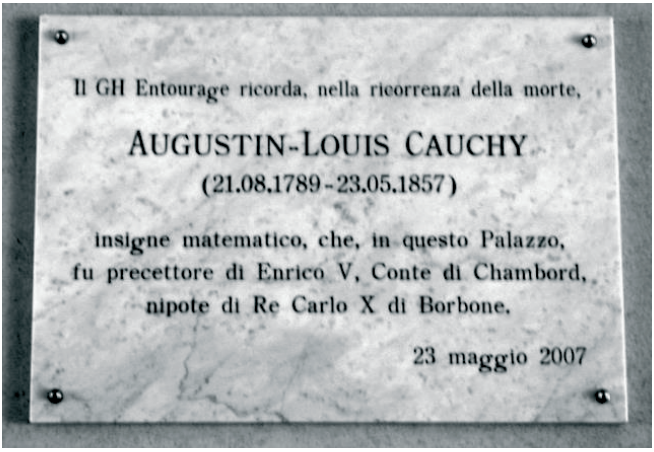

Figure 3: Memorial tablet to Cauchy. Palazzo Strassoldo, Piazza Sant'Antonio, Gorizia.

Figure 4: Močnik's coat of arms. VIRTUTE ET OPERA — BY VIRTUE AND DEEDS

1.2 Important key milestones of Franc Mocnik

1814 — Born in Cerkno
1821–1824 — primary school in Idrija
1824–1832 — grammar school and lyceum in Ljubljana
1832–1836 — Roman–Catholic seminary in Gorica
1836–1840 — University of Graz (teaching in Gorica)
1840 — doctorate at the University of Graz
1836–1846 — teacher in the normal school in Gorica
1846–1849 — professor of elementary and commercial mathematics at the technical academy in Lviv
1849–1851 — professor of mathematics at the University of Olomouc
1851–1860 — school counselor and school inspector for primary schools, Ljubljana
1860–1869 — school counselor and school inspector for primary and real schools, Graz
1862 — Order of Franz Joseph
1869–1871 — provincial school inspector of the first degree for Styria
1871 — retired
1871 — Order of the Iron Crown Third Class
1892 — died in Graz

2 Franc Hocevar

2.1 Life and work

Franc Hočevar was born in Metlika – Möttling (Ger.) in the Slovenian province of Bela Krajina – Weißkrain, or Weiße Mark (Ger.), which is nowadays in the southern part of Slovenia, near the Croatian border. After completing primary school, he studied at the grammar school in Ljubljana, where he had a very good teacher of mathematics. Thereafter, he went on to study mathematics and physics at the University of Vienna and passed his proof of ability for teaching these subjects. After that, he became an assistant at the Technische Hochschule. For his doctoral studies, he defended his dissertation at the University of Vienna entitled, "Über einige bestimmte Integrale" or, "On Some Definite Integrals" (more on this in [2]). At that time, several well-known scientists were working in Vienna, among them L. Boltzmann (1844–1906) and J. Petzval (1807–1891). Hočevar's doctoral advisor was no less than L. Boltzmann. Around this time he also passed his time of probation in the grammar school by the name of Theresianische Akademie in Vienna. Because he was unable to find a job in Vienna, he then moved to Innsbruck where he taught at the local grammar school. During this time, he earned the position of the Privatdozent at the University of Innsbruck, and later became an extraordinary and ordinary professor at the Deutsche Technische Hochschule in Brno – Brünn (Ger.). During his tenure at the University of Innsbruck the well-known mathematicians L. Gegenbauer (1849–1903) and O. Stolz (1842–

1905) also worked there. He was later moved from Brno to the Technische Hochschule in Graz where he was elected several times for the position of Dean of the Faculty of Mechanical and Electrotechnical Engineering. In the last year of his life he was nominated to the Court Counselor. More about Hočevar's life can be found in [5, 7, 8, 9].

Figure 5: Memorial tablet to Franc Hočevar.

During his lifetime, Franc Hočevar also wrote some textbooks in geometry and arithmetic, incorporating his rich experiences from having taught in grammar schools. Like Močnik's textbooks, Hočevar's were also written in the German language and then translated into other languages of the Monarchy. Unfortunately, they were not translated into Slovenian since the Slovenian language was only used as a teaching language at very few teaching schools at that time. Some of Hočevar's textbooks were translated in the Croatian language and used up until World War II. One of his books was also translated and adapted into English.

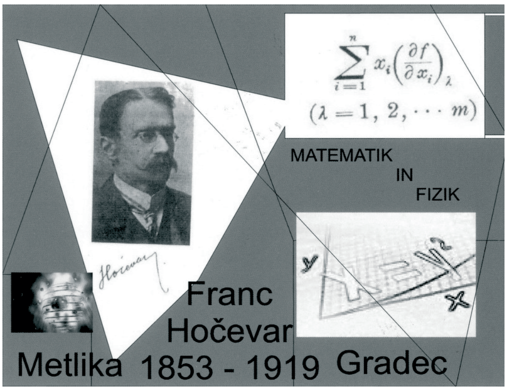

Figure 6: Memorial postcard to Franc Hočevar.

A memorial tablet was belatedly unveiled in honor of Franc Hočevar 90 years after his death. This is because his textbooks were never translated into Slovenian and because he was never as well known as Franc Močnik.

Figure 7: Memorial stamp and mark to Franc Hočevar.

2.2 Important key milestones of Franc Hocevar

1853 — Born in Metlika — Möttling (Ger.)

1864–1871 — grammar school in Ljubljana

1871–1875 — University of Vienna

1874–1879 — assistant, Technische Hochschule in Vienna

1875 — doctorate at the University of Vienna: Über einige bestimmte Integrale, advisor L. Boltzmann

1875–1876 — time of probation in the grammar school of the Theresianische Akademie in Vienna

1879–1891 — teacher in the grammar school in Innsbruck

1883 — Privatdozent at the University of Innsbruck

1891 — extraordinary professor, Deutsche Technische Hochschule in Brno – Brünn (Ger.)

1894 — ordinary professor, Deutsche Technische Hochschule in Brno

1895 — Technische Hochschule in Graz, dean of the Faculty of mechanical and electrotechnical engineering

1919 — court counsellor

1919 — died in Graz

2.3 Articles and discussions

We can find abstracts and publishing details of Hočevar's articles and discussions at "Zentralblatt für Mathematik Online Database" and at "Gallica–Math: Œuvres Complètes". Lists of his works were published at many places, for example in [5, 7]. His name, Franc, was written mostly in German as "Franz" but he preserved his surname throughout in the Slovenian form as "Hočevar".

Über die unvollständige Gammafunktion (1876)
 – On the incomplete gamma function

Über die Ermittelung des Wertes einiger bestimmter Integrale (1876)
 – On the calculation of the value of some definite integrals

Über eine partielle Differentialgleichung erster Ordnung (1887)
 – On a partial differential equation of the first order

Über die Integration eines Systems simultaner Differentialgleichungen (1878)
 – On the integration of a system of simultaneous differential equations

Über die Lösung von dynamischen Problemen mittels der Hamiltonschen partiellen Differentialgleichung (1879)
 – On the solution of dynamic problems by using the Hamilton partial differential equation

Über die Erweiterung eines geometrischen Lehrsatzes von Varignon (1881)
 – On the extension of a geometric theorem of Varignon

Über einige Versuche mit einer Holtzschen Influenzmaschine (1881)
 – On some experiments by using a Holtz's influence machine

Über das Kombinieren zu einer bestimmten Summe (1881)
 – On combinations according to a certain sum

Zur Lehre der Teilbarkeit der ganzen Zahlen (1881)
 – On the theory of divisibility of integer numbers

Zur Integration der Jacobischen Differentialgleichung $Ldx+Mdy+N(xdy-ydx)=0$ (1882)
 – On the integration of the Jacobi differential equation $Ldx+Mdy+N(xdy-ydx)=0$

Über die Wheatstonesche Brücke (1882)
 – On the Wheatstone bridge

Über die Anwendung von exakten Methoden auf die analytische Geometrie der Ebene und zur Ableitung der goniometrischen Grundformeln (1884)
 – On the application of exact methods within planar analytic geometry and on the deriving of basic goniometrical formulas

Bemerkungen zur Simpsonschen Methode der mechanischen Quadratur (1884)
 – Remarks on the Simpson method of mechanical squaring

Über einige elementare Aufgaben der Approximationsrechnung (1890)
– On some elementary problems of the approximation calculus

Über die Konvergenz bestimmter Integrale mit unendlichen Grenzen (1893)
– On the convergence of definite integrals with infinite limits

Das Associationsgesetz der unendlichen Reihen und Produkte (1895)
– The associative law for infinite series and products

Über den arithmetischen Unterricht im Obergymnasium (1901)
– On the arithmetic lessons in the upper grammar school

Sur les formes décomposables en facteur linéaires — Über die Zerlegbarkeit algebraischer Formen in lineare Faktoren (1904)
– On the decomposability of algebraic forms into linear factors

Sind die Elemente der Infinitesimalrechnung an den Mittelschulen einzuführen oder nicht? (1906)
– Are elements of the calculus in secondary schools to introduce or not?

Über die Bestimmung der linearen Teiler einer algebraischen Form (1907)
– On the determination of the linear divisor for an algebraic form

Über die Bestimmung der quadratischen Teiler algebraischer Formen (1907)
– On the determination of the quadratic divisors for algebraic forms

Über den Zusammenhang zwischen den irreduziblen Teilern einer Form und einem linearen System ihrer Nullstellen (1913)
– On the relationship between the irreducible divisors of a form and a linear system of their zeros

2.4 Description of some Hocevar's articles

1. In the article "Über die unvollständige Gammafunktion" found Hočevar a series expansion of the incomplete gamma function. It is defined for positive numbers a and x by integral

$$\gamma(a,x) = \int_0^x t^{a\;\;1} e^{\;\;t}\, dt. \tag{A}$$

Here, the integrand is integrated over a bounded interval $(0, x)$ and not over $(0, \infty)$ like in the case of the complete gamma function defined by

$$(a) = \int_0^\infty t^{a\;\;1} e^{\;\;t}\, dt,$$

therefore, the word incomplete is used. If we integrate (A) by parts we get:

$$\gamma(a,x) = \frac{1}{a} t^a e^{\;\;t} \Big|_0^x + \frac{1}{a}\int_0^x t^a e^{\;\;t}\, dt = \frac{1}{a} x^a e^{\;\;x} + \frac{1}{a}\gamma(a+1,x).$$

In this way we obtain the recursion

$$\gamma(a,x) = \frac{1}{a} x^a e^{\;\;x} + \frac{1}{a}\gamma(a+1,x).$$

Step by step we have the following expansion

$$\gamma(a,x) = \frac{1}{a} x^a e^{-x} \left(1 + \frac{x}{a+1} + \frac{x^2}{(a+1)(a+2)} + \dots \right)$$

which is applicable for small x and big a. It can be written also by using the confluent hypergeometric function

$$M(a,b,z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(b)_n n!} \quad ((a)_0 = 1, (a)_n = a(a+1)\dots(a+n-1))$$

in the form

$$\gamma(a,x) = \frac{1}{a} x^a e^{-x} M(1, a+1, x).$$

In Hočevar's time, some results regarding $\gamma(a,x)$ published by A.-M. Legendre (1752–1833) and O. X. Schlömilch (1823–1901), F. E. Prym (1841–1915), F.-M. E. Vallier (1849–1921) were known.

2. It is well known that an integer number written in decimal form is divisible by 11 if the alternating sum of its digits is divisible by 11. In the article "Zur Lehre der Teilbarkeit der ganzen Zahlen", Hočevar generalized this rule for any integer n which is represented to the base b:

$$n = a_r a_{r-1} \dots a_2 a_1 a_0 (b) = a_r b^r + a_{r-1} b^{r-1} + \dots + a_1 b + a_0. \tag{B}$$

Here, $a_r, a_{r-1}, \dots, a_1, a_0$ are digits of n represented to the base b, $0 \le a_k < b$. In (B), we partition digits from the right to the left side in groups so that each one contains q digits:

$$n = \dots | a_{3q-1} \dots a_{2q-1} a_{2q} | a_{2q-1} \dots a_{q-1} a_q | a_{q-1} \dots a_1 a_0 |.$$

Thereafter, we define integers

$$m_0 = a_{q-1} \dots a_1 a_0 (b), \quad m_1 = a_{2q-1} \dots a_{q-1} a_q (b),$$

$$m_2 = a_{3q-1} \dots a_{2q-1} a_{2q} (b), \dots$$

and the alternating sum

$$m = m_0 - m_1 + m_2 \pm \dots = \sum_{r \ge 0} (-1)^r m_r.$$

Proposition. *If $b^q + 1$ divides m than $b^q + 1$ divides n, too.*

Proof. We write:

$$n = \sum_{k \ge 0} a_k b^k = \sum_{r \ge 0} \sum_{s=0}^{q-1} a_{qr-s} b^{qr-s} = \sum_{r \ge 0} b^{rq} \sum_{s=0}^{q-1} a_{qr-s} b^s = \sum_{r \ge 0} b^{rq} m_r.$$

Consider the difference:

$$n - m = \sum_{r \geq 1} m_r (b^{qr} - (-1)^r).$$

For odd indices r, say $r = 2j+1$, the factors in terms of the above sum have form $(b^q)^{2j \;\; 1}+1$. They can be factored into

$$(b^q)^{2j \;\; 1} + 1 = (b^q + 1)P(b, q, j)$$

where $P(b, q, j)$ is an integer.

For even indices r, say $r = 2j$, we have on the right-hand side of the sum factors $(b^q)^{2j} - 1$ and we can also factor them

$$(b^q)^{2j} - 1 = (b^{2q})^j - 1 = (b^{2q} - 1)Q(b, q, j) = (b^q + 1)(b^q - 1)Q(b, q, j),$$

where $Q(b, q, j)$ is an integer.

Therefore, $n - m$ is divisible by $b^q + 1$ and if $b^q + 1$ divides m than $b^q + 1$ also divides n. ■

For $b = 10$ and $q = 1$ we obtain the divisibility rule of n by 11.

3. In his contribution “Zur Integration der Jacobischen Differentialgleichung $L\,dx + M\,dy + N(x\,dy - y\,dx)$”, he obtained the integral of the Jacobi differential equation

$$\frac{dx}{a_1x + b_1y + c_1 - x(a_3x + b_3y + c_3)}$$
$$= \frac{dy}{a_2x + b_2y + c_2 - y(a_3x + b_3y + c_3)},$$

where all coefficients are real.

In modern notations, he associated to the above equation a matrix:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

If A has different eigenvalues λ_k $(k = 1, 2, 3)$ there exist linearly independent eigenvectors $v_k = [\;\; _k, \beta_k, \gamma_k]^t$ $(k = 1, 2, 3)$, and the solution of the given differential equation has the form

$$(\;\; _1x + \beta_1y + \gamma_1)^{\;\; 2 \;\;\; 3}(\;\; _2x + \beta_2y + \gamma_2)^{\;\; 3} \;\;\; (\;\; _3x + \beta_3y + \gamma_3)^{\;\;\;\; 2} = const.$$

Hočevar discussed also cases $\lambda_1 = \lambda_2 \neq \lambda_3$ and $\lambda_1 = \lambda_2 = \lambda_3$.

He found that the integral of the equation is algebraic, if the real parts of all eigenvalues of A are equal.

In this way, the Jacobi differential equation is treated by V. V. Stepanov (1889–1950) in his work “Course of differential equations” with the remark that D. F. Egorov (1869–1931) at the Moscow university used also the same method.

Figure 8: Franc Močnik and Franc Hočevar.

4. In the article "Über die Integration eines Systems simultaner Differentialgleichungen" Hočevar discussed the system

$$\frac{dx_1}{X_1 - x_1 X} = \ldots = \frac{dx_n}{X_n - x_n X} = \frac{dz}{X_{n+1} - zX}$$

of differential equations where X denotes a homogeneous function of an arbitrary order h and $X_1, \ldots, X_{n+1}$ are linear homogeneous functions in variables $x_1, \ldots, x_n, z$. He proved that if h is an integer and

$$X = a_1 x_1^h + \ldots + a_n x_n^h + a_{n+1} z^h,$$

the system can be integrated in a closed form.

Here we have described two mathematicians who had a great influence in the fields of educational literature and mathematics in Central Europe. Franc Močnik and Franc Hočevar both wrote many textbooks that were used for many years within the primary and elementary schools of the Austrian-Hungarian Empire. Moreover, both of them had lectured at good academies and technical universities where students from many provinces from the Monarchy studied. In this way, they significantly contributed to new knowledge and progress of many generations.

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