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THE RECEPTION OF BOLYAI'S GEOMETRY IN THE AUSTRO-HUNGARIAN EMPIRE

KATI MUNKACSY

Abstract: There is plenty of detailed literature on the history of non-Euclidean geometry. I would like to mention some important mathematicians such as Euclid, Playfair, Saccheri, Lambert, Bolyai, Lobachevsky, Gauß, Riemann, Klein, Halsted, Beltrami, and Hilbert who were interested in the fundaments of geometry. We know less about the reception of Farkas Bolyai's work in Hungary's neighbor countries, in the Austrian-Hungarian Empire. With the help of my colleagues I collected some data on publications and university lectures on the reception of hyperbolic geometry in Vienna, Prague, Beograd and Kolozsvár.

1 Introduction

1.1 The birth of Non-Euclidean geometry

Janos Bolyai introduced non-Euclidean geometry, or "new geometry" as he called it, in an appendix of the book "Tentamen ..." by his father Farkas Bolyai.

We know that the *Appendix*[1] had been published on June 20, 1831, since on that day Farkas Bolyai sent its reprint to C. F. Gauß who, on reading it, wrote to his friend saying: *I regard this young geometer Bolyai as a genius of the first order.*

To Farkas Bolyai, however, C. F. Gauß wrote: *To praise it would amount to praising myself. For the entire content of the work ... coincides almost exactly with my own meditations which have occupied my mind for the past thirty or thirty-five years.*

I. N. Lobachevsky had published a similar piece of work in 1829 in Kazan.

1.2 Was there one "geometry" or many?

One

Beltrami thought that hyperbolic geometry is not an independent, new theory, but a part of differential geometry. The hyperbolic plane is a special kind of surface with constant curvature. What he suggested was that J. Bolyai and I. N. Lobachevsky had not really introduced new concepts at all, it is not an alternative to Euclidean geometry (Beltrami, 1868).

Many

After Gauß's death, Bolyai's papers on geometries[2] were published. Today we call these three geometries Euclidean, hyperbolic (=non-Euclidean), and absolute. Absolute

[1] Bolyai Joannes: *Appendix, Scientiam Spatii absolute veram exhibens; a veritate aut falsitate axiomatis XI. Euclidei (a priori haud unquam decidenda) independentem; adjecta ad casum falsitatis quadratura circuli geometrica*, Maros Vásárhely, 1831. In: Bolyai Farkas: *Tentamen juventutem studiosam in elementa matheseos purae, elementaris ac sublimioris, methodo intuitiva, evidentiaque huic propria, introducendi. Cum Appendice triplici. I., II.*, Maros Vásárhely, 1832–1833.

geometry is the part that the first two geometries have in common. Most of the *Appendix* deals with the absolute version of geometry.

The first International Congress of Mathematicians was held in Zürich from 9 to 11 August 1897. The congress decided that the new geometry should be named Bolyai-Lobachevsky geometry. C. Burali-Forti attended the Congress and presented a paper *The postulates for the geometry of Euclid and of Lobachevsky* to the Geometry section of the Congress.

2 Bolyai's Geometry as one of the three elementary geometries

2.1 Researches and Lectures on Bolyai's Geometry in the Monarchy

Prague[3]

The hyperbolic geometry came to Prague from Russia. Eduard Weyr (1852–1903) was the first Czech professor of mathematics who wrote on non-Euclidean geometry in the Czech lands. In 1896, he published two short articles on the celebration in Russia on occasion of Lobachevsky's birth in 1796 and where he gave the first Czech analysis of Lobachevsky's work. Eduard Weyr translated some interesting and important parts from the proceedings which were published by the University in Kazan (i.e. the parts form the lectures of F. M. Suvorov (1845–1911) and A. V. Vasiljev (1853–1929)).[4]

Vienna

Until 1900 non-Euclidean elementary geometry was not taught, but perhaps part of it was included in differential geometry, theory of surfaces, projective geometry, and spherical trigonometry. Gustav Kohn's lecture held in Vienna in 1905 was the first one

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[2] J. Bolyai: *La science absolue de l'espace* ..., Traduit par C. J. Houël, 1867.
J. Bolyai: *Sulla scienza dello spazio assolutamente vera* ..., Versione dal latino per G. Battaglini, Giornale di Matematica, 6, Napoli, 1868.
J. Bolyai: *The science absolute of space* ..., Translated from the Latin (1832) by G. B. Halsted, Austin, Texas, 1891, 1892, 1893, 1896.
J. Bolyai: *Appendix, Prilozsenyije* ..., Perevod V. F. Kagana, Moskva –Leningrad, 1950.

[3] E. Weyr: *Oslava stoleté ročnice dne narození N. I. Lobačevského cís. Kazaňskou universitou* [The 100th anniversary of the birth of N. I. Lobachevsky organized by University in Kazan], Časopis pro pěstování mathematiky a fysiky 25(1896), pp. 1–38. (There is a very short review of this article in the journal Jahrbuch über die Fortschritte der Mathematik und Physik 27(1896), pp. 15) – article and review are on www pages of EMS.)
E. Weyr: *Oslava stoleté ročnice dne narození N. I. Lobačevského cís. Kazaňskou universitou* [The 100th anniversary of the birth of N. I. Lobachevsky organized by University in Kazan], Živa 6(1896), pp. 6–10. (It is a short extract from the article mentioned above.)
E. Weyr: *Slavnostní odkrytí pomníku N. I. Lobačevskému v Kazani 1. září 1896* [The ceremonial revelation of N. I. Lobachevsky's monument in Kazan on September 1, 1896], Časopis pro pěstování mathematiky a fysiky 26(1897), pp. 249–254.
E. Weyr: *Založení ceny na počet Lobačevského a odhalení jeho pomníku* [The foundation of N. I. Lobachevsky's Price and the ceremonial opening of his monument], Časopis pro pěstování mathematiky a fysiky 26(1897), pp. 31–32. (A very short review of this article can be found in the journal Jahrbuch über die Fortschritte der Mathematik und Physik 28 (1897), pp. 13 – articles and reviews are on www pages of EMS.)
V. Hauner: *Geometrie neeuklidovská a její poměr k teorii poznání* [Non-Euclidean geometry and its relation to the theory of recognition], Česká Mysl, vol. IV
V. Hauner: *Geometrie neeuklidovská: Theorie Riemannova* [Non-Euclidean geometry: Riemann's theory], Česká Mysl, vol. IX

[4] Thanks to Martina Bečvářová for her help to collect the information described above.

dedicated explicitely to non-Euclidean geometry. It is interesting to mention that G. Kohn was in Berlin as a „student" of Otto Stolz, 1870/1871.[5]

Graz

There were lectures on hyperbolic geometry in Graz, as well, by Johannes Frischauf in the 1880's, but I do not have more information about this.

Budapest

We do not have earlier data than following Béla Kerékjártó: *The Foundations of Geometry, Projective geometry*, Budapest, 1937, 1944.

Beograd

The University of Beograd was established in 1905, until 1946 there were no lectures on geometry. Researches and lectures on hyperbolic geometry started after 1946.[6]

3 First seminar on Bolyai's geometry in Kolozsvar-Cluj

Gyula Vályi (1855–1913), mathematician at the University of Kolozsvár, taught a course on Bolyai geometry in 1891–1892. What were his sources for this seminar?[7]

The scientific source: Gyula Vályi learnt the role of the new theories of geometry in contemporary mathematics during his scholarship in Berlin from 1878 up 1880.

The personal source: Gyula Vályi had a copy of *Tentamen* (the 1st edition, 1832) dedicated by Farkas Bolyai to his father, Károly Vályi, who was a student of Farkas Bolyai. This book was not available, neither in the libraries nor in the book shops. Luckily the *Tentamen* – the book with *Appendix* – was preserved as a relic in the Vályi family. We know all of this from a personal letter of professor Réthy Mór.[8]

There is a chain of teachers and their students between Farkas Bolyai and Barna Szénássy, who was a great Hungarian mathematics historian. Lajos David (University professor of mathematics in Kolozsvar and in Debrecen), was a student of Gyula Vályi, and Barna Szénássy (University professor of mathematics in Debrecen) was the student of Lajos David.[9] This chain of tradition solved the problem that neither the book nor the manuscript was available.

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[5] Thanks to Hellmuth Stachel and Christa Binder for their information mentioned above.

[6] Thanks to Mileva Pranovic for her help to find information published above.

[7] *Csak két előadása volt, amely kevéssé változott. Egyik az elemi függvénytan, melyben egykori mesterét, Weierstrasst követte; ebbe volt beleszőve az elliptikus függvények elmélete is. Másik ilyen előadása Bolyai János Appendixe, melyben különösen kezdetben ragaszkodott e mestermű paragrafusaihoz sorrendben is, tartalomban is. A másik – matematikai kultúránk fejlődése szempontjából az előbbinél még jelentősebb – kollégiuma Bolyai János Appendixéről szólt. Ezt először az 1891/92. tanév második félévében tartotta, ettől kezdve csaknem változatlan formában négyévenként többször megismételte. Élvezetes, szép olvasmány az előadásról készült, megfakult, százkét lapos litografált jegyzet. Eszerint Vályi Gyula kollégiumának mintegy harmadát a történelmi előzmények ismertetésére fordította, majd az Appendixet kommentálta, a paragrafusok sorrendjében haladva előre. A bizonyításokat kiegészítette, a rendkívül tömör fogalmazást – magyarázó részek közbeiktatásával – feloldotta. Helyenként – az abszolút és hiperbolikus geometria összehasonlítása céljából – kölcsönzött Lobacsevszkij eredményeiből is.*

[8] Thanks to Tibor Weszely for his help to collect these pieces of information.

[9] For more information see web pages: Dávid Lajos
http://www.magyaronline.net/ujforum/index.php?showtopic=495&pid=7700&mode=threaded&show=&st=&
and Kalmár László – Dávid Lajos web pages: http://www.math.u-szeged.hu/polygon/kalmarium.html.

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4 Conclusion

Was the implementation of Bolyai's geometry successful? My hypothesis is that research was successful but the teaching not.

Why was the teaching process unsuccessful? The main reason I see in the lack of good teaching tools and models. The implementation of Bolyai's geometry was not solved in a satisfying way. And today, elementary geometry is not in the centre of interest of mathematicians. Educators think that by nature Euclidean geometry is in the students' head and that we should not disturb them with other types of geometries. Therefore teachers cannot really teach Bolyai's geometry.

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[2] Weszely T.: *150 éve született Vályi Gyula* Természet Világa, 2005/1. http://www.termeszetvilaga.hu/szamok/tv2005/tv0501/weszely.html.

[10] *Csak két előadása volt, amely kevéssé változott. Egyik az elemi függvénytan, melyben egykori mesterét, Weierstrasst követte; ebbe volt beleszőve az elliptikus függvények elmélete is. Másik ilyen előadása Bolyai János Appendixe, melyben különösen kezdetben ragaszkodott e mestermű paragrafusaihoz sorrendben is, tartalomban is. A másik – matematikai kultúránk fejlődése szempontjából az előbbinél még jelentősebb – kollégiuma Bolyai János Appendixéről szólt. Ezt először az 1891/92. tanév második félévében tartotta, ettől kezdve csaknem változatlan formában négyévenként többször megismételte. Élvezetes, szép olvasmány az előadásról készült, megfakult, százkét lapos litografált jegyzet. Eszerint Vályi Gyula kollégiumának mintegy harmadát a történelmi előzmények ismertetésére fordította, majd az Appendixet kommentálta, a paragrafusok sorrendjében haladva előre. A bizonyításokat kiegészítette, a rendkívül tömör fogalmazást – magyarázó részek közbeiktatásával – feloldotta. Helyenként – az abszolút és hiperbolikus geometria összehasonlítása céljából – kölcsönzött Lobacsevszkij eredményeiből is.*

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