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FULLY DISTRIBUTED CONSENSUS FOR HIGH-ORDER STRICT-FEEDBACK NONLINEAR MULTIAGENT SYSTEMS WITH SWITCHED TOPOLOGIES

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This paper studies the distributed consensus problem of high-order strict-feedback nonlinear multiagent systems. By employing the adaptive backstepping technique and switched system theory, a novel protocol is proposed for MASs with switched topologies. Global information such as the number of agents and communication topology is not used. In addition, the communication topology between agents can be switched between possible topologies at any time. Based on the Lyapunov function method, the proposed adaptive protocol guarantees the complete consensus of multiagent systems without restricting the dwell time of the switched signal. Finally, two numerical examples are provided to illustrate the effectiveness and advantages of the given protocol.

Keywords: fully distributed consensus, multiagent systems, adaptive control, nonlinear systems, arbitrary switching

Classification: 93A14, 93C10, 93C40

1. INTRODUCTION

In recent years, multiagent systems (MASs) have attracted more and more scholars' attention because of its wide application in such fields as driverless vehicle, multirobot systems, satellite formation flying and sensor networks [10, 15, 35, 48]. Consensus is one of the critical issues in the research of MASs. The main work of consensus control is to propose a protocol so that the state error between any two agents can converge to zero or a small adjustable neighborhood [47, 20, 50].

At the early stage of the research, abundant achievements have been made for the first-order or second-order integrator MASs [9, 22, 49]. However, all physical systems are inherently nonlinear in reality, so the research of nonlinear MAS has received extensive attention [4, 2, 11, 33, 44]. The consensus problem for second-order nonlinear MASs has been investigated in works [1, 18, 21]. When the agent is described by a high-order nonlinear dynamics, corresponding results were given in some other works [29, 43]. Note that in the above works, the nonlinear term and input exist in the same channel in agent dynamics. Some scholars proposed many consensus protocols for strict-feedback nonlinear (lower-triangular) MASs [12, 41, 42]. For instance, Hua et al. [12] proposed a finite

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time consensus protocol for high-order strict-feedback MASs with uncertain nonlinear dynamics. For strict-feedback MASs satisfying Lipschitz condition, You et al. [41] developed a self-triggered consensus protocol with only output information of neighbored agents. Subsequently, for a class of high-order strict-feedback nonlinear MASs with time-varying gain in paper [42], a fixed-time consensus protocol was presented. However, the above results are based on the assumption that the communication topology between agents is fixed.

In practical applications, interaction topologies among agents are often unreliable due to the limited communication range or the influence of obstacles. The switching of topology leads to the change of the Laplacian matrix corresponding to the MAS, which will inevitably affect the selection of some parameters in the protocol. Until now, many researches take topology switching into account. By constructing topology-dependent multiple Lyapunov functions, Wen et al. [38]proposed an observer-based protocol for nonlinear MASs with directed switched topology. Zou et al. [52]considered a class of second-order nonlinear MASs with switched communication topology and designed global adaptive consensus protocol. For a class of time-delay MASs with switched topology satisfying the average dwell time, Chen et al. [3] proposed a distributed protocol by using only relative position. Although some progress has been made on the consensus of MASs with switched topology, the above results [3, 38, 52] are based on the fact that the switched signals of MAS communication topology meet the dwell time. When the communication topology of the MAS is arbitrarily switched among possible topologies, and the switching interval does not meet the average dwell time, the above protocols may be invalid. Olfati–Saber et al. [24] constructed a common Lyapunov function to deal with the consensus problem of linear MAS under directed networks with switching topology. Considering that the agent dynamics is a single-input stabilizable model, Valcher et al. [32] constructed a common quadratic positive definite Lyapunov function. A constant state-feedback matrix is derived that makes the MAS achieves consensus. However, the feedback matrix is determined by the eigenvalues of Laplacian matrices corresponding to all possible topologies. Liu et al. [23] studied the leaderless consensus of MASs with Lipschitz nonlinearities under directed arbitrary switched topologies. Under the condition that the directed graph is strongly connected and balanced, the consensus of MAS is realized. Similarly, the solution of the feedback matrix involves eigenvalue information of Laplacian matrices of topologies. Unfortunately, few attention has been paid to the problem of fully distributed protocol for high-order strict-feedback nonlinear MASs with arbitrary switching topologies.

Another point to note is that, in most of the above-mentioned results, the controller parameters depend on the eigenvalues of the Laplacian matrix. For a single agent in MASs, some global information may not be available, such as the communication topology and the number of agents. For the distributed protocol design of MAS, each agent can only obtain the state information of itself and adjacent agents. This protocol that does not use global information is called fully distributed consensus protocol. For the linear MASs with directed topology, Li et al. [17] developed the distributed consensus protocol without using the smallest real part of the nonzero eigenvalues of the Laplacian matrix. Feng et al. [7] proposed an adaptive fully distributed consensus protocol for second-order nonlinear MASs. By using distributed adaptive protocols, Li et al. [16] solved fully distributed consensus problems for both linear and Lipschitz nonlinear MASs. The fully distributed consensus problem was investigated for a class of high-order nonlinear MASs with unmatched disturbances in paper [40]. Wang et al. [36] addressed the fully distributed cooperative control problem of nonlinear strict-feedback MASs under directed and time-invariant communication graphs. By designing a distributed adaptive protocol, the containment control problem of high-order nonlinear MAS under directed communication topology is solved by Wang et al. [34]. When the topology is directed, and there are different unknown control directions, Huang et al. [13]. proposed a fully distributed adaptive control approach to solve the consensus problem of high-order nonlinear MASs. Although many methods have been proposed to solve the fully distributed consensus control of high-order nonlinear MASs, switching topologies are not taken into account in the above results. The topology switching will cause the change of unknown parameters, so it is necessary to redesign the adaptive law to estimate the time-varying unknown parameters. From all the above discussions, it can be seen that there are few results of eliminating dwell time constraints when fully distributed control and switching topology are considered at the same time.

To the best of our knowledge, the fully distributed consensus for high-order strict-feedback nonlinear MASs under arbitrary switched topologies is still an open topic. Therefore, we will investigate this issue in this paper. The main contributions of this paper are as follows:

1. A novel fully distributed consensus protocol is proposed for high-order strict-feedback MASs with switched topologies. By using the information of the agent itself and its neighbors, the control law of each agent is determined. The selection of protocol parameters does not depend on the global information of MAS, such as the number of agents and communication topology. It is worth noting that due to topology switching, we propose an innovative adaptive law to approximate time-varying unknown parameters.

2. The protocol proposed in this paper can realize the consensus of MASs when the communication topology between agents is switched, and the fixed topology [12, 41, 42] can be regarded as a special case. Further more, by skillfully constructing the error system and selecting the Lyapunov function the dwell time constraint of the switched signal [3, 38, 52] is avoided.

3. The protocol proposed in this paper is globally effective, and MASs can achieve complete consensus under the affect of this protocol. Globally effectiveness means that the protocol can be used for MASs with arbitrary initial states. The state error between agents can approach 0 rather than a small neighborhood asymptotically.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Algebraic graph theory preliminaries

In this paper, the communication between agents is described by a graph. For an MAS consisting of N follower agents, each follower can correspond to a node. Due to the connection between agents is not fixed, a time-varying graph $G_{s(t)} = \{V, E(t), A(t)\}$

is introduced. $V = \{\mathbf{v}_i,\ i \in \mathcal{V}\}$ denotes the set of nodes. V is fixed because the number of agents doesn't change. $E(t) \subseteq V \times V$ is the set of edges. $A(t) = [\alpha_{ij}]_{N\times N}$ represents the adjacency matrix , where each element corresponds to the weight of communication between two agents. If follower i can receive information from follower j at time t, then $(j,i) \in E(t)$, and the corresponding weight $\alpha_{ij}(t) > 0$; otherwise, $(j,i) \notin E(t)$ and $\alpha_{ij}(t) = 0$. Since the communication topology will change under the action of the switched signal, $E(t)$ and $A(t)$ are time-varying. $s(t) : [t_0, +\infty) \to \underline{S} = \{1, \ldots, S\}$ is the switched signal. Each element in index set $\underline{S}$ corresponds to a different communication topology, S stands for the number of possible communication topologies. For graph $G_k,\ k \in \underline{S}$, if $\alpha_{ij} = \alpha_{ji}$ (the communication between the two agents is bidirectional) , the graph is called undirected graph. Furthermore, if there is at least one path between any two nodes, the graph is called undirected and connected. Defining $D(t) = diag\{\sum_{j=1}^{n} \alpha_{1j}(t), \ldots, \sum_{j=1}^{n} \alpha_{nj}(t)\}$, the Laplacian matrix of $G_{s(t)}$ can be obtained as $L(t) = D(t) - A(t)$. $\beta_i(t)$ represents the communication status between each follower agent and the leader agent. If follower i can receive information from the leader at time t, $\beta_i(t) > 0$; otherwise, $\beta_i(t) = 0$. Define $B(t) = diag\{\beta_1, \ldots, \beta_N\}$ and $H_{s(t)} = L(t) + B(t)$.

2.2. Problem formulation

An MAS consisting of $N+1$ agents is considered, where the agent labeled 0 is called a leader, and other agents (labeled $1, 2, \ldots, N$) are followers. Each follower agent is described by the following high-order strict-feedback nonlinear system:

$$\begin{aligned} &\dot{x}_{i,m} = x_{i,m+1} + f_m\left(\bar{x}_{i,m}, t\right), m = 1, \ldots, n-1 \\ &\dot{x}_{i.n} = u_i + f_n\left(\bar{x}_{i,n}, t\right), i \in \mathcal{V} = \{1, \ldots, N\} \end{aligned} \tag{1}$$

and the leader agent is described as:

$$\begin{aligned} &\dot{x}_{0,m} = x_{0,m+1} + f_m\left(\bar{x}_{0,m}, t\right), m = 1, \ldots, n-1 \\ &\dot{x}_{0,n} = f_n\left(\bar{x}_{0,n}, t\right) \end{aligned} \tag{2}$$

where $\bar{x}_{i,m} = \left(x_{i,1}, \ldots, x_{i,m}\right)^T \in \mathbb{R}^m$ and $\bar{x}_{0,m} = \left(x_{0,1}, \ldots, x_{0,m}\right)^T \in \mathbb{R}^m$. $x_{i,\alpha}$ and $x_{0,\alpha}$ $(\alpha = 1, \ldots, n)$ are the αth state component of the follower i and the leader, respectively. $f_\alpha(\cdot)$ $(\alpha = 1, \ldots, n)$ are continuous nonlinear functions. $u_i \in \mathbb{R}$ is the input which needs to be designed.

Remark 2.1. In many works such as [9, 22, 49], the consensus of linear MASs was considered. However, the proposed protocols may be ineffective for MAS (1)−(2) due to the presence of nonlinear factors. If $f_m = 0, m = 1, \ldots, n-1$, then the underlying MAS degenerates to the model investigated in [8, 14, 46]($\dot{x}_{i,m} = \dot{x}_{i,m+1}, \dot{x}_{i.n} = u_i + f\left(\bar{x}_{i,n}, t\right), i \in \Gamma = \{0, \ldots, N\}$). That is, only nonlinear factors with the same channel as the input were considered in [8, 14, 46]. Furthermore, if $n = 2$, then the underlying MAS degenerates to a second-order nonlinear MAS ($\dot{x}_{i,1} = \dot{x}_{i,2}, \dot{x}_{i.2} = u_i + f\left(\bar{x}_{i,2}, t\right), i \in \Gamma = \{0, \ldots, N\}$), which has been widely studied, such as in [5, 6, 28, 30].

The objective of this paper is to design a fully distributed consensus protocol based on local information such that the MAS with switched topologies can achieve global asymptotic consensus. Besides, the switched signal is not constrained by the dwell time.

To this end, the following assumptions and lemmas are required.

Assumption 1. There are known positive constants ρ_m $(m = 1, \dots, n)$ such that

$$|f_m(\bar{x}_{i,m}, t) - f_m(\bar{x}_{0,m}, t)| \leq \rho_m \sum_{k=1}^{m} |x_{i,k} - x_{0,k}|. \tag{3}$$

Remark 2.2. Assumption 1 is widely used in the research of MASs [19, 27, 51], which is called the Lipschitz condition. This condition imposes the limitation on the growth of the nonlinear function. With this assumption, it is conducive to design a fully distributed protocol to realize the complete consensus for MASs instead of bounded consensus. Some classical systems satisfy the above assumption, including Lorenz system, Chen system, Lü system, Chua's circuit, and so on [26]. The agents considered in this paper are homogeneous, so the Lipschitz constants of each order are the same for all agents. In the future, we will work on heterogeneous multi-agent systems.

Assumption 2. For all $k \in \underline{S}$, the graph G_k is undirected and connected.

Remark 2.3. Assumption 2 is commonly used in consensus control of nonlinear systems [31, 37, 39]. Different from the situation considered in these papers, the number of all possible communication topologies and every communication topology are not available. So it is difficult to design a fully distributed consensus protocol for the underlying MASs. This paper only assumes that the possible communication topologies of MASs are undirected and connected, and it is not necessary to know the exact topology information and switched signal information. In addition, the switching signal does not need to satisfy the dwell time constraint.

Lemma 2.4. (See the work of Zhang [45]) Under Assumption 2, $\forall k \in \{1, 2, \dots, S\}$, H_k is a positive definite and symmetric matrix.

Lemma 2.5. (See the work of Qian [25]) For any $\beta_1 > 0, \beta_2 > 0$ and $\eta > 0$, the following inequality can be obtained

$$|p|^{\beta_1}|q|^{\beta_2} \leq \frac{\beta_1}{\beta_1+\beta_2}\eta|p|^{\beta_1+\beta_2} + \frac{\beta_2}{\beta_1+\beta_2}\eta^{-\frac{\beta_1}{\beta_2}}|q|^{\beta_1+\beta_2}. \tag{4}$$

3. MAIN RESULTS

3.1. Fully distributed consensus protocol design

In this subsection, the fully distributed consensus protocol for high-order strict-feedback nonlinear MASs will be proposed by using the adaptive backstepping technique and Lyapunov functional method.

First we construct the following error system

$$\begin{aligned} e_{i,m} &= x_{i,m} - x_{0,m} \\ e_{i,n} &= x_{i,n} - x_{0,n} \end{aligned} \tag{5}$$

the dynamical equation of the error system can be obtained as

$$\begin{aligned} \dot{e}_{i,m} &= e_{i,m+1} + \bar{f}_{i,m} \\ \dot{e}_{i,n} &= u_i + \bar{f}_{i,n} \end{aligned} \tag{6}$$

where $\bar{f}_{i,m} = f_m(\bar{x}_{i,m}, t) - f_m(\bar{x}_{0,m}, t)$.

By constructing the state errors of each follower and the leader, we transform the consensus problem of the MASs into the stability problem of the error system. The control goal of this paper is to construct u_i to make the error system asymptotically stable.

Step 1: Letting $z_{i,1} = e_{i,1}$, we have

$$\dot{z}_{i,1} = e_{i,2} + \bar{f}_{i,1}. \tag{7}$$

Selecting the Lyapunov function candidate as

$$V_1 = \frac{1}{2}\sum_{i=1}^{N} z_{i,1}^2. \tag{8}$$

Differentiating V_1 with respect to time, one has

$$\dot{V}_1 = \sum_{i=1}^{N} z_{i,1}\dot{z}_{i,1} = \sum_{i=1}^{N} z_{i,1}\left(e_{i,2} + \bar{f}_{i,1}\right). \tag{9}$$

Based on Assumption 1, one gets

$$\begin{aligned} z_{i,1}\bar{f}_{i,1} &\le |z_{i,1}||\bar{f}_{i,1}| = |z_{i,1}||f_1(x_{i,1}, t) - f_1(x_{0,1}, t)| \\ &\le \rho_1|z_{i,1}||x_{i,1} - x_{0,1}| = \rho_1 z_{i,1}^2. \end{aligned} \tag{10}$$

Substituting (10) into (9), we have

$$\dot{V}_1 \le \rho_1\sum_{i=1}^{N} z_{i,1}^2 + \sum_{i=1}^{N} z_{i,1}e_{i,2}^* + \sum_{i=1}^{N} z_{i,1}(e_{i,2} - e_{i,2}^*) \tag{11}$$

where $e_{i,2}^*$ is a virtual signal.

Letting $e_{i,2}^* = -\gamma_1 z_{i,1}$ and choosing $\gamma_1 > \rho_1$, we obtain

$$\begin{aligned} \dot{V}_1 &\le -(\gamma_1 - \rho_1)\sum_{i=1}^{N} z_{i,1}^2 + \sum_{i=1}^{N} z_{i,1}\left(e_{i,2} - e_{i,2}^*\right) \\ &= -a_{1,1}\sum_{i=1}^{N} z_{i,1}^2 + \sum_{i=1}^{N} z_{i,1}\left(e_{i,2} - e_{i,2}^*\right) \end{aligned} \tag{12}$$

where $a_{1,1} = \gamma_1 - \rho_1$ is a positive constant.

Step 2: Letting

$$z_{i,2} = e_{i,2} - e_{i,2}^{*} = e_{i,2} + \gamma_1 e_{i,1} \tag{13}$$

we have

$$\dot{z}_{i,2} = \dot{e}_{i,2} + \gamma_1 \dot{e}_{i,1} = e_{i,3} + \bar{f}_{i,2} + \gamma_1 e_{i,2} + \gamma_1 \bar{f}_{i,1}. \tag{14}$$

Choose the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{N} z_{i,2}^2. \tag{15}$$

By combining (12) and (14), we have

$$\dot{V}_2 \leq -a_{1,1}\sum_{i=1}^{N} z_{i,1}^2 + \sum_{i=1}^{N} z_{i,1}z_{i,2} + \sum_{i=1}^{N} z_{i,2}\left(e_{i,3} + \bar{f}_{i,2} + \gamma_1 e_{i,2} + \gamma_1 \bar{f}_{i,1}\right). \tag{16}$$

With Lemma 2.5, we could obtain that

$$z_{i,1}z_{i,2} \leq \frac{1}{2\iota_{2,1}} z_{i,1}^2 + \frac{\iota_{2,1}}{2} z_{i,2}^2 \tag{17}$$

where $\iota_{2,1}$ is a positive constant.

According to Assumption 1, we can obtain that

$$\begin{aligned} z_{i,2}\bar{f}_{i,2} &\leq \rho_2 |z_{i,2}| \left(|e_{i,1}| + |e_{i,2}|\right) \\ &= \rho_2 |z_{i,2}| \left(|z_{i,1}| + |z_{i,2} - \gamma_1 z_{i,1}|\right) \\ &\leq \rho_2 |z_{i,1}| |z_{i,2}| + \rho_2 |z_{i,2}| |z_{i,2}| + \rho_2 \gamma_1 |z_{i,1}| |z_{i,2}| \\ &\leq \left(\frac{1}{2\iota_{2,2}} + \frac{1}{2\iota_{2,3}}\right) z_{i,1}^2 + \left(\frac{\iota_{2,2}\rho_2^2}{2} + \rho_2 + \frac{\iota_{2,3}\rho_2^2\gamma_1^2}{2}\right) z_{i,2}^2 \end{aligned} \tag{18}$$

where $\iota_{2,2}, \iota_{2,3}$ are positive constants.

From (13), one has

$$\begin{aligned} z_{i,2}\gamma_1 e_{i,2} &= \gamma_1 z_{i,2}\left(z_{i,2} - \gamma_1 z_{i,1}\right) \\ &= \gamma_1 z_{i,2}^2 - \gamma_1^2 z_{i,1} z_{i,2} \\ &\leq \frac{1}{2\iota_{2,4}} z_{i,1}^2 + \left(\gamma_1 + \frac{\iota_{2,4}\gamma_1^4}{2}\right) z_{i,2}^2 \end{aligned} \tag{19}$$

and

$$\begin{aligned} z_{i,2}\gamma_1 \bar{f}_{i,1} &\leq \gamma_1 \rho_1 |z_{i,1}| |z_{i,2}| \\ &\leq \frac{1}{2\iota_{2,5}} z_{i,1}^2 + \frac{\iota_{2,5}\gamma_1^2\rho_1^2}{2} z_{i,2}^2 \end{aligned} \tag{20}$$

where $\iota_{2,4}, \iota_{2,5}$ are positive constants.

From (16) – (20), we have

$$\begin{aligned}\dot{V}_2 \leq & -\left(a_{1,1} - \frac{1}{2\iota_{2,1}} - \frac{1}{2\iota_{2,2}} - \frac{1}{2\iota_{2,3}} - \frac{1}{2\iota_{2,4}} - \frac{1}{2\iota_{2,5}}\right)\sum_{i=1}^{N} z_{i,1}^2 \\ & + \left(\frac{\iota_{2,1}}{2} + \frac{\iota_{2,2}\rho_2^2}{2} + \rho_2 + \frac{\iota_{2,3}\rho_2^2\gamma_1^2}{2} + \gamma_1 + \frac{\iota_{2,4}\gamma_1^4}{2} + \frac{\iota_{2,5}\gamma_1^2\rho_1^2}{2}\right)\sum_{i=1}^{N} z_{i,2}^2 + \sum_{i=1}^{N} z_{i,2}e_{i,3} \\ = & -a_{1,2}\sum_{i=1}^{N} z_{i,1}^2 + b_2\sum_{i=1}^{N} z_{i,2}^2 + \sum_{i=1}^{N} z_{i,2}e_{i,3}^* + \sum_{i=1}^{N} z_{i,2}(e_{i,3} - e_{i,3}^*)\end{aligned} \tag{21}$$

where $e_{i,3}^*$ is a virtual signal, and

$$\begin{aligned}a_{1,2} &= a_{1,1} - \frac{1}{2\iota_{2,1}} - \frac{1}{2\iota_{2,2}} - \frac{1}{2\iota_{2,3}} - \frac{1}{2\iota_{2,4}} - \frac{1}{2\iota_{2,5}} \\ b_2 &= \frac{\iota_{2,1}}{2} + \frac{\iota_{2,2}\rho_2^2}{2} + \rho_2 + \frac{\iota_{2,3}\rho_2^2\gamma_1^2}{2} + \gamma_1 + \frac{\iota_{2,4}\gamma_1^4}{2} + \frac{\iota_{2,5}\gamma_1^2\rho_1^2}{2}.\end{aligned} \tag{22}$$

We can choose the appropriate values of $\iota_{2,1}, \iota_{2,2}, \ldots, \iota_{2,5}$ to make $a_{1,2} > 0$. Note that ρ_1, ρ_2 and γ_1 are known, we can get the value of b_2 by (22).

Letting $e_{i,3}^* = -\gamma_2 z_{i,2}$ and choose $\gamma_2 > b_2$, we have

$$\begin{aligned}\dot{V}_2 &\leq -a_{1,2}\sum_{i=1}^{N} z_{i,1}^2 - (\gamma_2 - b_2)\sum_{i=1}^{N} z_{i,2}^2 + \sum_{i=1}^{N} z_{i,2}\left(e_{i,3} - e_{i,3}^*\right) \\ &= -a_{1,2}\sum_{i=1}^{N} z_{i,1}^2 - a_{2,2}\sum_{i=1}^{N} z_{i,2}^2 + \sum_{i=1}^{N} z_{i,2}\left(e_{i,3} - e_{i,3}^*\right)\end{aligned} \tag{23}$$

where $a_{2,2} = \gamma_2 - b_2$ is a positive constant.

Step m $(m = 3, \ldots, n-1)$**:** Assume that there are virtual signals

$$\begin{aligned}e_{i,m}^* &= -\gamma_{m-1}z_{i,m-1} \\ z_{i,m-1} &= e_{i,m-1} - e_{i,m-1}^* \\ \gamma_{m-1} &> b_{m-1}\end{aligned} \tag{24}$$

such that the Lyapunov function constructed in step $m-1$ satisfies

$$\dot{V}_{m-1} \leq -\sum_{p=1}^{m-1}\left(a_{p,m-1}\sum_{i=1}^{N} z_{i,p}^2\right) + \sum_{i=1}^{N} z_{i,m-1}\left(e_{i,m} - e_{i,m}^*\right) \tag{25}$$

where b_{m-1} is the coefficient of $\sum_{i=1}^{N} z_{i,m-1}^2$ in the step $m-1$. The specific expression will be given in the following derivation.

Remark 3.1. In the steps 1 and 2, we have proved that the derivatives of the constructed Lyapunov functions satisfy the inequality constraint (25). Supposed that (23) and (25) are true, we just need to prove that the step m is also valid. Through mathematical induction, it can be obtained that the virtual control laws can be designed as the form of (23) for all $n-1$ order dynamics of the system.

Letting $z_{i,m} = e_{i,m} - e^*_{i,m}$, we have

$$\begin{aligned}
\dot{z}_{i,m} &= \dot{e}_{i,m} - \dot{e}^*_{i,m} \\
&= e_{i,m+1} + \bar{f}_{i,m} + \gamma_{m-1}\dot{z}_{i,m-1} \\
&= e_{i,m+1} + \bar{f}_{i,m} + \gamma_{m-1}(\dot{e}_{i,m-1} - e^*_{i,m-1}) \\
&\vdots \\
&= e_{i,m+1} + \bar{f}_{i,m} + \gamma_{m-1}\dot{e}_{i,m-1} + \gamma_{m-1}\gamma_{m-2}\dot{e}_{i,m-2} + \cdots + \prod_{\varepsilon=1}^{m-1}\gamma_\varepsilon \dot{e}_{i,1} \\
&= e_{i,m+1} + \bar{f}_{i,m} + \gamma_{m-1}(e_{i,m} + \bar{f}_{i,m-1}) + \gamma_{m-1}\gamma_{m-2}(e_{i,m-1} + \bar{f}_{i,m-2}) \\
&\quad + \cdots + \prod_{\varepsilon=1}^{m-1}\gamma_\varepsilon(e_{i,2} + \bar{f}_{i,1}) \\
&= e_{i,m+1} + \bar{f}_{i,m} + \sum_{p=1}^{m-1}\prod_{\varepsilon=p}^{m-1}\gamma_\varepsilon(e_{i,p+1} + \bar{f}_{i,p}).
\end{aligned} \tag{26}$$

Consider the following Lyapunov function candidate

$$V_m = V_{m-1} + \frac{1}{2}\sum_{i=1}^{N} z_{i,m}^2. \tag{27}$$

From (25) and (26), we have

$$\begin{aligned}
\dot{V}_m \leq &-\sum_{p=1}^{m-1}\left(a_{p,m-1}\sum_{i=1}^{N} z_{i,p}^2\right) + \sum_{i=1}^{N} z_{i,m-1}z_{i,m} \\
&+ \sum_{i=1}^{N} z_{i,m}\sum_{p=1}^{m-1}\prod_{\varepsilon=p}^{m-1}\gamma_\varepsilon\left(e_{i,p+1} + \bar{f}_{i,p}\right) + \sum_{i=1}^{N} z_{i,m}\left(e_{i,m+1} + \bar{f}_{i,m}\right).
\end{aligned} \tag{28}$$

Based on Lemma 2.5, we have

$$z_{i,m-1}z_{i,m} \leq \frac{1}{2\iota_{m,1}} z_{i,m-1}^2 + \frac{\iota_{m,1}}{2} z_{i,m}^2. \tag{29}$$

where $\iota_{m,1}$ ia a positive constant.

According to Assumption 1, we have

$$
\begin{aligned}
z_{i,m}\bar{f}_{i,m} &\le \rho_m |z_{i,m}| \sum_{p=1}^{m} |e_{i,p}| \\
&= \rho_m |z_{i,m}| \sum_{p=1}^{m} |z_{i,p} - \gamma_{p-1} z_{i,p-1}| \\
&\le \sum_{p=1}^{m} \rho_m |z_{i,m}| |z_{i,p}| + \sum_{p=1}^{m-1} \rho_m \gamma_p |z_{i,m}| |z_{i,p}| \\
&= \rho_m |z_{i,m}| |z_{i,1}| + \cdots + \rho_m |z_{i,m}| |z_{i,m-1}| + \rho_m |z_{i,m}| |z_{i,m}| \\
&\quad + \rho_m \gamma_1 |z_{i,m}| |z_{i,1}| + \cdots + \rho_m \gamma_{m-1} |z_{i,m}| |z_{i,m-1}| \\
&\le \frac{1}{2\iota_{m,2}} z_{i,1}^2 + \frac{\iota_{m,2}\rho_m^2}{2} z_{i,m}^2 + \cdots + \frac{1}{2\iota_{m,m}} z_{i,m-1}^2 + \frac{\iota_{m,m}\rho_m^2}{2} z_{i,m}^2 \\
&\quad + \rho_m z_{i,m}^2 + \frac{1}{2\iota_{m,m+1}} z_{i,1}^2 + \frac{\iota_{m,m+1}\rho_m^2\gamma_1^2}{2} z_{i,m}^2 \\
&\quad + \cdots + \frac{1}{2\iota_{m,2m-1}} z_{i,1}^2 + \frac{\iota_{m,2m-1}\rho_m^2\gamma_{m-1}^2}{2} z_{i,m}^2 \\
&= \sum_{p=1}^{m-1} \left(\frac{1}{2\iota_{m,p+1}} + \frac{1}{2\iota_{m,p+m}} \right) z_{i,p}^2 + g_{m,1} z_{i,m}^2
\end{aligned} \tag{30}
$$

where $\iota_{m,p+1}$,$\iota_{m,p+m}$ $(p = 1, \ldots, m-1)$ are positive constants, and

$$
g_{m,1} = \sum_{p=1}^{m-1} \left(\frac{\iota_{m,p+1}\rho_m^2}{2} + \frac{\iota_{m,p+m}\rho_m^2\gamma_p^2}{2} \right) + \rho_m. \tag{31}
$$

In view of $z_{i,m} = e_{i,m} - e_{i,m}^*$, one has

$$
\begin{aligned}
& z_{i,m} \sum_{p=1}^{m-1} \prod_{\varepsilon=p}^{m-1} \gamma_\varepsilon e_{i,p+1} \\
& = z_{i,m} \sum_{p=1}^{m-1} \prod_{\varepsilon=p}^{m-1} \gamma_\varepsilon \left(z_{i,p+1} - \gamma_p z_{i,p} \right) \\
& \le \gamma_1 \prod_{\varepsilon=1}^{m-1} \gamma_\varepsilon |z_{i,1}| |z_{i,m}| + \cdots + \gamma_{m-1}\gamma_{m-1} |z_{i,m-1}| |z_{i,m}| \\
& \quad + \prod_{\varepsilon=1}^{m-1} \gamma_\varepsilon |z_{i,2}| |z_{i,m}| + \cdots + \gamma_{m-2}\gamma_{m-1} |z_{i,m-1}| |z_{i,m}| + \gamma_{m-1} |z_{i,m}| |z_{i,m}|
\end{aligned}
$$

$$\begin{aligned}
&\le \frac{1}{2\iota_{m,2m}} z_{i,1}^2 + \frac{\iota_{m,2m}\gamma_1^2\left(\prod\limits_{\varepsilon=1}^{m-1}\gamma_\varepsilon\right)^2}{2} z_{i,m}^2 + \cdots + \frac{1}{2\iota_{m,3m-2}} z_{i,m-1}^2 \\
&\quad + \frac{\iota_{m,3m-2}\gamma_{m-1}^2\gamma_{m-1}^2}{2} z_{i,m}^2 + \frac{1}{2\iota_{m,3m-1}} z_{i,2}^2 + \frac{\iota_{m,3m-1}\left(\prod\limits_{\varepsilon=1}^{m-1}\gamma_\varepsilon\right)^2}{2} z_{i,m}^2 \qquad (32)\\
&\quad + \cdots + \frac{1}{2\iota_{m,4m-4}} z_{i,m-1}^2 + \frac{\iota_{m,4m-4}\left(\gamma_{m-2}\gamma_{m-1}\right)^2}{2} z_{i,m}^2 + \gamma_{m-1} z_{i,m}^2 \\
&= \frac{1}{2\iota_{m,2m}} z_{i,1}^2 + \sum_{p=2}^{m-1}\left(\frac{1}{2\iota_{m,2m-1+p}} + \frac{1}{2\iota_{m,3m-3+p}}\right) z_{i,p}^2 + g_{m,2} z_{i,m}^2
\end{aligned}$$

where $\iota_{m,2m}$, $\iota_{m,2m-1+p}$,$\iota_{m,3m-3+p}$ $(p = 2, \ldots, m-1)$ are positive constants, and

$$g_{m,2} = \sum_{p=1}^{m-1} \frac{\iota_{m,2m-1+p}\gamma_1^2\left(\prod\limits_{\varepsilon=p}^{m-1}\gamma_\varepsilon\right)^2}{2} + \sum_{p=2}^{m-1} \frac{\iota_{m,3m-3+p}\left(\prod\limits_{\varepsilon=p}^{m-1}\gamma_\varepsilon\right)^2}{2} + \gamma_{m-1}. \qquad (33)$$

We can also get that

$$\begin{aligned}
&z_{i,m}\sum_{p=1}^{m-1}\prod_{\varepsilon=p}^{m-1}\gamma_\varepsilon \bar{f}_{i,p} \\
&\le \sum_{p=1}^{m-1}\prod_{\varepsilon=p}^{m-1}\gamma_\varepsilon\rho_p\left(|e_{i,1}| + \cdots + |e_{i,p}|\right)|z_{i,m}| \\
&= \sum_{p=1}^{m-1}\sum_{q=p}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q\,|e_{i,p}|\,|z_{i,m}| \\
&\le \sum_{q=1}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q\,|z_{i,1}|\,|z_{i,m}| + \sum_{p=2}^{m-1}\sum_{q=p}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q\left(|z_{i,p}| + \gamma_{p-1}|z_{i,p-1}|\right)|z_{i,m}| \qquad (34)\\
&= \sum_{p=1}^{m-1}\sum_{q=p}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q\,|z_{i,p}|\,|z_{i,m}| + \sum_{p=1}^{m-2}\sum_{q=p+1}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q\gamma_p\,|z_{i,p}|\,|z_{i,m}| \\
&= \sum_{p=1}^{m-2}\left(\sum_{q=p}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q + \sum_{q=p+1}^{m-1}\prod_{\varepsilon=q}^{m-1}\gamma_\varepsilon\rho_q\gamma_p\right)|z_{i,p}|\,|z_{i,m}| + \gamma_{m-1}\rho_{m-1}\,|z_{i,m-1}|\,|z_{i,m}| \\
&\le \sum_{p=1}^{m-2}\left(\frac{1}{2\iota_{m,4m-4+p}} + \frac{1}{2\iota_{m,5m-6+p}}\right) z_{i,p}^2 + \frac{1}{2\iota_{m,6m-7}} z_{i,m-1}^2 + g_{m,3} z_{i,m}^2
\end{aligned}$$

where $\iota_{m,4m-4+p}$,$\iota_{m,5m-6+p}$ $(p=1,\ldots,m-2)$, $\iota_{m,6m-7}$ are positive constants, and

$$g_{m,3}=\sum_{p=1}^{m-2}\left(\frac{\iota_{m,4m-4+p}\left(\sum\limits_{q=p}^{m-1}\prod\limits_{\varepsilon=q}^{m-1}\gamma_{\varepsilon}\rho_{q}\right)^{2}}{2}+\frac{\iota_{m,5m-6+p}\left(\sum\limits_{q=p+1}^{m-1}\prod\limits_{\varepsilon=q}^{m-1}\gamma_{\varepsilon}\rho_{q}\gamma_{p}\right)^{2}}{2}\right)+\frac{\iota_{m,6m-7}\big(\gamma_{m-1}\rho_{m-1}\big)^{2}}{2}. \tag{35}$$

From (27) − (35), we have

$$\begin{aligned}\dot{V}_m\le&-\sum_{p=1}^{m-1}\left(a_{p,m-1}\sum_{i=1}^{N}z_{i,p}^2\right)+\frac{1}{2\iota_{m,1}}\sum_{i=1}^{N}z_{i,m-1}^2\\&+\sum_{p=1}^{m-1}\left(\left(\frac{1}{2\iota_{m,p+1}}+\frac{1}{2\iota_{m,p+m}}\right)\sum_{i=1}^{N}z_{i,p}^2\right)+\frac{1}{2\iota_{m,2m}}\sum_{i=1}^{N}z_{i,1}^2\\&+\sum_{p=2}^{m-1}\left(\left(\frac{1}{2\iota_{m,2m-1+p}}+\frac{1}{2\iota_{m,3m-3+p}}\right)\sum_{i=1}^{N}z_{i,p}^2\right)\\&+\sum_{p=1}^{m-2}\left(\left(\frac{1}{2\iota_{m,4m-4+p}}+\frac{1}{2\iota_{m,5m-6+p}}\right)\sum_{i=1}^{N}z_{i,p}^2\right)+\frac{1}{2\iota_{m,6m-7}}z_{i,m-1}^2\\&+\left(\frac{\iota_{m,1}}{2}+g_{m,1}+g_{m,2}+g_{m,3}\right)\sum_{i=1}^{N}z_{i,m}^2\\=&-\left(a_{1,m-1}-\frac{1}{2\iota_{m,2}}-\frac{1}{2\iota_{m,m+1}}-\frac{1}{2\iota_{m,2m}}-\frac{1}{2\iota_{m,4m-3}}-\frac{1}{2\iota_{m,5m-5}}\right)\sum_{i=1}^{N}z_{i,1}^2\\&-\sum_{p=2}^{m-2}\left(\left(a_{p,m-1}-\frac{1}{2\iota_{m,p+1}}-\frac{1}{2\iota_{m,p+m}}-\frac{1}{2\iota_{m,2m-1+p}}\right.\right.\\&\left.\left.-\frac{1}{2\iota_{m,3m-3+p}}-\frac{1}{2\iota_{m,4m-4+p}}-\frac{1}{2\iota_{m,5m-6+p}}\right)\sum_{i=1}^{N}z_{i,p}^2\right)\\&-\left(a_{m-1,m-1}-\frac{1}{2\iota_{m,1}}-\frac{1}{2\iota_{m,m}}-\frac{1}{2\iota_{m,2m-1}}\right.\\&\left.-\frac{1}{2\iota_{m,3m-2}}-\frac{1}{2\iota_{m,4m-4}}-\frac{1}{2\iota_{m,6m-7}}\right)\sum_{i=1}^{N}z_{i,m-1}^2\\&+\left(\frac{\iota_{m,1}}{2}+g_{m,1}+g_{m,2}+g_{m,3}\right)\sum_{i=1}^{N}z_{i,m}^2\\&+\sum_{i=1}^{N}z_{i,m}e_{i,m+1}^*+\sum_{i=1}^{N}z_{i,m}\left(e_{i,m+1}-e_{i,m+1}^*\right).\end{aligned} \tag{36}$$

There exist $\iota_{i,k}(k = 1, \ldots, 6m-7)$ such that

$$\begin{aligned} a_{1,m} &= a_{1,m-1} - \frac{1}{2\iota_{m,2}} - \frac{1}{2\iota_{m,m+1}} - \frac{1}{2\iota_{m,2m}} - \frac{1}{2\iota_{m,4m-3}} - \frac{1}{2\iota_{m,5m-5}} > 0 \\ a_{p,m} &= a_{p,m-1} - \frac{1}{2\iota_{m,p+1}} - \frac{1}{2\iota_{m,p+m}} - \frac{1}{2\iota_{m,2m-1+p}} \\ &\quad - \frac{1}{2\iota_{m,3m-3+p}} - \frac{1}{2\iota_{m,4m-4+p}} - \frac{1}{2\iota_{m,5m-6+p}} > 0 \\ a_{m-1,m} &= a_{m-1,m-1} - \frac{1}{2\iota_{m,1}} - \frac{1}{2\iota_{m,m}} - \frac{1}{2\iota_{m,2m-1}} \\ &\quad - \frac{1}{2\iota_{m,3m-2}} - \frac{1}{2\iota_{m,4m-4}} - \frac{1}{2\iota_{m,6m-7}} > 0. \end{aligned} \tag{37}$$

When all values of $\iota_{i,k}(k = 1, \ldots, 6m-7)$ are selected, the value of b_m can be determined with the known $\rho_k(k = 1, \ldots, m)$ and $\gamma_k(k = 1, \ldots, m-1)$, where

$$b_m = \frac{\iota_{m,1}}{2} + g_{m,1} + g_{m,2} + g_{m,3}. \tag{38}$$

Then we construct the virtual signal as

$$\begin{aligned} e^*_{i,m+1} &= -\gamma_m z_{i,m} \\ z_{i,m} &= e_{i,m} - e^*_{i,m} \\ \gamma_m &> b_m. \end{aligned} \tag{39}$$

Substituting (37)−(39) into (36), we can get

$$\begin{aligned} \dot{V}_m &\le -\sum_{p=1}^{m-1}\left(a_{p,m}\sum_{i=1}^{N} z_{i,p}^2\right) - (\gamma_m - b_m)\sum_{i=1}^{N} z_{i,m}^2 + \sum_{i=1}^{N} z_{i,m}\left(e_{i,m+1} - e^*_{i,m+1}\right) \\ &\le -\sum_{p=1}^{m}\left(a_{p,m}\sum_{i=1}^{N} z_{i,p}^2\right) + \sum_{i=1}^{N} z_{i,m}\left(e_{i,m+1} - e^*_{i,m+1}\right). \end{aligned} \tag{40}$$

Step n: Choose the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2}\sum_{i=1}^{N} z_{i,n}^2 \tag{41}$$

where $z_{i,n} = e_{i,n} - e^*_{i,n} = e_{i,n} + \gamma_{n-1} z_{i,n-1}$.

From step $n-1$, we can see that $\dot{V}_{n-1}$ satisfies (40), then the time derivative of V_n is given by

$$\begin{aligned} \dot{V}_n &\le -\sum_{p=1}^{n-1}\left(a_{p,n-1}\sum_{i=1}^{N} z_{i,p}^2\right) + \sum_{i=1}^{N} z_{i,n-1} z_{i,n} \\ &\quad + \sum_{i=1}^{N} z_{i,n} \sum_{p=1}^{n-1}\prod_{\varepsilon=p}^{n-1}\gamma_\varepsilon\left(e_{i,p+1} + \bar{f}_{i,p}\right) + \sum_{i=1}^{N} z_{i,n}\left(u_i + \bar{f}_{i,n}\right). \end{aligned} \tag{42}$$

Similar to the derivation of (27) − (40), we can get

$$\dot{V}_n \leq -\sum_{p=1}^{n-1}\left(a_{p,n}\sum_{i=1}^{N} z_{i,p}^2\right) + b_n\sum_{i=1}^{N} z_{i,n}^2 + \sum_{i=1}^{N} z_{i,n}u_i \tag{43}$$

where $a_{p,n}(p = 1,\ldots,n-1), b_n$ are positive constants.

Construct the virtual control law as

$$\begin{aligned} u_i{}^* &= -\gamma_n z_{i,n} \\ \gamma_n &> b_n \end{aligned} \tag{44}$$

it can be note that

$$u_i{}^* = -\gamma_n z_{i,n} = -\sum_{p=1}^{n}\prod_{\varepsilon=p}^{n}\gamma_\varepsilon e_{i,p} = -\sum_{p=1}^{n}\prod_{\varepsilon=p}^{n}\gamma_\varepsilon\left(x_{i,p} - x_{0,p}\right). \tag{45}$$

For each follower agent, its control input will depend on the states of the leader agent if the input is given as the virtual control law $u_i{}^*$. However, in the MASs, only some follower agents ($b_i > 0$) can obtain the states of the leader agent.

To solve this problem, we design a new variable for each agent as follows

$$\begin{aligned} q_i &= \sum_{j=1}^{N} a_{i,j}(t)\left(z_{i,n} - z_{j,n}\right) + b_i(t)z_{i,n} \\ &= \sum_{j=1}^{N} a_{i,j}(t)\left(e_{i,n} + \sum_{p=1}^{n-1}\prod_{\varepsilon=p}^{n-1}\gamma_\varepsilon e_{i,p} - e_{j,n} - \sum_{p=1}^{n-1}\prod_{\varepsilon=p}^{n-1}\gamma_\varepsilon e_{j,p}\right) \\ &\quad + b_i(t)\left(e_{i,n} + \sum_{p=1}^{n-1}\prod_{\varepsilon=p}^{n-1}\gamma_\varepsilon e_{i,p}\right) \\ &= \sum_{j=1}^{N} a_{i,j}(t)\left(x_{i,n} - x_{j,n} + \sum_{p=1}^{n-1}\prod_{\varepsilon=p}^{n-1}\gamma_\varepsilon\left(x_{i,p} - x_{j,p}\right)\right) \\ &\quad + b_i(t)\left(x_{i,n} - x_{0,n} + \sum_{p=1}^{n-1}\prod_{\varepsilon=p}^{n-1}\gamma_\varepsilon\left(x_{i,p} - x_{0,p}\right)\right). \end{aligned} \tag{46}$$

By introducing variable q_i, the input and adaptive law are designed as

$$u_i = c_1\left(\sum_{j=1}^{N} a_{ij}(t)\left(\hat{\theta}_j q_j - \hat{\theta}_i q_i\right) - b_i(t)\hat{\theta}_i q_i\right) \tag{47}$$

$$\dot{\hat{\theta}}_i = c_1 q_i^2 \tag{48}$$

where $c_1 > 1$ is a constant, $\hat{\theta}_i$ is the estimation of θ for the ith agent and θ will be given later.

Choose the Lyapunov function as

$$V = V_n + \frac{1}{2}\sum_{i=1}^{N}\left(\theta - \hat{\theta}_i\right)^2 \tag{49}$$

where $\theta = \frac{a_{n,n}+b_n}{\min\left\{\lambda_{\min^2}(H_q)|q\in\underline{S}\right\}}$,, $a_{n,n}$ is a positive constant. According to Lemma 2.4,we can obtain $\lambda_{\min}(H_k) > 0$.

From (43) and (49), we have

$$\dot{V} \le -\sum_{p=1}^{n-1}\left(a_{p,n}\sum_{i=1}^{N} z_{i,p}^2\right) + b_n\sum_{i=1}^{N} z_{i,n}^2 + \sum_{i=1}^{N} z_{i,n}u_i - \theta\sum_{i=1}^{N}\dot{\hat{\theta}}_i + \sum_{i=1}^{N}\hat{\theta}_i\dot{\hat{\theta}}_i. \tag{50}$$

From (47), we can get that

$$\begin{aligned}
-\theta\sum_{i=1}^{N}\dot{\hat{\theta}}_i &= -\theta c_1\sum_{i=1}^{N} q_i^2 \\
&= -\theta c_1 q^T q \\
&= -\theta c_1 (H(t)z_n)^T\,(H(t)z_n) \\
&= -\frac{a_{n,n}+b_n}{\min\{\lambda_{\min}^2(H_k)|k\in S\}}c_1 z_n^T H^2(t) z_n \\
&\le -\left(a_{n,n}+b_n\right) z_n^T z_n \\
&= -\left(a_{n,n}+b_n\right)\sum_{i=1}^{N} z_{i,n}^2
\end{aligned} \tag{51}$$

where $q = [q_1, \ldots, q_N]^T, z_n = [z_{1,n}, \ldots, z_{N,n}]^T$ and $q = H(t)z_n$ are used.

Note that

$$\sum_{i=1}^{N} z_{i,n}u_i = z_n^T u = -c_1 z_n^T H(t)\bar{q} = -c_1 q^T\bar{q} = -c_1\sum_{i=1}^{N}\hat{\theta}_i q_i^2 = -\sum_{i=1}^{N}\hat{\theta}_i\dot{\hat{\theta}}_i \tag{52}$$

where $u = [u_1, \ldots, u_N]^T, \bar{q} = [\hat{\theta}_1 q_1, \ldots, \hat{\theta}_N q_N]^T$.

Substituting (50)−(52) into (49), we can obtain

$$\begin{aligned}
\dot{V} &\le -\sum_{p=1}^{n-1}\left(a_{p,n}\sum_{i=1}^{N} z_{i,p}^2\right) + b_n\sum_{i=1}^{N} z_{i,n}^2 - \sum_{i=1}^{N}\hat{\theta}_i\dot{\hat{\theta}}_i - \left(a_{n,n}+b_n\right)\sum_{i=1}^{N} z_{i,n}^2 + \sum_{i=1}^{N}\hat{\theta}_i\dot{\hat{\theta}}_i \\
&\le -a_{\min}\sum_{i=1}^{N}\sum_{p=1}^{n} z_{i,p}^2
\end{aligned} \tag{53}$$

where $a_{\min} = \min\{a_{p,n}|p = 1, \ldots, n\}$.

Remark 3.2. Through the state information and communication weight of neighbor agents, each agent calculates the variable q_i and obtains the adaptive variable $\hat{\theta}_i$. Then, agent i shares these parameters with neighbor agents through communication and calculates its own control input u_i based on $\hat{\theta}_i$, $\hat{\theta}_j$, q_i, q_j, communication weight, and a given constant c_1. Note that all the above parameters are local information that can be obtained for each agent, so the designed control protocol is fully distributed.

3.2. Consensus analysis

Theorem 3.3. Consider the MAS (1)–(2) under Assumptions 1 and 2. The proposed input (47) with the adaptive law (48) and the virtual signal (39) ensures that the global asymptotic consensus can be achieved, and the the auxiliary dynamical variable $\hat{\theta}_i, i = \{1, \ldots, N\}$ is bounded.

Proof. The inequality (53) indicates that V is bounded and so is each $\hat{\theta}_i$. Note that $\dot{\hat{\theta}}_i \geq 0$, it can be seen from (48) that each $\hat{\theta}_i$ is monotonically increasing and eventually approaches a fixed value.

By directing integration of (53) at the interval $[0, t]$ yields

$$V(t) - V(0) \leq -a_{\min} \int_0^t \sum_{i=1}^{N} \sum_{p=1}^{n} z_{i,p}^2 \, \mathrm{d}\tau. \tag{54}$$

It can be concluded that $\int_0^{+\infty} z_{i,p}^2 \, \mathrm{d}\tau$ is bounded. Then we can prove that the derivations of $z_{i,p}$ is also bounded. According to the Barbalat's Lemma, it yields that $z_{i,p}$ is asymptotically converge to 0.

It is note that

$$\begin{aligned}
\lim_{t\to\infty} z_{i,1} &= \lim_{t\to\infty} e_{i,1} = \lim_{t\to\infty} (x_{i,1} - x_{0,1}) = 0 \\
\lim_{t\to\infty} z_{i,2} &= \lim_{t\to\infty} \left(e_{i,2} - e_{i,2}^*\right) = \lim_{t\to\infty} (e_{i,2} + \gamma_1 e_{i,1}) \\
&= \lim_{t\to\infty} (x_{i,2} - x_{0,2} + \gamma_1 (x_{i,1} - x_{0,1})) = 0 \\
&\ \vdots \\
\lim_{t\to\infty} z_{i,n} &= \lim_{t\to\infty} \left(e_{i,n} - e_{i,n}^*\right) = \lim_{t\to\infty} \left(x_{i,n} - x_{0,n} + \cdots + \prod_{\varepsilon=1}^{n-1} \gamma_\varepsilon (x_{i,1} - x_{0,1})\right) = 0
\end{aligned} \tag{55}$$

which means that $\lim_{t\to\infty} |x_{i,p} - x_{0,p}| \to 0, \ i = 1, \ldots, N, \ p = 1, \ldots, n.$ □

Remark 3.4. The constructed Lyapunov function is independent of the communication topology, so the switching of the topology will not lead to the jump of the function. Note that the Lyapunov function is non-increasing in the whole time domain, by Barbalat's Lemma, it can be seen that V asymptotically converges to 0. This shows that the MAS finally achieves consensus and the adaptive parameters are bounded. In addition, we relax the restrictions on switching signals.

3.3. Algorithm

In this subsection, we will give the parameter selection guidance and the controller implementation steps.

1. By modeling the physical structure of the agent, n (the order of the agent dynamical equation) and ρ_i (the Lipschitz constant of the nonlinear term contained in each order) are obtained. c_1 is used to adjust the overall size of the control input, as long as $c_1 > 1$ is established.

2. Select the appropriate γ_1 according to the value of ρ_1, and calculate the value of $a_{1,1}$.

3. According to $a_{p,m-1}$ $(p = 1, \ldots, m-1)$ determined in step $m-1$ and formula (37), $\iota_{m,\delta}$ $(\delta = 1, \ldots, 6m-7)$ are selected appropriately so that $a_{p,m} > 0$ $(p = 1, \ldots, m-1)$.

4. With $\gamma_1, \cdots \gamma_{m-1}$ determined by the previous $m-1$ steps and $\rho_1, \ldots, \rho_m$, the value of b_m can be computed according to (38). Then we can select appropriate γ_m.

5. Repeat steps 3 and 4 until $\gamma_1, \ldots, \gamma_{n-1}$ are determined.

6. Each agent obtains state information $\bar{x}_{j,n}$ by communicating with its neighbor agents to calculate variable q_i. Then the adaptive parameter $\hat{\theta}_i$ is updated.

7. Agent i sends q_i and $\hat{\theta}_i$ to the neighbors and receives q_j and $\hat{\theta}_j$ from them, so as to calculate its own control quantity u_i.

It can be seen that the specific implementation of the protocol proposed in this paper is divided into off-line part and online part. Steps $1-5$ are the off-line part, which is determined by the characteristics of the agent itself. When $\gamma_1, \ldots, \gamma_{n-1}$ are determined, they are stored in all agents. No matter how many agents there are and what the switching signal is, there is no need to re-select them. Step 6 and step 7 are the online part. The agent updates its own control input according to the real-time information sent by the neighbor agents. At the same time, the gain of the controller will be updated autonomously through the adaptive law, without the need to re-select according to the current topology information. As such, the proposed protocol is called a fully distributed one.

Remark 3.5. Considering that the homogeneous MAS is studied in this paper, the selection of $\gamma_1, \ldots, \gamma_{n-1}$ is the same for each agent because of $\gamma_1, \ldots, \gamma_{n-1}$ are related to Lipschitz constants. The selection of c_1 is independent of the dynamics of the agent. In theory, the control goal can be achieved as long as $c_1 > 1$ is met. In order to facilitate theoretical analysis and practical application, we choose the same value for c_1 of each agent.

4. EXAMPLES

Example 4.1. We consider an MAS consisting of five followers and a leader. Each agent is described as

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2} + \sin(x_{i,1})\\ \dot{x}_{i,2} &= x_{i,3} + 2\cos(x_{i,1} + x_{i,2})\\ \dot{x}_{i,3} &= u_i + \sin(x_{i,3})\cos(x_{i,1} + x_{i,2})\end{aligned} \tag{56}$$

where $i = 0, 1, 2, 3, 4, 5$. Agent 0 is the leader, so the input $u_0 = 0$.

The possible communication topologies and switched signal are shown in Figs. 1 and 2. The initial conditions of agents's states are set as follows:

$$\begin{aligned}[x_{0,1}, x_{0,2}, x_{0,3}] &= [3, -5, 1] & [x_{1,1}, x_{1,2}, x_{1,3}] &= [-3, -7.5, -1]\\ [x_{2,1}, x_{2,2}, x_{2,3}] &= [-1, 0.75, 0.5] & [x_{3,1}, x_{3,2}, x_{3,3}] &= [5, 1, -0.5]\\ [x_{4,1}, x_{4,2}, x_{4,3}] &= [7, -1, 1.5] & [x_{5,1}, x_{5,2}, x_{5,3}] &= [4, 2, 2]\end{aligned}$$

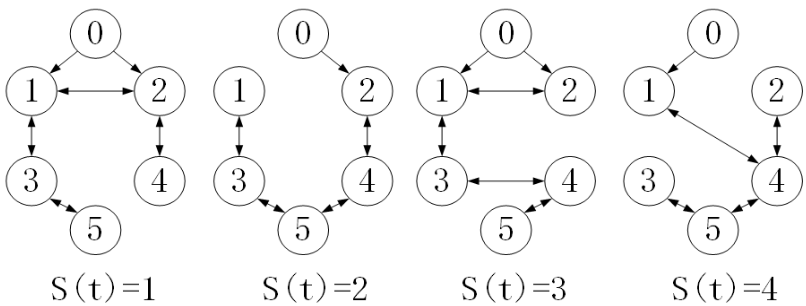

Fig. 1. Possible networks.

The initial values of dynamical variables are set as $\hat{\theta}_i(0) = 0$ $(i = 1, \dots, 5)$. The design parameters are chosen as $c_1 = 1.1, \gamma_1 = 2, \gamma_2 = 64$. The states of agents are shown in Figs. 3-5, and the dynamical variables $\hat{\theta}_i$ $(i = 1, \dots, 5)$ are shown in Fig. 6. We can see that the consensus can be reached, and $\hat{\theta}_i$, $(i = 1, \dots, 5)$, are bounded.

Example 4.2. The adaptive consensus protocol for second-order nonlinear MASs was proposed [52], but the implementation of the protocol requires that the switching signal meet certain dwell time constraints. We make the protocol proposed in this paper work on the MAS given in paper [52] for comparison. Consider a MAS consisting of five followers and one leader. The dynamics of the agent and possible topologies are the same as those in the comparative paper [52]. It can be seen from the paper that the nonlinear function in the dynamics satisfies Assumption 1, and the topology satisfies Assumption 2. The switching signal circulates as shown in Fig. 7, which does not meet the dwell time constraint required in the comparison paper. The design parameters are chosen as $c_1 = 1.1$ and $\gamma_1 = 5$. The initial values of dynamical variables are set as

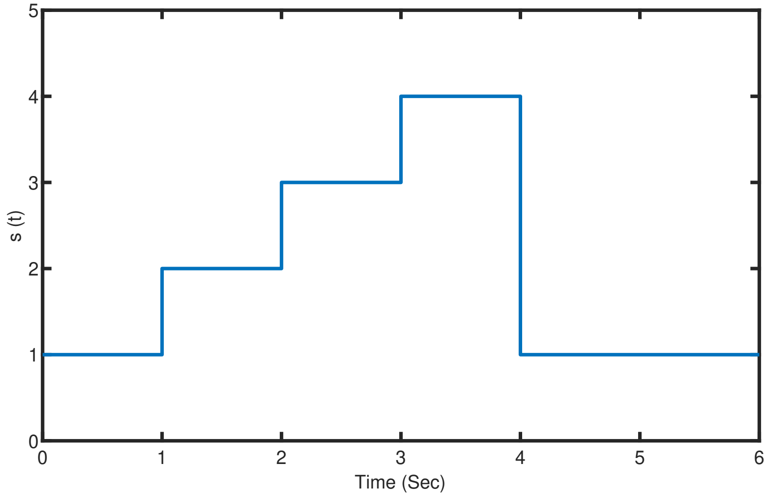

Fig. 2. Switched signal.

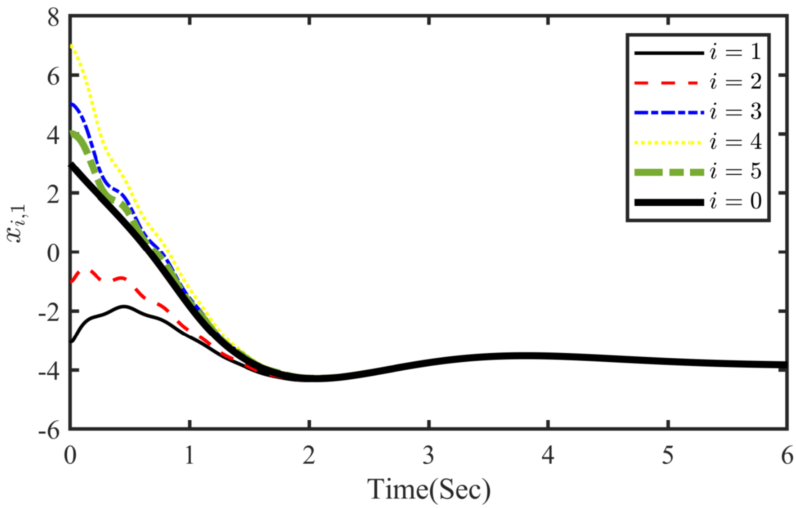

Fig. 3. $x_{i,1}, i = 0, 1, \ldots, 5$.

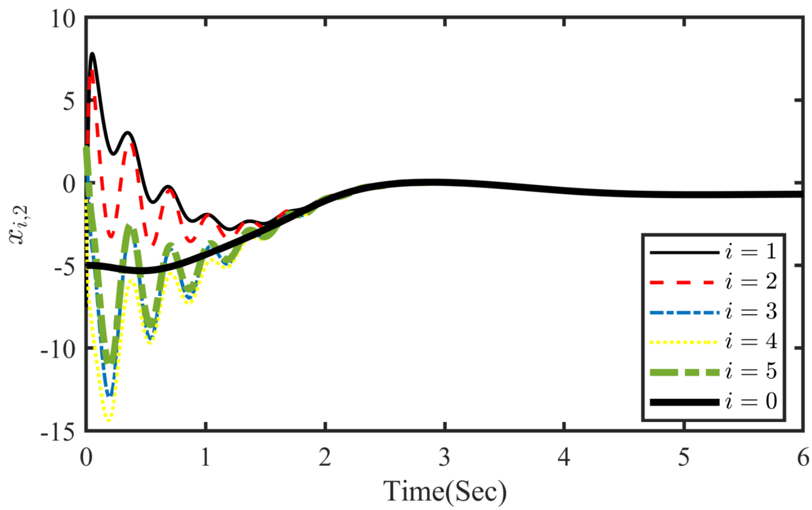

Fig. 4. $x_{i,2}, i = 0, 1, \ldots, 5.$

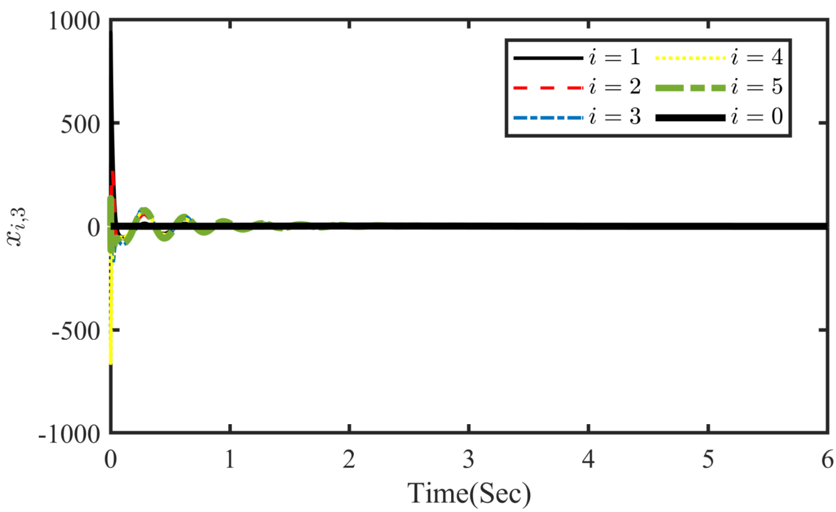

Fig. 5. $x_{i,3}, i = 0, 1, \ldots, 5.$

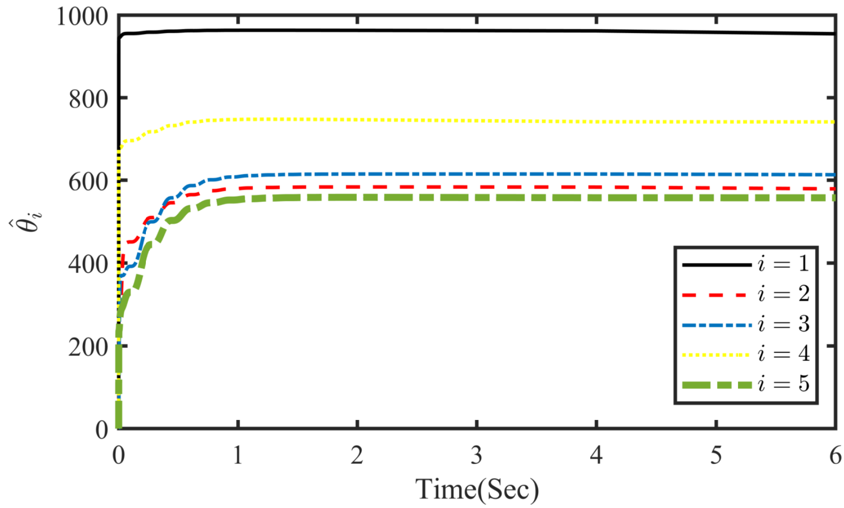

Fig. 6. $\hat{\theta}_i, i = 1, \ldots, 5$.

$\hat{\theta}_i(0) = 0$ $(i = 1, \ldots, 5)$. The parameters of the dwell time protocol are consistent with those in the comparative paper.

Fig. 8 and Fig. 9 show the velocity trajectories of the MAS under the two protocols, respectively. It can be seen that under the same conditions, the MAS achieves consensus faster under the action of the protocol proposed in this paper. We intercept the velocity trajectories of the MAS at the interval $t \in [9, 10]$ under the action of the two protocols in Fig. 10 and Fig. 11. We can see that the protocol proposed in this paper has better control accuracy.

5. CONCLUSION

In this paper, the distributed consensus problem was investigated for high-order strict-feedback nonlinear MASs. The communication topology of the considered systems can be switched at any time, and the topology information is unknown for each agent. In view of the adaptive backstepping technique, a novel adaptive protocol was proposed for MASs. The implementation of this protocol only uses the local information. By applying the Lyapunov function method, the sufficient conditions on complete consensus of the MASs were established. Finally, simulation examples were given to verify the effectiveness and advantages of the proposed protocol. Based on the work of this paper, how to realize the fully distributed complete consensus for high-order strict-feedback MASs with unknown Lipschitz constants and arbitrary switched topologies is a future research direction.

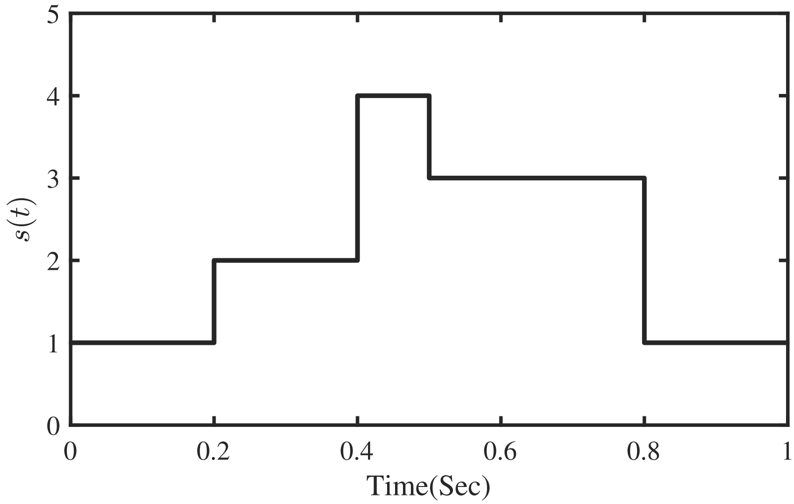

Fig. 7. Switched signal within 1 second.

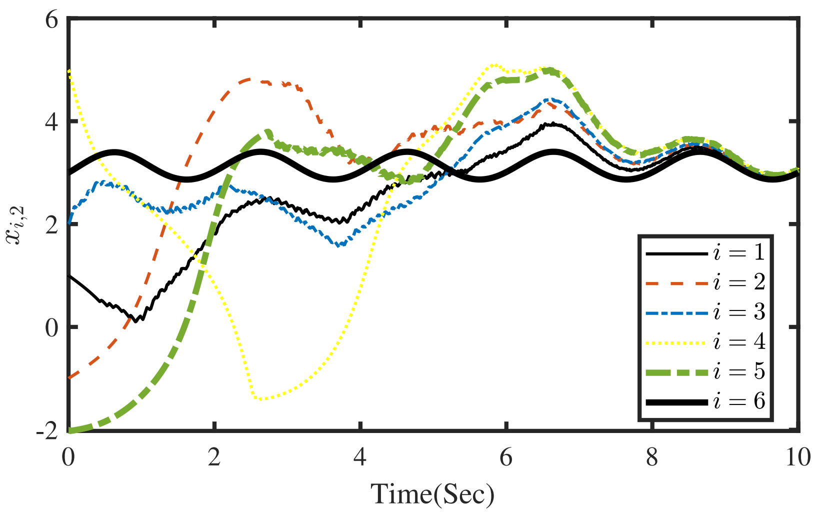

Fig. 8. Velocity trajectories under dwell time protocol.

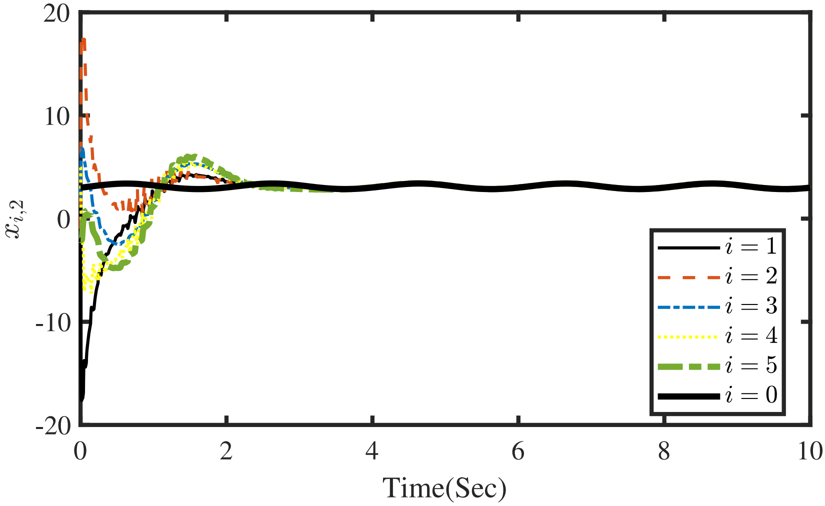

Fig. 9. Velocity trajectories under arbitrary switched protocol.

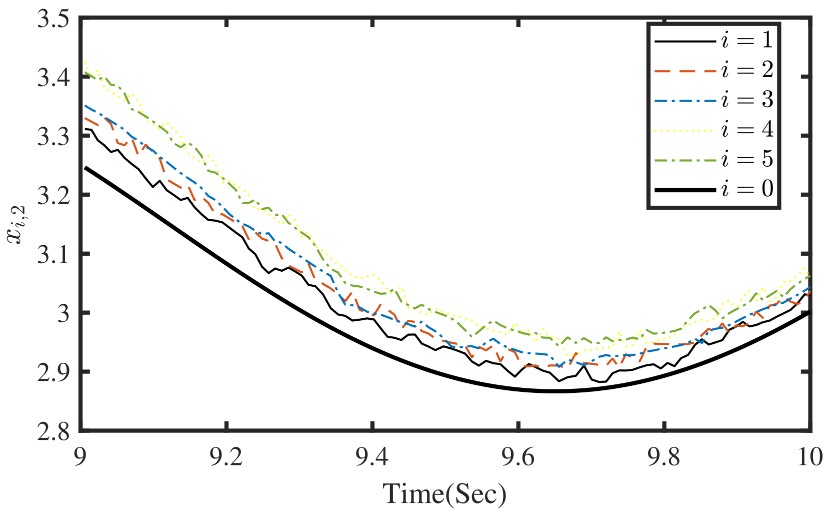

Fig. 10. Velocity trajectories under dwell time switched protocol between the 9th and 10th seconds.

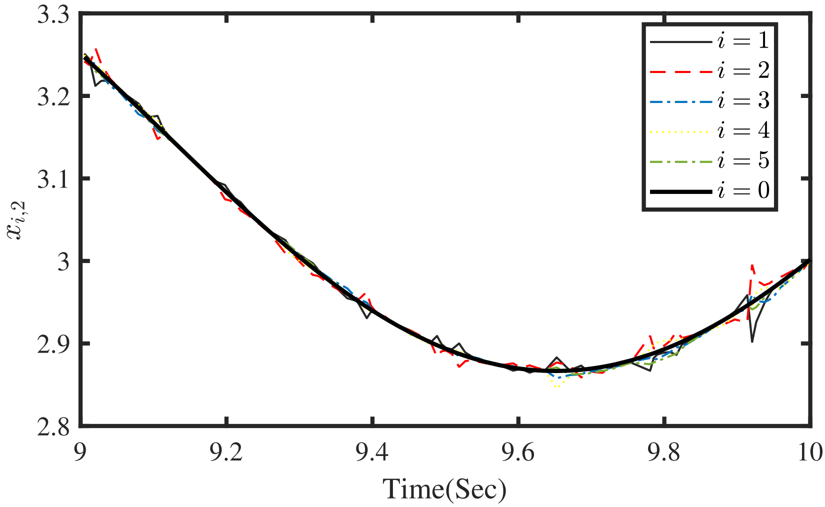

Fig. 11. Velocity trajectories under arbitrary switched protocol between the 9th and 10th seconds.

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