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φ-MULTIPLIERS ON A CLASS OF TOPOLOGICAL ALGEBRAS

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Abstract. In this paper, we generalize the concept of φ-multipliers on Banach algebras to a class of topological algebras. Then the characterizations of φ-multipliers are investigated in these algebras.

Keywords: fundamental topological algebra; infrasequential topological algebra; approximate identity; φ-multiplier; uniformly bounded

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1. Introduction

Throughout this paper, if A is a unital algebra, then e stands for its unit element. Also all topological algebras are considered Hausdorff. The concept of multipliers of a Banach algebra was introduced by Helgason (see [9]) as follows:

Let A be a semisimple Banach algebra considered as an algebra of continuous functions over its regular maximal ideal space Δ. A function g over Δ such that $gA \subset A$ is called a multiplier of A.

The general theory of multipliers have been studied extensively in Banach algebras without order by Birtal, Wang and Larsen (see [7], [17], [13]).

Husain in [12] has generalized the notion of a multiplier on a topological algebra. Multipliers have extensive applications in many areas of mathematics, such as harmonic analysis, differential equations, representation theory, optimal contorol, quantum mechanic and engineering. In view of applications, it is important to study multipliers on more generalized topological algebras. In [3] the concept of multipliers was generalized to φ-multipliers.

Let A be a Banach algebra and $\varphi\colon A \to A$ be an algebra homomorphism. A linear continuous map $T\colon A \to A$ is called a left (or right) φ-multiplier on A if

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$T(xy) = T(x)\varphi(y)$ (or $T(xy) = \varphi(x)T(y)$, respectively) for all $x, y \in A$. We say T is a φ-multiplier on A if it is both a left and a right φ-multiplier on A.

This paper is devoted to establishing some fundamentally important results about φ-multipliers on a class of topological algebras. The paper is divided into the following sections. In Section 2, we gathered a collection of definitions and known results and in Section 3 we derive some results concerning φ-multipliers on fundamental topological algebras with bounded elements. In Section 4, the characterizations of φ-multipliers are investigated in infrasequential fundamental F-algebras.

2. Definitions and known results

In this section, we present a collection of definitions and known results, which are included in our list of references.

Definition 2.1. Let A be an algebra. A map $T\colon A \to A$ is said to be a left (or right) multiplier on A if

$$T(xy) = T(x)y \quad (\text{or } T(xy) = xT(y), \text{ respectively})$$

for all $x, y \in A$. It is called a multiplier on A if T is both a left multiplier and a right multiplier.

Definition 2.2. An algebra A is said to be semiprime if $\{0\}$ is the only two-sided ideal J such that $J^2 = \{0\}$ (see [8], Definition IV.30.3). In other words, A is semiprime if and only if $aAa = \{0\}$ implies $a = 0$.

Definition 2.3. Let A be a unital algebra. The set of all invertible elements of A is denoted by $\mathrm{Inv}(A)$.

Definition 2.4. A complete metrizable topological vector space is called an F-space.

Definition 2.5. A complete metrizable topological algebra is called an F-algebra.

Definition 2.6. Let B be a topological vector space and A be a topological algebra. Then B is said to be a topological A-bimodule if it is an A-bimodule and the module multiplications are separately continuous.

Definition 2.7. A set B in a topological algebra A is said to be uniformly bounded if there exists $k > 0$ such that the set $\{(x/k)^n\colon\ x \in B, n \in \mathbb{N}\}$ is a bounded subset of A.

Definition 2.8. A net $(e_\alpha)_{\alpha\in I}$ in a topological algebra is called an approximate identity if, for all $x \in A$,

$$\lim_\alpha e_\alpha x = \lim_\alpha x e_\alpha = x;$$

$(e_\alpha)_{\alpha\in I}$ is said to be uniformly bounded if there exists $k > 0$ such that $\{(e_\alpha/k)^n\colon \alpha \in I,\ n \in \mathbb{N}\}$ is a bounded set in A.

Definition 2.9. A topological algebra A is said to be a fundamental topological algebra if there exists $b > 1$ such that for every sequence (a_n) of A the convergence of $b^n(a_n - a_{n-1})$ to zero in A implies that (a_n) is a Cauchy sequence. This class of topological algebras was introduced in [4].

Theorem 2.1 (Cohen's factorization theorem)**.** *An F-algebra with uniformly bounded approximate identity which is fundamental can be factorized, see* [4].

Definition 2.10 ([11])**.** Let A be a topological algebra.

(i) A is said to be strongly sequential if there exists a neighbourhood U of 0 such that for all $x \in U$ we have $x^n \to 0$ as $n \to \infty$.

(ii) A is said to be infrasequential if for each bounded set $B \subseteq A$ there exists $\lambda > 0$ such that for all $x \in B$ we have $(\lambda x)^n \to 0$ as $n \to \infty$.

Proposition 2.1 ([11])**.** *With reference to the above definitions,* (i) $\Rightarrow$ (ii), *but the reverse implication need not hold.*

Definition 2.11. An element a of a topological algebra A is called to be bounded if there exists $k > 0$ such that the set $\{(a/k)^n\colon\ n \in \mathbb{N}\}$ is bounded in A. If every element of A is bounded, then A is called a topological algebra with bounded elements.

Proposition 2.2 ([1])**.** *Every unital sequentially complete fundamental topological algebra with bounded elements is a topological algebra with the exponential map.*

Definition 2.12. Let A be a unital topological algebra. The exponential function in A is defined by

$$E(x) = \exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad x \in A,\ x^0 = e,$$

whenever the series on the right converges. We denote by D_E the domain of E and by R_E its range. Since $0 \in D_E$, $e \in R_E$, both D_E, R_E are nonempty.

By $E(A)$ we mean the set of all elements $x \in A$ for which $E(x)$ can be defined. For further information we refer to [6].

Definition 2.13. Let A be a unital topological algebra. The connected component of the group $\mathrm{Inv}(A)$ containing e is denoted by $\mathrm{Inv_c}(A)$; $\mathrm{Inv_c}(A)$ is called the principal component of $\mathrm{Inv}(A)$.

3. φ-MULTIPLIERS ON TOPOLOGICAL ALGEBRAS WITH BOUNDED ELEMENTS

In this section, we derive some results concerning φ-multipliers on fundamental topological algebras with bounded elements.

Definition 3.1. Let A be an algebra and B be an A-bimodule. A map $T\colon A \to B$ is said to be a left (or right) φ-multiplier if

$$T(xy) = T(x)\varphi(y) \quad (\text{or } T(xy) = \varphi(x)T(y), \text{ respectively}) \quad \text{for all } x, y \in A,$$

where $\varphi\colon A \to A$ is a homomorphism. It is called a φ-multiplier if T is both a left φ-multiplier and a right φ-multiplier.

In case A has a unity e and B is a unital A-bimodule, $T\colon A \to B$ is a left (or right) φ-multiplier if and only if T is of the form

$$T(x) = T(e)\varphi(x) \quad (\text{or } T(x) = \varphi(x)T(e), \text{ respectively}) \quad \text{for all } x \in A.$$

Also T is a φ-multiplier if and only if

$$T(x) = \varphi(x)T(e) = T(e)\varphi(x) \quad \text{for all } x \in A.$$

Example 3.1. Let A be a commutative algebra. For each $x \in A$, if we put $L_x(y) = x\varphi(y)$ for all $y \in A$, where $\varphi\colon A \to A$ is a homomorphism, then L_x is a φ-multiplier. To see this, let $y, z \in A$. Then

$$L_x(yz) = x\varphi(yz) = x(\varphi(y)\varphi(z)) = (x\varphi(y))\varphi(z) = L_x(y)\varphi(z).$$

Hence L_x is a left φ-multiplier. Similarly, L_x is a right φ-multiplier. Therefore L_x is a φ-multiplier.

Lemma 3.1. *Let A be a unital sequentially complete fundamental topological algebra with bounded elements. Then $R_E \subseteq \mathrm{Inv_c}(A)$.*

Proof. By [1] (4.1), we have $E(A) = A$. Since A is connected and E continuous, R_E is connected. Clearly $E(-x) = E(x)^{-1}$, $x \in A$. So $R_E \subseteq \mathrm{Inv}(A)$. Since $e \in R_E$ and R_E is connected, $R_E \subseteq \mathrm{Inv_c}(A)$. $\square$

Theorem 3.1. *Let A be a unital sequentially complete fundamental topological algebra with bounded elements and B be a topological vector space. If $T\colon A\times A \to B$ is a continuous bilinear map with the property that*

$$x \in \mathrm{Inv_c}(A) \Rightarrow T(x, x^{-1}) = T(e, e),$$

then

$$T(x, x) = T(x^2, e) \quad \text{and} \quad T(x, e) = T(e, x), \quad x \in A.$$

Proof. Let $x \in A$ and $\lambda \in \mathbb{C}$. Since $R_E \subseteq \mathrm{Inv_c}(A)$, we have

$$
\begin{aligned}
T(e,e) &= T(\exp(\lambda x), \exp(-\lambda x)) = T\Big(\exp(\lambda x), \sum_{i=0}^{\infty} \frac{(-1)^i \lambda^i x^i}{i!}\Big) \\
&= \sum_{i=0}^{\infty} \frac{(-1)^i \lambda^i}{i!} T(\exp(\lambda x), x^i) = \sum_{i=0}^{\infty} \frac{(-1)^i \lambda^i}{i!} T\Big(\sum_{j=0}^{\infty} \frac{(\lambda^j x^j}{j!}, x^i\Big) \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^i \lambda^{i+j}}{i!j!} T(x^j, x^i) = T(e,e) + \sum_{r=1}^{\infty} \lambda^r \Big(\sum_{i+j=r} \frac{(-1)^i}{i!j!} T(x^j, x^i)\Big).
\end{aligned}
$$

Continuity of T implies that

$$\sum_{r=1}^{\infty} \lambda^r \Big(\sum_{i+j=r} \frac{(-1)^i}{i!j!} T(x^j, x^i)\Big) = 0 \quad \text{for any } \lambda \in \mathbb{C}.$$

Consequently,

$$\sum_{i+j=r} \frac{(-1)^i}{i!j!} T(x^j, x^i) = 0 \quad \text{for all } x \in A,\ r \in \mathbb{N}. \tag{3.1}$$

For $r = 1$, we get $T(x,e) - T(e,x) = 0$ and hence

$$T(x,e) = T(e,x) \quad \text{for all } x \in A. \tag{3.2}$$

By taking $r = 2$ in (3.1), we obtain

$$\frac{1}{2} T(x^2, e) - T(x,x) + \frac{1}{2} T(e, x^2) = 0 \quad \text{for all } x \in A.$$

So by (3.2), we have

$$T(x,x) = T(x^2, e), \quad x \in A. \tag{3.3}$$

$\square$

Theorem 3.2. *Let A be a unital sequentially complete fundamental topological algebra with bounded elements and B be a topological vector space. Suppose that B is a topological A-bimodule and $T\colon A \to B$ is a continuous linear map with the property that*

$$x \in \mathrm{Inv_c}(A) \Rightarrow \varphi(x) T(x^{-1}) = T(e),$$

where $\varphi : A \to A$ is a continuous homomorphism. Then

$$T(x) = \varphi(x) T(e) \quad \textit{for all } x \in A,$$

i.e., T is a right φ-multiplier.

P r o o f. Define a continuous bilinear map $S\colon A\times A\to B$ by $S(x,y)=\varphi(x)T(y)$. Then

$$S(x,x^{-1})=\varphi(x)T(x^{-1})=T(e)=S(e,e)\quad\text{for all } x\in \mathrm{Inv_c}(A).$$

By applying Theorem 3.1, we get

$$S(e,x)=S(x,e)\quad\text{for all } x\in A.$$

So $\varphi(e)T(x)=\varphi(x)T(e)$. Thus,

$$T(x)=\varphi(x)T(e)\quad\text{for all } x\in A.$$

□

A left φ-multiplier can be obtained by the same argument.

Corollary 3.1. *Let A be a sequentially complete infrasequential fundamental topological algebra and B be a topological vector space such that B is a topological A-bimodule. Suppose that $T\colon A\to B$ is a continuous linear map with the property that*

$$x\in \mathrm{Inv_c}(A)\Rightarrow \varphi(x)T(x^{-1})=T(e),$$

where $\varphi\colon A\to A$ is a continuous homomorphism. Then T is a right φ-multiplier.

P r o o f. Obviously, the definition of Husain of infrasequential topological algebras (see [11]) implies that every infrasequential fundamental topological algebra is a topological algebra with bounded elements. So the result follows from Theorem 3.2.

□

Theorem 3.3. *Let A be a semiprime sequentially complete fundamental topological algebra with bounded elements and $T\colon A\to A$ be a continuous linear map with the property that $x\in \mathrm{Inv_c}(A)\Rightarrow xT(x^{-1})+T(x^{-1})x=2T(e)$. Then T is a multiplier.*

P r o o f. Define a continuous bilinear map $T\colon A\times A\to A$ by

$$T(x,y)=xT(y)+T(y)x.$$

Then

$$T(x,x^{-1})=T(e,e)\quad\text{for all } x\in \mathrm{Inv_c}(A).$$

Replacing x by $x+y$ in (3.3) of Theorem 3.1, we get

$$T(x,y)+T(y,x)=T(xy+yx,e)=T(e,xy+yx)\quad\text{for all } x,y\in A.$$

So

$$xT(y)+T(y)x+yT(x)+T(x)y=2T(xy+yx).$$

Letting $y = x$, we have

$$2(xT(x) + T(x)x) = 2T(2x^2).$$

Hence

$$xT(x) + T(x)x = 2T(x^2) \quad \text{for all } x \in A.$$

By [16], Theorem 1, T is a multiplier. $\square$

Corollary 3.2. *Let A be a semiprime sequentially complete infrasequential fundamental topological algebra and $T\colon A \to A$ be a continuous linear map with the property that $x \in \mathrm{Inv_c}(A) \Rightarrow xT(x^{-1}) + T(x^{-1})x = 2T(e)$. Then T is a multiplier.*

4. Some properties of φ-multipliers

In this section, we investigate the characterizations of φ-multipliers in infrasequential fundamental F-algebras.

Theorem 4.1. *Let A be a fundamental F-algebra with a left (right) uniformly bounded approximate identity and B be an F-space which is a topological A-bimodule. Then any left (right) φ-multiplier $T\colon A \to B$ is linear, where $\varphi\colon A \to A$ is a homomorphism. Moreover, if φ is continuous, then so is T.*

P r o o f. Let $T\colon A \to B$ be a left φ-multiplier. By Cohen's factorization theorem for fundamental F-algebras (see [4], (4.1)), given any sequence $(x_n)_n \subseteq A$ with $x_n \to 0$, there exist $x \in A$ and $(y_n)_n \subseteq A$ with $y_n \to 0$ such that $x_n = xy_n$ for all $n \geqslant 1$. Let $x_1, x_2 \in A$ and $\lambda, \mu \in \mathbb{C}$. Put $(x_n) = (x_1, x_2, 0, 0, \ldots)$. Then there exist $x, y_1, y_2 \in A$ such that $x_1 = xy_1$, $x_2 = xy_2$. Since φ is linear, we have

$$\begin{aligned} T(\lambda x_1 + \mu x_2) &= T(x(\lambda y_1 + \mu y_2)) = T(x)\varphi(\lambda y_1 + \mu y_2) = T(x)(\lambda\varphi(y_1) + \mu\varphi(y_2)) \\ &= \lambda T(xy_1) + \mu T(xy_2) = \lambda T(x_1) + \mu T(x_2). \end{aligned}$$

By the above argument, we have

$$T(x_n) = T(xy_n) = T(x)\varphi(y_n) \to 0 \quad \text{as } n \to \infty.$$

So T is continuous. $\square$

Lemma 4.1. *Let A be an infrasequential topological algebra and $B \subseteq A$. Then B is bounded if and only if it is uniformly bounded.*

P r o o f. Let B be bounded. The definition of infrasequential algebras implies that there exists $\lambda > 0$ such that for all $x \in B$, we have $(\lambda x)^n \to 0$ as $n \to \infty$. Put $k = \lambda^{-1}$. Then for all $x \in B$, $(k^{-1}x)^n \to 0$ as $n \to \infty$. Now, the set $\{k^{-n}x^n\colon x \in B, n \in \mathbb{N}\}$ is bounded. The converse is obvious. $\square$

Corollary 4.1. *Let A be a strongly sequential topological algebra and $B \subseteq A$. Then B is bounded if and only if it is uniformly bounded.*

P r o o f. Since every strongly sequential topological algebra is an infrasequential topological algebra (see [11]), the result follows from Lemma 4.1. $\square$

Theorem 4.2. *Let A be an infrasequential fundamental F-algebra with a uniformly bounded approximate identity and $\varphi\colon A \to A$ be a dense range homomorphism. If T is a φ-multiplier on A with closed range, then $T(A)$ is also an infrasequential fundamental F-algebra.*

P r o o f. Clearly $T(A)$ is a linear subspace of A. We first show that it is an ideal in A. Let $a \in A$ and $b \in T(A)$. Then $\varphi(c_n) \to a$ and $b = T(x)$ for some $(c_n)_n \subseteq A$ and $x \in A$. So

$$ab = \lim_{n\to\infty} \varphi(c_n)T(x) = \lim_{n\to\infty} T(c_n x).$$

Since $\overline{T(A)} = T(A)$, $ab \in T(A)$. Similarly, $ba \in T(A)$. Now we prove that $T(A)$ is fundamental. Let $(y_n)_n \subseteq T(A)$ and $b > 1$ such that $b^n(y_n - y_{n-1}) \to 0$ in $T(A)$. So we have $b^n(y_n - y_{n-1}) \to 0$ in A. By the fundamentality of A we deduce that $(y_n)_n$ is Cauchy in A. Since A is complete, $(y_n)_n$ converges to some $y \in A$. But $\overline{T(A)} = T(A)$, and so $(y_n)_n$ is Cauchy in $T(A)$. Thus $T(A)$ is fundamental. By [11], Proposition 4, $T(A)$ is infrasequential. $\square$

Proposition 4.1. *Let A be a fundamental F-algebra with a uniformly bounded approximate identity $(e_\alpha)_{\alpha\in I}$ and $T\colon A \to A$ be a φ-multiplier with closed range, where φ is a continuous homomorphism from A onto A such that $\varphi \circ T = T \circ \varphi$. If $T^2(A) = T(A)$, then $T(A)$ has an approximate identity.*

P r o o f. Let $T^2(A) = T(A)$. Then there exists a net $(u_\alpha)_{\alpha\in I}$ in A such that $T(e_\alpha) = T^2(u_\alpha)$ for each $\alpha \in I$. We show that $(s_\alpha)_{\alpha\in I} = (\varphi(T(u_\alpha)))_{\alpha\in I}$ is an approximate identity for $T(A)$. Indeed, for any $x \in A$, we have

$$T(x) = \lim_\alpha T(xe_\alpha) = \lim_\alpha \varphi(x)T^2(u_\alpha) = \lim_\alpha T(x)\varphi(T(u_\alpha)) = \lim_\alpha T(x)s_\alpha.$$

Similarly, $T(x) = \lim\limits_\alpha s_\alpha T(x)$. On the other hand,

$$(s_\alpha)_{\alpha\in I} = (\varphi(T(u_\alpha)))_{\alpha\in I} = (T(\varphi(u_\alpha)))_{\alpha\in I} \subseteq T(A).$$

So $(s_\alpha)_{\alpha\in I}$ is an approximate identity for $T(A)$. $\square$

Theorem 4.3. *Let A be an infrasequential fundamental F-algebra with a uniformly bounded approximate identity and $T\colon A \to A$ be a φ-multiplier with closed range, where φ is a homomorphism from A onto A such that $\varphi \circ T = T \circ \varphi$. If $T(A)$ has a bounded approximate identity, then $T^2(A) = T(A)$.*

P r o o f. Let $(t_\alpha)_{\alpha\in I}$ be a bounded approximate identity for $T(A)$. By Lemma 4.1 it is uniformly bounded. Then by Cohen's factorization theorem for fundamental F-algebras (see [4], (4.1)), $AA = A$ and $T(A)T(A) = T(A)$. Since $\varphi \circ T = T \circ \varphi$ and φ is onto, we obtain

$$\begin{aligned} T^2(A) &= T(T(AA)) = T(T(A)\varphi(A)) = \varphi(T(A))T(\varphi(A)) \\ &= \varphi(T(A))\varphi(T(A)) = \varphi(T(A)) = T(\varphi(A)) = T(A). \end{aligned}$$

□

Theorem 4.4. *Let A be a semiprime infrasequential fundamental F-algebra with a uniformly bounded approximate identity and $T\colon A \to A$ be a φ-multiplier with closed range, where φ is an idempotent isomorphism from A onto A such that $\varphi\circ T = T \circ \varphi$. If $T(A)$ has a bounded approximate identity, then T is injective if and only if it is surjective.*

P r o o f. We first show that $\ker T^2 = \ker T$. If $T^2x = 0$, then

$$\begin{aligned} T\varphi(x)\varphi(a)T\varphi(x) &= T\varphi(x)\varphi^2(a)T\varphi(x) = T(\varphi(x)T(ax)) \\ &= T^2(xax) = (T^2x)\varphi(ax) = 0 \end{aligned}$$

for any $a \in A$. Since φ is onto and A semiprime, we get $T\varphi(x) = 0$. So we have $T\varphi(x) = \varphi(Tx) = 0$. Since φ is injective, $Tx \in \ker\varphi = \{0\}$. Hence $Tx = 0$. This implies that $\ker T^2 \subseteq \ker T$. Since the reverse inclusion is trivial, it follows that $\ker T^2 = \ker T$ for any φ-multiplier T. On the other hand, $T^2(A) = T(A)$ by Theorem 4.3.

By [10], Proposition 38.4, it follows that $A = T(A) \oplus \ker T$. Let T be surjective. Since $T(A)\cap\ker T = A\cap\ker T = \{0\}$ implies $\ker T = \{0\}$, we see that T is injective. Conversely, suppose that $\ker T = \{0\}$. It follows from $A = T(A) \oplus \ker T$ that $A = T(A)$, i.e., T is surjective. □

Theorem 4.5. *Let A be a semiprime infrasequential fundamental F-algebra with a uniformly bounded approximate identity and $T\colon A \to A$ be a φ-multiplier, where φ is a continuous idempotent isomorphism from A onto A such that $\varphi \circ T = T \circ \varphi$. If T is surjective and $T(A)$ has a bounded approximate identity, then T^{-1} is a φ^{-1}-multiplier. Moreover, T^{-1} and φ^{-1} are continuous.*

P r o o f. By Theorem 4.4, T is an isomorphism. The Open Mapping Theorem (see [15], 2.11) implies that T and φ are homeomorphisms. If $x, y \in A$, then

$$T^{-1}(x)\varphi^{-1}(y) = T^{-1} \circ T(T^{-1}(x)\varphi^{-1}(y)) = T^{-1}(T \circ T^{-1}(x)\varphi \circ \varphi^{-1}(y)) = T^{-1}(xy).$$

Similarly, we get $\varphi^{-1}(x)T^{-1}(y) = T^{-1}(xy)$. Hence T^{-1} is a φ^{-1}-multiplier. $\square$

R e m a r k 4.1. Every complete metrizable FLM algebra (see [5]) is a strongly sequential algebra (see [2], [14]). So by [11] it is an infrasequential algebra. Thus all the above statements which are true for infrasequential fundamental topological algebras, also hold for complete metrizable FLM algebras.

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