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A q-CONGRUENCE FOR A TRUNCATED ${}_4\varphi_3$ SERIES

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Abstract. Let $\Phi_n(q)$ denote the nth cyclotomic polynomial in q. Recently, Guo, Schlosser and Zudilin proved that for any integer $n>1$ with $n\equiv 1 \pmod 4$,

$$\sum_{k=0}^{n-1}\frac{(q^{-1};q^2)_k^2(q^{-2};q^4)_k}{(q^2;q^2)_k^2(q^4;q^4)_k}q^{6k}\equiv 0 \pmod{\Phi_n(q)^2},$$

where $(a;q)_m=(1-a)(1-aq)\dots(1-aq^{m-1})$. In this note, we give a generalization of the above q-congruence to the modulus $\Phi_n(q)^3$ case. Meanwhile, we give a corresponding q-congruence modulo $\Phi_n(q)^2$ for $n\equiv 3 \pmod 4$. Our proof is based on the 'creative microscoping' method, recently developed by Guo and Zudilin, and a ${}_4\varphi_3$ summation formula.

Keywords: basic hypergeometric series; Watson's transformation; q-congruence; supercongruence; creative microscoping

MSC 2020: 33D15, 11A07, 11B65

1. Introduction

In 1997, Van Hamme in [18], Equation (H.2) established the following supercongruence:

$$\sum_{k=0}^{(p-1)/2}\frac{(\frac12)_k^3}{k!^3}\equiv\begin{cases}\Gamma_p(\frac14)^4 \pmod{p^2} & \text{if } p\equiv 1 \pmod 4,\\ 0 \pmod{p^2} & \text{if } p\equiv 3 \pmod 4,\end{cases}\tag{1.1}$$

where $(a)_n=a(a+1)\dots(a+n-1)$ is the Pochhammer symbol and $\Gamma_p(x)$ is the p-adic Gamma function. For refinements of (1.1) modulo p^3 or p^4, see [10], [12].

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In 2019, Guo and Zudilin in [6], Theorem 2 gave a q-analogue of (1.1) as follows: Modulo $\Phi_n(q)^2$,

$$\sum_{k=0}^{(n-1)/2}\frac{(q;q^2)_k^2(q^2;q^4)_k}{(q^2;q^2)_k^2(q^4;q^4)_k}q^{2k}\equiv\begin{cases}\dfrac{(q^2;q^4)_{(n-1)/4}^2}{(q^4;q^4)_{(n-1)/4}^2}q^{(n-1)/2} & \text{if } n\equiv 1 \pmod 4,\\ 0 & \text{if } n\equiv 3 \pmod 4.\end{cases}\tag{1.2}$$

Here and throughout the paper, the *q-shifted factorial* is defined by $(a;q)_0=1$ and $(a;q)_n=(1-a)(1-aq)\ldots(1-aq^{n-1})$ for $n\geqslant 1$. For simplicity, we sometimes compactly write $(a_1,a_2,\ldots,a_m;q)_n=(a_1;q)_n(a_2;q)_n\ldots(a_m;q)_n$ for $n\geqslant 0$. Moreover, $[n]=1+q+\ldots+q^{n-1}$ denotes the *q-integer*, and $\Phi_n(q)$ stands for the nth *cyclotomic polynomial* in q.

Recently, Mao and Pan in [13] (see also Sun in [15], Theorem 1.3) showed that if $p\equiv 1 \pmod 4$ is a prime, then

$$\sum_{k=0}^{(p+1)/2}\frac{(-\frac{1}{2})_k^3}{k!^3}\equiv 0 \pmod{p^2}.\tag{1.3}$$

And Guo, Schlosser, and Zudilin in [4] proved the following result: For any integer $n>1$ with $n\equiv 1 \pmod 4$,

$$\sum_{k=0}^{(n+1)/2}\frac{(q^{-1};q^2)_k^2(q^{-2};q^4)_k}{(q^2;q^2)_k^2(q^4;q^4)_k}q^{6k}\equiv 0 \pmod{\Phi_n(q)^2}.\tag{1.4}$$

In this note, we shall give a generalization of (1.4) modulo $\Phi_n(q)^3$ and also a corresponding congruence modulo $\Phi_n(q)^2$ for $n\equiv 3 \pmod 4$ as follows.

Theorem 1. *Let n be a positive odd integer. Then*

$$\begin{aligned}&\sum_{k=0}^{(n+1)/2}\frac{(q^{-1};q^2)_k^2(q^{-2};q^4)_k}{(q^2;q^2)_k^2(q^4;q^4)_k}q^{6k}\\ &\quad\equiv\begin{cases}\dfrac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}}\Omega_n(q) \pmod{\Phi_n(q)^3} & \text{if } n\equiv 1 \pmod 4,\\ \dfrac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}}\Omega_n(q) \pmod{\Phi_n(q)^2} & \text{if } n\equiv 3 \pmod 4,\end{cases}\end{aligned}\tag{1.5}$$

where

$$\Omega_n(q)=\frac{1+q^{2n-2}+2q^{2n-1}+4q^{2n}+2q^{2n+1}+q^{2n+2}+q^{4n}}{(1+q^{n-1})(1+q^{n+1})}.\tag{1.6}$$

It is easy to see that $[n](q;q^4)_{(n-1)/2}\equiv 0 \pmod{\Phi_n(q)^2}$ and $[3](q^7;q^4)_{(n-1)/2}$ is relatively prime to $\Phi_n(q)$ for $n\equiv 1 \pmod 4$. Thus, the q-congruence (1.5) implies (1.4).

Letting n be an odd prime and letting $q\to 1$ in Theorem 1, we obtain the following conclusion, which was first proved by Guo and Zudilin, see [7].

Corollary 2. *Let p be an odd prime. Then*

$$\sum_{k=0}^{(p+1)/2}\frac{(-\frac12)_k^3}{k!^3}\equiv\begin{cases} p\dfrac{(\frac14)_{(p-1)/2}}{(\frac74)_{(p-1)/2}} \pmod{p^3} & \text{if } p\equiv 1 \pmod 4,\\ p\dfrac{(\frac14)_{(p-1)/2}}{(\frac74)_{(p-1)/2}} \pmod{p^2} & \text{if } p\equiv 3 \pmod 4.\end{cases}$$

Some other recent q-analogues of supercongruences can be found in [2], [3], [5], [8], [9], [11], [14], [17], [19], [20], [21] with various techniques. In particular, Guo and Zudilin in [5] developed a method called '*creative microscoping*' to prove quite a few q-supercongruences. We shall use this method to prove Theorem 1 in Section 3 (we need to give a related summation formula in the next section at first).

2. A SUMMATION FORMULA

Following Gasper and Rahman (see [1]), the *basic hypergeometric series* ${}_{r+1}\varphi_r$ is defined as

$${}_{r+1}\varphi_r\left[\begin{matrix} a_1,a_2,\dots,a_{r+1}\\ b_1,b_2,\dots,b_r\end{matrix};\,q,\,z\right]=\sum_{k=0}^{\infty}\frac{(a_1,a_2,\dots,a_{r+1};q)_k z^k}{(q,b_1,\dots,b_r;q)_k}.$$

We shall use Watson's ${}_8\varphi_7$ transformation formula, see [1], Appendix (III.18)

$$\begin{aligned}(2.1)\quad {}_8\varphi_7&\left[\begin{matrix} a, & qa^{1/2}, & -qa^{1/2}, & b, & c, & d, & e, & q^{-m}\\ & a^{1/2}, & -a^{1/2}, & aq/b, & aq/c, & aq/d, & aq/e, & aq^{m+1}\end{matrix};\,q,\,\frac{a^2q^{m+2}}{bcde}\right]\\ &=\frac{(aq,aq/de;q)_m}{(aq/d,aq/e;q)_m}\,{}_4\varphi_3\left[\begin{matrix} aq/bc,d,e,q^{-m}\\ aq/b,aq/c,deq^{-m}/a\end{matrix};\,q,\,q\right],\end{aligned}$$

and the q-Paff-Saalschütz formula, see [1], Appendix (II.12)

$$(2.2)\qquad {}_3\varphi_2\left[\begin{matrix} a,b,q^{-m}\\ c,abq^{1-m}/c\end{matrix};\,q,\,q\right]=\frac{(c/a,c/b;q)_m}{(c,c/ab;q)_m}$$

to give a new ${}_4\varphi_3$ summation formula, which plays an important part in our proof of Theorem 1.

Theorem 3. *Let n be a positive odd number. Then*

$$\sum_{k=0}^{(n+1)/2}\frac{(q^{-1-n};q^2)_k(q^{-1+n};q^2)_k(q^{-2};q^4)_k}{(q^{2-n};q^2)_k(q^{2+n};q^2)_k(q^4;q^4)_k}q^{6k}$$
$$=\frac{(1-q^n)(q;q^4)_{(n-1)/2}(1+q^{2n-2}+2q^{2n-1}+4q^{2n}+2q^{2n+1}+q^{2n+2}+q^{4n})}{(1-q^3)(q^7;q^4)_{(n-1)/2}(1+q^{n-1})(1+q^{n+1})}.$$

Proof. Letting $a=q^{-1/2}$, $b=-q^{-1/2}$, $c=q^{m-1}$, $d=q^{-1/4}$ and $e=-q^{-1/4}$ in Watson's transformation formula (2.1), we obtain

$$(2.3)\qquad {}_4\varphi_3\left[\begin{matrix}q^{-1/2},-q^{-1/2},q^{m-1},q^{-m}\\ -q,q^{3/2-m},q^{1/2+m}\end{matrix};q,\,q^3\right]$$
$$=\frac{(q^{1/2},-q;q)_m}{(q^{3/4},-q^{3/4};q)_m}{}_4\varphi_3\left[\begin{matrix}q^{-1/4},-q^{-1/4},-q^{2-m},q^{-m}\\ -q,q^{3/2-m},-q^{-m}\end{matrix};q,\,q\right].$$

Moreover, replacing c by cq in (2.2), we get

$$(2.4)\qquad {}_3\varphi_2\left[\begin{matrix}a,b,q^{-m}\\ cq,abq^{-m}/c\end{matrix};q,\,q\right]=\frac{(cq/a,cq/b;q)_m}{(cq,cq/ab;q)_m}.$$

It is not difficult to see that

$${}_4\varphi_3\left[\begin{matrix}a,b,xq,q^{-m}\\ cq,x,abq^{1-m}/c\end{matrix};q,\,q\right]=\frac{(1-c)(ab-cxq^m)}{(1-x)(ab-c^2q^m)}{}_3\varphi_2\left[\begin{matrix}a,b,q^{-m}\\ c,abq^{1-m}/c\end{matrix};q,\,q\right]$$
$$+\frac{(c-x)(ab-cq^m)}{(1-x)(ab-c^2q^m)}{}_3\varphi_2\left[\begin{matrix}a,b,q^{-m}\\ cq,abq^{-m}/c\end{matrix};q,\,q\right]$$

by comparing the kth summands in the summations. Substituting (2.2) and (2.4) into the last equation, we obtain

$$(2.5)\qquad {}_4\varphi_3\left[\begin{matrix}a,b,xq,q^{-m}\\ cq,x,abq^{1-m}/c\end{matrix};q,\,q\right]=\Omega(q;a,b,c,x,m),$$

where

$$\Omega(q;a,b,c,x,m)$$
$$=\frac{(c/a,c/b;q)_m}{(qc,c/ab;q)_m}\Big(\frac{(1-cq^m)(ab-cxq^m)}{(1-x)(ab-c^2q^m)}+\frac{(c-x)(ab-c)(a-cq^m)(b-cq^m)}{(1-x)(a-c)(b-c)(ab-c^2q^m)}\Big).$$

Replacing c by cq in (2.5), we have

$$(2.6)\qquad {}_4\varphi_3\left[\begin{matrix}a,b,xq,q^{-m}\\ cq^2,x,abq^{-m}/c\end{matrix};q,\,q\right]=\Omega(q;a,b,cq,x,m).$$

It is also routine to verify the relation

$$
{}_5\varphi_4\left[\begin{matrix} a, b, xq, yq, q^{-m} \\ cq^2, x, y, abq^{1-m}/c \end{matrix}; q, q\right] = \frac{(1-cq)(ab-cyq^m)}{(1-y)(ab-c^2q^{m+1})}\,{}_4\varphi_3\left[\begin{matrix} a, b, xq, q^{-m} \\ cq, x, abq^{1-m}/c \end{matrix}; q, q\right]
+ \frac{(cq-y)(ab-cq^m)}{(1-y)(ab-c^2q^{m+1})}\,{}_4\varphi_3\left[\begin{matrix} a, b, xq, q^{-m} \\ cq^2, x, abq^{-m}/c \end{matrix}; q, q\right].
$$

Substituting (2.5) and (2.6) into the last equation, we get

$$
\text{(2.7)}\quad {}_5\varphi_4\left[\begin{matrix} a, b, xq, yq, q^{-m} \\ cq^2, x, y, abq^{1-m}/c \end{matrix}; q,\ q\right] = \frac{(1-cq)(ab-cyq^m)}{(1-y)(ab-c^2q^{m+1})}\Omega(q; a, b, c, x, m)
+ \frac{(cq-y)(ab-cq^m)}{(1-y)(ab-c^2q^{m+1})}\Omega(q; a, b, cq, x, m).
$$

Evaluating the series on the right-hand side of (2.3) by the case $a = q^{-1/4}$, $b = -q^{-1/4}$, $c = -q^{-1}$, $x = -q^{-m}$, $y = -q^{1-m}$ of (2.7), we gain

$$
{}_4\varphi_3\left[\begin{matrix} q^{-1/2}, -q^{-1/2}, q^{m-1}, q^{-m} \\ -q, q^{3/2-m}, q^{1/2+m} \end{matrix}; q, q^3\right] = \frac{(1-q^{m-1/2})(1-q)(q^{-3/4}, -q^{-3/4}; q)_m}{(1+q^{m-1/2})(1+q^{m-1})(q^{3/4}, -q^{3/4}; q)_m}
\times\Big(\frac{(1-q^{m-1})(1-q^{2m-3/2})(1+q^{2m})}{(1+q^m)(1-q^{-3/2})(1-q)}
+ \frac{q^m(1+q^{m-1}+q^m+2q^{m-1/2}+q^{2m-1})}{(1-q^{3/2})(1-q^{-1/2})}\Big).
$$

Employing the substitutions $q \mapsto q^2$ and $m \mapsto \frac{1}{2}(n+1)$ in the above ${}_4\varphi_3$ summation, we arrive at Theorem 3. □

3. PROOF OF THEOREM 1

The following simple q-congruence (see [3], Lemma 3.1 and Equation (5.4)) will be used in our proof.

Lemma 1. *Let n be a positive odd integer. Then for $0 \leqslant k \leqslant \frac{1}{2}(n+1)$ we have*

$$
\frac{(aq^{-1}; q^2)_{(n+1)/2-k}}{(q^2/a; q^2)_{(n+1)/2-k}} = (-a)^{(n+1)/2-2k}\frac{(aq^{-1}; q^2)_k}{(q^2/a; q^2)_k}q^{(n-1)^2/4+3k-1} \pmod{\Phi_n(q)}.
$$

We are going to prove Theorem 1 using the creative microscoping method, see [5]. That is, we need to establish the following parametric version of Theorem 1.

Theorem 4. *Let $n > 1$ be an odd integer. Then*

$$\sum_{k=0}^{(n+1)/2} \frac{(aq^{-1};q^2)_k(q^{-1}/a;q^2)_k(q^{-2};q^4)_k}{(aq^2;q^2)_k(q^2/a;q^2)_k(q^4;q^4)_k} q^{6k} \equiv \begin{cases} \dfrac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}} \Omega_n(q) \pmod{\Phi_n(q)(1-aq^n)(a-q^n)} \\ \qquad\qquad\qquad\qquad \text{if } n \equiv 1 \pmod 4, \\ \dfrac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}} \Omega_n(q) \pmod{(1-aq^n)(a-q^n)} \\ \qquad\qquad\qquad\qquad \text{if } n \equiv 3 \pmod 4, \end{cases} \tag{3.1}$$

where $\Omega_n(q)$ is given by (1.6).

Proof. For $a = q^{-n}$ or $a = q^n$, by Theorem 3 the left-hand side of (3.1) is equal to

$$\sum_{k=0}^{(n+1)/2} \frac{(q^{-1-n};q^2)_k(q^{-1+n};q^2)_k(q^{-2};q^4)_k}{(q^{2-n};q^2)_k(q^{2+n};q^2)_k(q^4;q^4)_k} q^{6k} = \frac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}} \Omega_n(q).$$

This shows that the q-congruence (3.1) holds modulo $1-aq^n$ and $a-q^n$. On the other hand, by Lemma 1 we can easily verify that for $n \equiv 1 \pmod 4$ $(n > 1)$ and $0 \leqslant k \leqslant \frac{1}{2}(n+1)$,

$$\begin{aligned} &\frac{(aq^{-1};q^2)_{(n+1)/2-k}(q^{-1}/a;q^2)_{(n+1)/2-k}(q^{-2};q^4)_{(n+1)/2-k}}{(aq^2;q^2)_{(n+1)/2-k}(q^2/a;q^2)_{(n+1)/2-k}(q^4;q^4)_{(n+1)/2-k}} q^{6((n+1)/2-k)} \\ &\qquad \equiv -\frac{(aq^{-1};q^2)_k(q^{-1}/a;q^2)_k(q^{-2};q^4)_k}{(aq^2;q^2)_k(q^2/a;q^2)_k(q^4;q^4)_k} q^{6k} \pmod{\Phi_n(q)}. \end{aligned}$$

Namely, the sum of the kth and $(\frac{1}{2}(n+1)-k)$th summands of the left-hand side of (3.1) vanishes modulo $\Phi_n(q)$. It follows that

$$\sum_{k=0}^{(n+1)/2} \frac{(aq^{-1};q^2)_k(q^{-1}/a;q^2)_k(q^{-2};q^4)_k}{(aq^2;q^2)_k(q^2/a;q^2)_k(q^4;q^4)_k} q^{6k} \equiv 0 \pmod{\Phi_n(q)} \tag{3.2}$$

for $n \equiv 1 \pmod 4$ $(n > 1)$. Since $[n] \equiv 0 \pmod{\Phi_n(q)}$ for $n > 1$, we have proved that the q-congruence (3.1) is also true modulo $\Phi_n(q)$ for $n \equiv 1 \pmod 4$. Finally, noticing that the polynomials $1-aq^n$, $a-q^n$ and $\Phi_n(q)$ are pairwise relatively prime, we complete the proof of the theorem. $\square$

Proof of Theorem 1. Theorem 1 is obviously true for $n = 1$ (both sides are equal to 1). For $n > 1$, note that the limit of $(1-aq^n)(a-q^n)$ as $a \to 1$ contains the factor $\Phi_n(q)^2$ and the limits of the denominators on both sides of (3.1) as $a \to 1$ are relatively prime to $\Phi_n(q)$. Letting $a \to 1$ in (3.1), we obtain (1.5) immediately. $\square$

4. Two open problems

Swisher in [16], Equation (H.3) has made two interesting conjectures on supercongruences generalizing (1.1) as follows:

$$\sum_{k=0}^{(p^r-1)/2}\frac{(\frac12)_k^3}{k!^3}\equiv -\Gamma_p(\tfrac14)^4\sum_{k=0}^{(p^{r-1}-1)/2}\frac{(\frac12)_k^3}{k!^3}\pmod{p^{3r}},\quad p\equiv 1\pmod 4,$$
$$\sum_{k=0}^{(p^r-1)/2}\frac{(\frac12)_k^3}{k!^3}\equiv p^2\sum_{k=0}^{(p^{r-2}-1)/2}\frac{(\frac12)_k^3}{k!^3}\pmod{p^{3r-1}},\quad p\equiv 3\pmod 4,\ r\geqslant 2,\ p>3.$$

We did not find Swisher-type general patterns for (1.3). Nevertheless, we have the following supercongruences conjectures for generalizations of (1.3).

Conjecture 1. *Let p be a prime with $p\equiv 1 \pmod 4$ and let $r\geqslant 1$. Then*

$$\sum_{k=0}^{(p^r+1)/2}\frac{(-\frac12)_k^3}{k!^3}\equiv 0\pmod{p^{2r}}\quad\text{and}\quad\sum_{k=0}^{p^r-1}\frac{(-\frac12)_k^3}{k!^3}\equiv 0\pmod{p^{2r}}.$$

Conjecture 2. *Let p be a prime with $p\equiv 3 \pmod 4$ and let $r\geqslant 1$. Then*

$$\sum_{k=0}^{(p^r+1)/2}\frac{(-\frac12)_k^3}{k!^3}\equiv p^r\frac{(\frac14)_{(p^r-1)/2}}{(\frac74)_{(p^r-1)/2}}\pmod{p^{r+1}},$$
$$\sum_{k=0}^{p^r-1}\frac{(-\frac12)_k^3}{k!^3}\equiv p^r\frac{(\frac14)_{(p^r-1)/2}}{(\frac74)_{(p^r-1)/2}}\pmod{p^{r+1}}.$$

Any proof of the above two conjectures will be very interesting.

Since $(q^{-1};q^2)_k^2(q^{-2};q^4)_k\equiv 0 \pmod{\Phi_n(q)^3}$ for k in the range $\frac12(n+1)<k\leqslant n-1$, we see that (1.5) can also be written as

$$\sum_{k=0}^{n-1}\frac{(q^{-1};q^2)_k^2(q^{-2};q^4)_k}{(q^2;q^2)_k^2(q^4;q^4)_k}q^{6k}\equiv\begin{cases}\dfrac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}}\Omega_n(q)\pmod{\Phi_n(q)^3} & \text{if } n\equiv 1\pmod 4,\\[2mm] \dfrac{[n](q;q^4)_{(n-1)/2}}{[3](q^7;q^4)_{(n-1)/2}}\Omega_n(q)\pmod{\Phi_n(q)^2} & \text{if } n\equiv 3\pmod 4.\end{cases}\tag{4.1}$$

Note that for any prime $p \equiv 1 \pmod 4$ and integer $r \geqslant 1$, we have

$$\frac{(\frac{1}{4})_{(p^r-1)/2}}{(\frac{7}{4})_{(p^r-1)/2}} \equiv 0 \pmod p.$$

Thus, letting $n = p^r$ and $q \to 1$ in (1.5) and (4.1), we obtain the following supercongruences: for any prime $p \equiv 1 \pmod 4$ and integer $r \geqslant 2$,

$$\sum_{k=0}^{(p^r+1)/2} \frac{(-\frac{1}{2})_k^3}{k!^3} \equiv 0 \pmod{p^3} \quad \text{and} \quad \sum_{k=0}^{p^r-1} \frac{(-\frac{1}{2})_k^3}{k!^3} \equiv 0 \pmod{p^3}.$$

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