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FINITE-TIME TOPOLOGICAL IDENTIFICATION OF COMPLEX NETWORK WITH TIME DELAY AND STOCHASTIC DISTURBANCE

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The topology identification issue of complex stochastic network with delay and stochastic disturbance is mainly introduced in this paper. It is known the time delay and stochastic disturbance are ubiquitous in real network, and they will impair the identification of network topology, and the topology capable of identifying the network within specific time is desired on the other hand. Based on these discussions, the finite-time identification method is proposed to solve similar issues problems. The validity of theoretical results is proved with the stochastic dynamical system stability theory and finite-time stability theory. Finally, a simple numerical simulation is proposed to verify the feasibility of the method.

Keywords: topology identification, finite-time, time delay, stochastic perturbations

Classification: 34H10, 93E15

1. INTRODUCTION

The complex network, as a hotspot of studies in subject fields in recent years, serves as an important part of network science, and the structure of complex network is the most important basic feature. Actually it is extremely hard to completely know or determine the structure of network and especially for large-scale network. Consequently, researches started to explore studies and see if it is possible to find a suitable method to identify the structure of network initiated. Presently, the method for identifying the network structure by external synchronization between the drive network and auxiliary network for complex stochastic network model is proposed in abundant references [14, 20, 21, 24]. The adaptive control method is proposed in Reference [8] to identify complex dynamics network with delays. The importance of linearly independent conditions in the identification of system parameters is raised in Reference [7]and Reference [19]. Subsequently, the importance of linearly independent conditions in the network identification is raised in Reference [3], in which it is explained that the inter-network node synchronization will impair the identification. Additionally, the noise is unavoidable in real network. The topological identification of complex dynamic network models with stochastic noise is proposed, and it is raised linearly independent conditions are easier to be implemented

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in noise-containing dynamics network in Reference [16]. And the signal of driving network is utilized in Reference [5]to identify the topologies of discrete complex dynamics network. Additionally, the topology of complex dynamics network is proposed in references from Reference [17] to Reference [15]to identify the noise based on the Granger Causality. On the other hand, the finite-time control method has been widely used in the synchronization issue of complex dynamic network and the consistency issue of multi-agent systems [1, 4, 12, 13, 18, 23], the system is further expected to be stabilized in limited in practical applications. The finite-time method is proposed in Reference [11] to identify deterministic complex dynamics network without considering the time delay. Whereas, topological identification of complex dynamics network models with multiple delays and stochastic disturbances are discussed in Reference [22], and the methods and models are extremely complex with universalities. The finite-time topologies are proposed in this paper based on some of abovementioned discussions to identify complex dynamics network with delays and stochastic disturbances, and the models and methods are simpler and more applicable than those listed in Reference.

The organizational structure of this paper is shown as follows. Some basic knowledge is mainly discussed in Section 2. The problem of finite-time topological identification of complex network with delay and stochastic disturbance is described in Section 3. The correctness of theoretical results is stated via numerical simulation in Section 4. The conclusion of the full paper is intensively stated in Section 5.

2. PREREQUISITE KNOWLEDGE

The theories relating to the finite-time stability of stochastic differential equations and stochastic dynamical systems are introduced in this Section, see Reference [2] and Reference [10].

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i. e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$empty sets). Let $B(t) = (B_1(t), ..., B_m)^{\mathrm{T}}$ is the $m-$dimensional Brownian motion defined on the probability space. Define an $n-$dimensional non-autonomous stochastic differential equation:

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t), \tag{1}$$

on $t \geq 0$ with initial value $x(0) = x_0 \in \mathbb{R}^n$, the function $f : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^{n\times m}$ are measurable functions and satisfy the local Lipschitz condition and the linear growth condition, that is $f(0, t) = 0,\ g(0, t) = 0$.

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+; \mathbb{R}_+)\mathbb{R}^n \times \mathbb{R}_+$ denote the family of all nonnegative functions $V(x, t)$ on $\mathbb{R}_+$ which are continuously twice differentiable in x and once differentiable in t. Define an operator $\mathcal{L}$ acting on $V \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+)$ functions by

$$\mathcal{L}V(x, t) = V_t(x, t) + V_x(x, t)f(x, t) + \frac{1}{2}trace[g^{\mathrm{T}}(x, t)V_{xx}(x, t)g(x, t)], \tag{2}$$

where

$$V_t(x, t) = \frac{\partial V(x, t)}{\partial t}, V_x(x, t) = \left(\frac{\partial V(x, t)}{\partial x_1}, ..., \frac{\partial V(x, t)}{\partial x_n}\right),$$

$$V_{xx}(x,t)=\left(\frac{\partial^2 V(x,t)}{\partial x_i x_j}\right)_{n\times n}.$$

Consequently, the following definitions are made:

Definition 2.1. (Chen and Jiao [2]) For the stochastic system (1), the origin $x=0$ is said to be globally stochastically finite-time attractive, if for $x_0\in\mathbb{R}^n$, the following conditions hold.

(i) Stochastic settling time function $T_0(x_0,B)=\inf\{T\geq 0: x(t,x_0)=0,\forall t\geq T\}$ exists with the probability in one, or

$$P\{\lim_{t\to T_0^-}||x(t,x_0)||=0\}=1.$$

(ii) Provided that $T_0(x_0,B)$ exists, then $E[T_0(x_0,B)]<\infty$.

Lemma 2.2. (Chen and Jiao [2]) Considering system (1). If there is a positive definite, twice continuously differentiable and radially unbounded Lyapunov function $V: R^n\to R_+$ and real numbers $\eta>0$ and $0<\rho<1$, such that

$$\mathcal{L}V\leq -\eta(V(x))^\rho$$

then the origin of system (1) is globally stochastically finite-time attractive, and $E[T_0(x_0,B)]\leq\frac{(V(x_0))^{1-\rho}}{\eta(1-\rho)}$.

Lemma 2.3. (Hardy et al. [6]) Let $a_1,a_2,\ldots,a_n>0$ and $0<r<p$, then

$$\left(\sum_{i=1}^n a_i^p\right)^{\frac{1}{p}}\leq\left(\sum_{i=1}^n a_i^r\right)^{\frac{1}{r}}.$$

3. MAJOR CONCLUSIONS

The topological identification of complex dynamics network with coupling delays and stochastic perturbations are mainly introduced in this Section.

A complex dynamics network model with the coupling delay consisting of N different nodes is defined:

$$\dot{x}_i(t)=f_i(x_i(t))+\sum_{j=1}^N a_{ij}\Gamma x_j(t-\tau)+\varphi_i(x_i(t))\dot{\omega}_i(t), i=1,2,\ldots,N, \tag{3}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^{\mathrm{T}}\in R^n$ denotes the state vector of node i, $f_i(x_i(t)):R^n\to R^n$ denotes nonlinear function of the continuously differentiable function. τ represents the the coupling extension between nodes, $\Gamma: R^n\to R^n$denotes internal coupling matrix. The external coupling matrix $A=(a_{ij})_{N\times N}$ denotes the topologies between nodes in the network, and it is defined as follows: If the link is found between node j and node $i(i\neq j)$, then $a_{ij}>0$, or $a_{ij}=0$. $\varphi_i: R^n\to R^{n\times m}$represents the matrix for noise intensity, and $\omega_i(t)=(\omega_{i1},\ldots,\omega_{im})^{\mathrm{T}}$ represents the m-dimensional Brownian Movement with the natural filter $\{\mathcal{F}_t\}_{t\geq 0}$ in the probability space $(\Omega,\mathcal{F},P)$.

Hereafter, no symmetry or irreducible property is required for the coupling configuration matrix A which is diffusive in $a_{ii} = -\sum_{j=1,j\neq i}^{N} a_{ij}$. The main goal is to identify these unknown or uncertain coupling strengths, namely its network topological structure.

On the other hand, the information f_i, Γ, φ_i of all nodes $(i = 1, \dots, N.)$ in the above drive network (3) is known in advance, and the status information $x_i(t), x_i(t-\tau)$ are observable and measurable, and the noise is frequently present in real network. Thus, known and observed are utilized to construct the following drive network:

$$\dot{y}_i(t) = f_i(y_i(t)) + \sum_{j=1}^{N} b_{ij}\Gamma x_j(t-\tau) + u_i(t) + \varphi_i(y_i(t))\dot{\omega}_i(t), \tag{4}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \dots, y_{in}(t))^{\mathrm{T}} \in R^n$ represent the response status of Node i, $u_i(t)$ represents the control input, and the coupling matrix $B = [b_{ij}]$ represents the estimate matrix of the drive network topology $A = [a_{ij}]$.

It is possible to define the synchronization errors $e_i(t) = y_i(t) - x_i(t), e_i(t-\tau) = y_i(t-\tau) - x_i(t-\tau)$ and $\sigma_i(e_i(t)) = \varphi_i(y_i(t)) - \varphi_i(x_i(t))$ between the drive network (3) and the response network (4).

Thereby concluding the following the error dynamical system:

$$\dot{e}_i(t) = f_i(y_i(t)) - f_i(x_i(t)) + \sum_{j=1}^{N} b_{ij}\Gamma x_j(t-\tau) - \sum_{j=1}^{N} a_{ij}\Gamma x_j(t-\tau) + u_i(t) + \sigma_i(e_i(t))\dot{\omega}_i(t). \tag{5}$$

The following hypothesis are given.

Hypothesis 3.1. It is assumed that the nonnegative constant $\alpha_i(i = 1, 2 \dots, N.)$, found, the nodal dynamics f_i makes the following formula be true.

$$\|f_i(x(t)) - f_i(y(t))\| \leq \alpha_i \|x(t) - y(t)\|, \qquad \forall\, x, y \in R^n. \tag{6}$$

Hypothesis 3.2. It is assumed that the function for noise intensity $\sigma_i(i = 1, 2 \dots, N.)$ satisfies Lipschitz and linear growth conditions, and the nonnegative constant β_i is found to make the following formula be true:

$$trace(\sigma_i^{\mathrm{T}}(e_i(t))\sigma_i(e_i(t))) \leq 2\beta_i e_i^{\mathrm{T}}(t)e_i(t) \tag{7}$$

and, $\sigma(0) \equiv 0$.

Hypothesis 3.3. It is assumed that each element a_{ij} in known coupling matrix $A = (a_{ij})_{N\times N}$ is bounded, namely,

$$|a_{ij}| \leq L, \tag{8}$$

where $i, j = 1, 2 \dots, N, L$ represent known constant.

Hypothesis 3.4. It is assumed that the vector $\{\Gamma x_j(t-\tau)\}_{j=1}^{N}$ is linearly independent for synchronous manifold $\{x_i(t), x_i(t-\tau)\}_{i=1}^{N}$.

The main results are given in the following theorem and corollaries.

Theorem 3.5. If the hypotheses through from Hypothesis 3.1 to Hypothesis 3.4 are considered to be true, and the following drive network is designed:

$$\begin{cases} u_i = -2\gamma \sum_{j=1}^{N} (|b_{ij}| + L)^{\theta+1} (\frac{e_i(t)}{\|e_i(t)\|^2}) - 2\gamma sign(e_i(t)) ||e_i(t)||^{\theta} - ke_i(t), \|e_i(t)\| \neq 0, \\ u_i = 0, \|e_i(t)\| = 0, \\ \dot{b}_{ij} = -e_i^{\mathrm{T}}(t)\Gamma x_j(t-\tau), \end{cases}$$

where $0 < \theta < 1,\ \gamma > 0$ is satisfied, k denoted a positive instant big enough. Then it is possible to identify the topologies of complex dynamics network including the delay and stochastic disturbance (3) within finite time.

Proof. Construct the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t), \tag{9}$$

where

$$V_1(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t). \tag{10}$$

$$V_2(t) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (b_{ij} - a_{ij})^2. \tag{11}$$

The $V(t)$ is derived along the trajectories of the system (5)

$$\begin{aligned} \mathcal{L}V =& \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) \dot{e}_i(t) + \sum_{i=1}^{N} \sum_{j=1}^{N} (b_{ij} - a_{ij}) \dot{b}_{ij} \\ =& \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) [f_i(y_i(t)) - f_i(x_i(t)) + \sum_{j=1}^{N} b_{ij} \Gamma x_j(t-\tau) - \sum_{j=1}^{N} a_{ij} \Gamma x_j(t-\tau) \\ &- 2\gamma \sum_{j=1}^{N} (b_{ij} + M)^{\theta+1} \left(\frac{e_i(t)}{\|e_i(t)\|^2} \right) - 2\gamma sign(e_i(t)) ||e_i(t)||^{\theta} - ke_i(t)] \\ &+ \frac{1}{2} \sum_{i=1}^{N} trace(\sigma_i^{\mathrm{T}} \sigma_i) + \sum_{i=1}^{N} \sum_{j=1}^{N} (b_{ij} - a_{ij})(-e_i^{\mathrm{T}}(t) \Gamma x_j(t-\tau)) \\ \leq& \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) [\alpha_i e_i(t) + \beta_i e_i(t) - ke_i(t) - 2\gamma sign(e_i) ||e_i(t)||^{\theta}] - 2\gamma \sum_{i=1}^{N} \sum_{j=1}^{N} (b_{ij} + M)^{\theta+1}. \end{aligned} \tag{12}$$

Deriving

$$\begin{aligned}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) sign(e_i)||e_i(t)||^{\theta} &= \sum_{i=1}^{N}(||e_i(t)||^{\theta})^{\mathrm{T}} sign(e_i)e_i(t)\\ &= \sum_{i=1}^{N}(||e_i(t)||^{\theta})^{\mathrm{T}}||e_i(t)||\\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}||e_{ij}||^{\theta+1}. \qquad (13)\end{aligned}$$

According to 2.3, the following formula is derived

$$\left(\sum_{i=1}^{N}\sum_{j=1}^{N}||e_{ij}||^2\right)^{\frac{1}{2}} \le \left(\sum_{i=1}^{N}\sum_{j=1}^{N}||e_{ij}||^{\theta+1}\right)^{\frac{1}{\theta+1}}. \qquad (14)$$

Hence,

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N}||e_{ij}||^{\theta+1} &\ge \left(\sum_{i=1}^{N}\sum_{j=1}^{N}||e_{ij}||^2\right)^{\frac{\theta+1}{2}}\\ &= \left(\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t)\right)^{\frac{\theta+1}{2}}\\ &= (2V_1(t))^{\frac{\theta+1}{2}}. \qquad (15)\end{aligned}$$

Similarly, as

$$|b_{ij}-a_{ij}| \le |b_{ij}|+|a_{ij}| \le |b_{ij}|+L \qquad (16)$$

and

$$\left(\sum_{i=1}^{N}\sum_{j=1}^{N}(|b_{ij}|+L)^2\right)^{\frac{1}{2}} \le \left(\sum_{i=1}^{N}\sum_{j=1}^{N}(|b_{ij}|+L)^{\theta+1}\right)^{\frac{1}{\theta+1}}. \qquad (17)$$

Getting

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N}(|b_{ij}|+L)^{\theta+1} &\ge \left(\sum_{i=1}^{N}\sum_{j=1}^{N}(|b_{ij}|+L)^2\right)^{\frac{\theta+1}{2}}\\ &\ge \left(\sum_{i=1}^{N}\sum_{j=1}^{N}(b_{ij}-a_{ij})^2\right)^{\frac{\theta+1}{2}}\\ &= (2V_2(t))^{\frac{\theta+1}{2}}. \qquad (18)\end{aligned}$$

If $k \geq \max_{1\leq i\leq N}\{\alpha_i\} + \max_{1\leq i\leq N}\{\beta_i\}$ s satisfied, then according to the formulas thorough from Formula(9) to Formula (18), deriving

$$\begin{aligned}\mathcal{L}V &\leq -2\gamma\left((2V_1(t))^{\frac{\theta+1}{2}} + (2V_2(t))^{\frac{\theta+1}{2}}\right)\\ &\leq -2\gamma\left(2V_1(t) + 2V_2(t)\right)^{\frac{\theta+1}{2}}\\ &= -2\gamma(2V(t))^{\frac{\theta+1}{2}}.\end{aligned} \tag{19}$$

According to Lemma 1, it is concluded that Error Dynamics System (4) is globally asymptotic stable with finite time, or

$$P\{\lim_{t\to T_0^-}||e(t,x_0)|| = 0\} = 1 \tag{20}$$

where

$$E[T_0] \leq \frac{(2V(0))^{\frac{1-\theta}{2}}}{\gamma(1-\theta)}. \tag{21}$$

As $V_2(0)$ is an unknown energy function, and

$$\begin{aligned}(2V(0))^{\frac{1-\theta}{2}} &\leq (2V_1(0))^{\frac{1-\theta}{2}} + (2V_2(0))^{\frac{1-\theta}{2}}\\ &\leq \left(\sum_{i=1}^{N} e_i^{\mathrm{T}}(0)e_i(0)\right)^{\frac{1-\theta}{2}} + \left(\sum_{i=1}^{N}\sum_{j=1}^{N}(|b_{ij}(0)| + L)^2\right)^{\frac{1-\theta}{2}}.\end{aligned} \tag{22}$$

The estimate on T_0 is acquired, or

$$E[T_0] \leq \frac{\sum\limits_{i=1}^{N}\|e_i(0)\|^{1-\theta} + \sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N}(|b_{ij}(0)| + L)^{1-\theta}}{\gamma(1-\theta)}. \tag{23}$$

Supposed that $\lim_{t\to T_0^-}\dot{e}_i(t)$ exist, it is gotten that $\lim_{t\to T_0^-}\dot{e}_i(t) = \mathbf{0}_n$, according to Formula (5) indicating $y(t)$ goes to $\Omega = \{y(t) : \sum_{j=1}^{N}(b_{ij} - a_{ij})\Gamma x_j(t-\tau) = 0\}(i,j = 1,2,\ldots,N.)$. According to Hypothesis 3.3, it is gotten that $\lim_{t\to T_0^-} b_{ij} = a_{ij}$. It indicates Coupling Matrix A is subject to the network estimate within finite time, then it is proven. □

Remark 3.6. It can be concluded from the proving on Theorem 3.5 that the parameters θ, γ will influence the convergence time controlled with finite time. Therefore appropriate parameters are to be chosen in practical application.

If the network is composed of the same nodes and free of coupling delay, and it dynamical equation is shown as follows:

$$\dot{x}_i(t) = g(x_i(t)) + \sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + \varphi_i(x_i(t))\dot{\omega}_i(t). \tag{24}$$

Thereby concluding the following inferences:

Corollary 3.7. Supposed the hypothesis thorough from Hypothesis 3.1 to Hypothesis 3.4, the drive network is calculated as follows:

$$\begin{cases} u_i = -2\gamma \sum_{j=1}^{N} (|b_{ij}| + L)^{\theta+1} (\frac{e_i(t)}{\|e_i(t)\|^2}) - 2\gamma sign(e_i(t)) ||e_i(t)||^{\theta} - ke_i(t), \|e_i(t)\| \neq 0, \\ u_i = 0, \|e_i(t)\| = 0, \\ \dot{b}_{ij} = -e_i^{\mathrm{T}}(t)\Gamma x_j(t). \end{cases}$$

here $0 < \theta < 1,\ \gamma > 0,\ k$ is a positive instant great enough. Then the unknown coupling matrix A of stochastic complex dynamics network (24) may be identified within finite time.

4. NUMERICAL EXPERIMENTS

In this section, a simple example is given to prove the validity of previous method.

Considering one dynamics network composed of 4 identical Lorzen systems [9] and the coupling matrix between them are shown as follows:

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

when $L = 2$ is calculated, Equation (3.3) is supposed to be true. The Loezen System is chosen as the node dynamics, and it is described as

$$f(x_i) = \begin{pmatrix} a(x_{i2} - x_{i1}) \\ cx_{i1} - x_{i1}x_{i3} - x_{i2} \\ x_{i1}x_{i2} - bx_{i3} \end{pmatrix} \tag{25}$$

when the parameters $a = 10, b = 8/3, c = -28$ are satisfied, a typical chaotic attractor is found in System (25), as shown in Figure 1

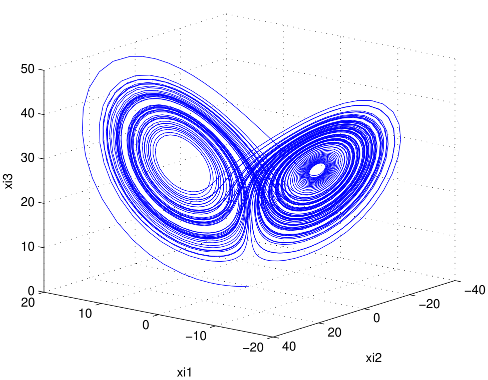

Fig. 1. Lorentz Attractor.

Here it is gotten Hypothesis 3.1 is true and α_i is calculated as $\alpha_i = 58$. Additionally, internal coupling matrix selected is $A = diag\{1, 0, 0\}$.

Let w_i to be the three-dimensional Brownian movement, take the function for noise intensity $\varphi_i(x_i(t)) = diag(0.1x_1(t), 0.1x_2(t), 0.1x_3(t))$ and $\varphi_i(y_i(t)) = diag(0.1y_1(t), 0.1y_2(t), 0.1y_3(t))$, then $\sigma_i(e_i(t)) = diag(0.1e_1(t), 0.1e_2(t), 0.1e_3(t))$, hence $trace(\sigma_i^{\mathrm{T}}(e_i(t))\sigma_i(e_i(t))) = 0.01e_i^{\mathrm{T}}(t)e_i(t)$ is true, satisfying Hypothesis 3.2.

Euler-Maruyama is used to calculate the stochastic delay differential equation with the time step valued in 0.01, and the initial values of nodes of driving network and auxiliary network are $x_i(0) = [0.1 \times (3i-2), 0.1 \times (3i-1), 0.1 \times 3i]^{\mathrm{T}}$, and $y_i(0) = [1.1 \times (3i-2), 1.1 \times (3i-1), 1.1 \times 3i]^{\mathrm{T}}$, respectively, define $b_{ij}(0) = 3$, $\theta = 0.5$, $\gamma = 1$ and $\tau = 0.01$, In the three-dimensional system, the feedback control gains are supposed to satisfy $k_1 = 60$, $k_2 = 120$ and $k_3 = 120$.

According to Theorem 3.5, it is possible to identify unknown coupling matrix A of driving network within finite time under designed driving network. Figure 2 represents the network identification diagram for network topology. Figure 3 shows that the error between coupling matrixes of driving network tends to zero over a period of time, and it also indicates that the coupling matrix of driving network is identified. Additionally, the finite time estimate is identified to satisfy $T_0 = 90.9030$ by calculating, and it is found in Figure 2 and Figure 3 that the results of identification are acquired when $T = 50$, demonstrating the finite time identification method is feasible.

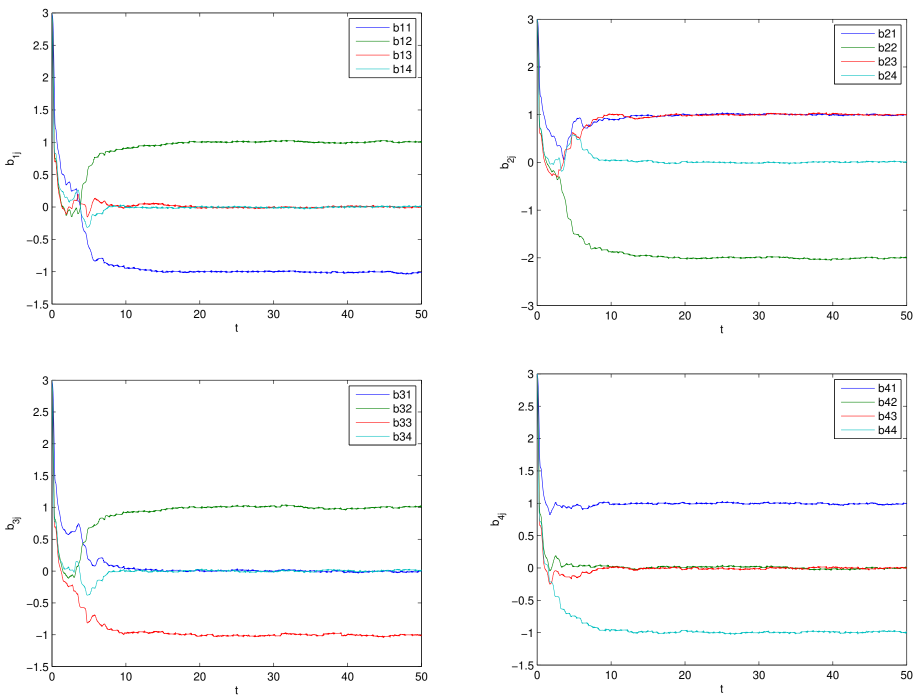

Fig. 2. Differences in Network Topologies.

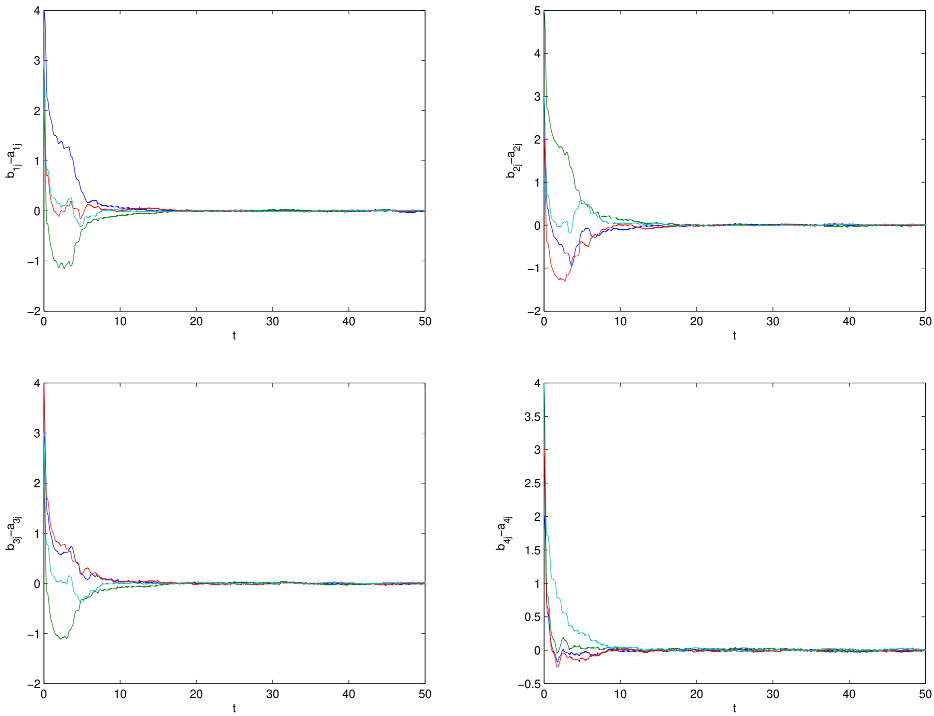

Fig. 3. Differences identifications in Network Topologies.

5. CONCLUSIONS

The topological identification issue of complex dynamics network with delay and stochastic disturbance is mainly explored in this paper., the method for finite time topology identification based on considered identification methods is posed . The correctness of theoretical results are proved by stochastic dynamical system stability and finite time stability. Finally, numerical examples are given to prove the feasibility of abovementioned method.

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