Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 70 (2020), No. 4, 1147–1160

Persistent URL: `http://dml.cz/dmlcz/148418`

Czechoslovak Mathematical Journal, 70 (145) (2020), 1147–1160

RECOLLEMENT OF COLIMIT CATEGORIES AND ITS APPLICATIONS

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Received May 29, 2019. Published online September 18, 2020.

Abstract. We give an explicit recollement for a cocomplete abelian category and its colimit category. We obtain some applications on Leavitt path algebras, derived equivalences and K-groups.

Keywords: colimit category; recollement; Leavitt path algebra; K_i group

MSC 2020: 18A30, 19D50

1. Introduction

In order to describe the derived categories of perverse sheaves over singular spaces, by using derived versions of Grothendieck's six functors, recollements were first introduced by Beilinson, Berstein and Deligne in 1982, see [6], [12]. Later, recollements of derived categories were employed to study stratifications of the derived categories of modules over blocks of the Bernstein-Gelfand-Gelfand category $\mathcal{O}$, see [9]. Further, recollements were used by Happel, see [14], to establish a relationship among finitistic dimensions of finite-dimensional algebras. Recently, they become of great interest in understanding the derived categories of the endomorphism rings of infinitely generated tilting modules, see [2]. It turns out that recollements are actually a very useful framework for investigating relationships among algebraic, geometric or topological objects, see [6].

On the other hand, we have gotten many results about limits and colimits, especially limits and colimits of some special categories. For example, Bergman studied

This research was supported by the NSFC (Grant No. 11871404) and the Middle-Age Teachers of Fujian Province (Grant No. JT180290 and FBJG20180021).

DOI: 10.21136/CMJ.2020.0240-19

the relationships between limits in a G-set and a fixed point set, and gave the condition for the fixed point of a G-set and limit being exchanged in [7]. Fuchs and Göbel in [11] researched a direct system and colimits in module categories. As we know, the colimit category is an extension of category. Xue and Yan in [20] studied some propositions on a colimit category, proving that such a colimit category of cocomplete abelian category is a cocomplete abelian category and a class of colimit categories which is equivalent to module categories. In contrast, the recollements of colimit categories are still in their infancy. As illustrated by [4], it started from recollements of functor categories on different levels. Nevertheless, it is also worth presenting some applications, including recollements of triangular matrix rings and derived categories. They get the recollement of triangulated categories by constructing two stable t-structures in categories and applying Miyachi's result, see [17]. Although the colimit category is just the functor category, we can discover a recollement of abelian categories by constructing six functors in details and the conclusion exactly, which will be shown in the following.

The aim of this paper is to study the recollement of a cocomplete abelian category $\mathcal{A}$ and its colimit category. Note that we consider the colimits of $\mathcal{A}$ being defined on the I-system.

Theorem 1.1. *Let $\mathcal{A}$ be a cocomplete abelian category, $\mathcal{A}^l$ be the colimit category of $\mathcal{A}$. There is a recollement of $\mathcal{A}^l$ relative to $\mathcal{A}^c$ and $\mathcal{A}$ being visualized by the diagram*

$$\begin{tikzcd} \mathcal{A}^c \arrow[r, "i_*=i_!"] & \mathcal{A}^l \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=i^*"] & \mathcal{A}. \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd}$$

Here $\mathcal{A}^c$ is a full subcategory of $\mathcal{A}^l$ with the poset I having a maximal element n such that the direct system A_i $(i \in I)$ has $A_n = 0$.

This paper is organized as follows. In Section 2, we briefly recall definitions and basic facts on the recollement and colimit category. We state and prove our main result in Section 3. However, before starting our proof, we first consider the left recollement of $\mathcal{A}^l$ relative to $\mathcal{A}^c$ and $\mathcal{A}$, and its dual theorem. Then we get the main result as an immediate consequence of them. In Section 4, by applying the main theorem, we obtain some results on Leavitt path algebras, derived equivalence and K_i groups.

In this section, we recall some definitions, notations and basic results of the recollement and colimit category. First, we recall the notations of recollements of abelian categories which are closely related to our proofs (we refer to [2] for details).

Definition 2.1. Let $\mathcal{A}$, $\mathcal{A}'$ and $\mathcal{A}''$ be abelian categories. There is a *recollement* of $\mathcal{A}$ relative to $\mathcal{A}'$ and $\mathcal{A}''$, diagrammatically expressed by

$$\begin{tikzcd} \mathcal{A}' \arrow[r, "i_*=i_!"] & \mathcal{A} \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=i^*"] & \mathcal{A}'' \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd}$$

and given by six additive functors

$$i_* = i_!\colon\ \mathcal{A}' \to \mathcal{A}; \quad j^* = j^!\colon\ \mathcal{A} \to \mathcal{A}''; \quad i^*, i^!\colon\ \mathcal{A} \to \mathcal{A}'; \quad j_*, j_!\colon\ \mathcal{A}'' \to \mathcal{A}$$

that satisfy the conditions:

(1) (i^*, i_*), $(i_!, i^!)$, $(j_!, j^!)$ and (j^*, j_*) are adjoint pairs.

(2) The functors i_*, $j_!$ and j_* are full, faithfull.

(3) $\operatorname{Ker} j^! = \operatorname{Im} i_*$.

Two weaker forms of recollements are introduced by Parshall and Scott, see [18]:

A left recollement is said to hold if the upper two rows of the recollement (as defined above) exist and the functors appearing in these two rows,

$$\begin{tikzcd} \mathcal{A}' \arrow[r, shift right, "i_*"'] & \mathcal{A} \arrow[l, shift right, "i^*"'] \arrow[r, shift left, "j_!"] & \mathcal{A}'', \arrow[l, shift left, "j^!"] \end{tikzcd}$$

satisfy all the conditions in the definition above which involve only these functors.

A right recollement is defined via the lower rows similarly.

Note that a recollement can be seen as the gluing of a left recollement and a right recollement, and if a left recollement and a right recollement satisfy that $i_* = i_!$ and $j^! = j^*$ then they can be glued to form a recollement.

Then we recall the definitions of a colimit category (see [16] in detail).

Let A_i $(i \in I)$ be a direct system of category $\mathcal{A}$, i.e., I is a poset for $i \leqslant j$, there is a morphism $\phi^i_j\colon\ A_i \to A_j$ which is compatible with the ordering (called I-system for simplicity). Denote it by $(A_i, \phi^i_j, A', \alpha_i)_{i\in I}$ or $(A_i, \phi^i_j, A', \alpha_i)$.

Let $\mathcal{A}$ be a category, $\mathbb{A} = (A_i, \phi^i_j, A', \alpha_i)$ and $\mathbb{B} = (B_i, \psi^i_j, B', \beta_i)$ two colimits in $\mathcal{A}$. A pair $\{f_i, f'\}$ is said to be a colimit morphism from $\mathbb{A}$ to $\mathbb{B}$, denoted by $f^l = \{f_i, f'\}$, if $f_i \in \operatorname{Hom}_{\mathcal{A}}(A_i, B_i)$, $f' \in \operatorname{Hom}_{\mathcal{A}}(A', B')$ are such that $\psi^i_j \cdot f_i = f_j \cdot \phi^i_j, \beta_i \cdot f_i = f' \cdot \alpha_i$.

Because $f' \in \operatorname{Hom}_{\mathcal{A}}(A', B')$ is uniquely determinant by $\{f_i\}$, then f' is called the *induced morphism* of $\{f_i\}$ and denoted by $f^l = \{f_i\}$.

Definition 2.2. Let $\mathcal{A}^l$ denote the class of all colimits in $\mathcal{A}$, we change this into a category by defining a morphism from the colimit $(A_i, \phi^i_j, A', \alpha_i)$ to the colimit $(B_i, \psi^i_j, B', \beta_i)$ in $\mathcal{A}$ as a family of morphisms $f_i \in \operatorname{Hom}_{\mathcal{A}}(A_i, B_i)$, $f' \in \operatorname{Hom}_{\mathcal{A}}(A', B')$ such that $\psi^i_j \cdot f_i = f_j \cdot \phi^i_j, \beta_i \cdot f_i = f' \cdot \alpha_i$. The composition defined by $(gf)_i = g_i \cdot f_i, (gf)' = g' \cdot f'$, is clearly associative. The category $\mathcal{A}^l$ is said to be the *colimit category*.

3. Main theorem

In this section, the index set I is a poset which has a maximal element, we denote it by n. Let $\mathcal{A}$ be a cocomplete abelian category, denote by $\mathcal{A}^l$ the colimit category of $\mathcal{A}$, and let $\mathcal{A}^c$ be the full subcategory of $\mathcal{A}^l$. Its definition is as follows:

$$\operatorname{obj}\mathcal{A}^c\colon\ \mathbb{A}^c = (A_i, \phi^i_j, A', \alpha_i) \in \operatorname{obj}\mathcal{A}^l, \quad \text{where } A_n = 0,$$
$$\operatorname{Hom}\mathcal{A}^c = \operatorname{Hom}\mathcal{A}^l, \ \text{and the exact sequences in } \mathcal{A}^c \text{ and } \mathcal{A}^l \text{ coincide.}$$

Since n is the maximal element in the poset I, $I \setminus \{n\}$ is a poset, we denote it by I_1. The entry A_i, $i \in I_1$, is a direct system of the category $\mathcal{A}$, hence the colimit of $\{A_i, \phi^i_j\}_{i \leqslant j \in I_1}$ denoted by $\mathcal{A}^l_1$ and $\mathcal{A}^l_1 \cong \mathcal{A}^c$. By [20], Theorem 3.2, $\mathcal{A}^c$ is a cocomplete abelian category.

By constructing a recollement of $\mathcal{A}^l$ relative to $\mathcal{A}^c$ and $\mathcal{A}$, we have the following consequence.

Theorem 3.1. *Let $\mathcal{A}$, $\mathcal{A}^l$, $\mathcal{A}^c$ be defined as above. There is a recollement of $\mathcal{A}^l$ relative to $\mathcal{A}^c$ and $\mathcal{A}$ being visualized by the diagram*

$$\begin{tikzcd}
\mathcal{A}^c \arrow[r, "i_*=i_!" description] & \mathcal{A}^l \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=i^*" description] & \mathcal{A}. \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"]
\end{tikzcd}$$

P r o o f. By the work of Parshall and Scott, we prove the theorem in two parts.

Part I: There is a left recollement of $\mathcal{A}^l$ relative to $\mathcal{A}^c$ and $\mathcal{A}$, diagrammatically expressed by

$$\begin{tikzcd}
\mathcal{A}^c \arrow[r, shift right, "i_*"'] & \mathcal{A}^l \arrow[l, shift right, "i^*"'] \arrow[r, shift left, "j_!"] & \mathcal{A}. \arrow[l, shift left, "j^!"]
\end{tikzcd}$$

Part II: There is a right recollement of $\mathcal{A}$ relative to $\mathcal{A}^l$ and $\mathcal{A}^c$, diagrammatically expressed by

$$\begin{tikzcd}
\mathcal{A}^c \arrow[r, shift left, "i_!"] & \mathcal{A}^l \arrow[l, shift left, "i^!"] \arrow[r, shift left, "j^*"] & \mathcal{A}. \arrow[l, shift left, "j_*"]
\end{tikzcd}$$

We prove only Part I. Apparently, it is the same to prove Part II.

(1) Define

$$j_!\colon \mathcal{A} \to \mathcal{A}^l,\ A \mapsto \mathbb{A}_n = (A_i, \phi^i_j, A', \alpha_i), \quad \text{where } A_i = \begin{cases} 0, & i \neq n, \\ A, & i = n, \end{cases}$$

and

$$f\colon A \to B \mapsto f^l = \{f_i, f\}\colon \mathbb{A}_n \to \mathbb{B}_n, \quad \text{where } f_i = \begin{cases} 0, & i \neq n, \\ f, & i = n, \end{cases}$$

then $j_!$ is full, faithfull and exact.

Indeed, it is easy to prove that $j_!$ is full and faithfull. Let $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ be exactly in $\mathcal{A}$. By the process of the proof Theorem 3.2 in [20], $0 \to \mathbb{A}_n \xrightarrow{f^l} \mathbb{B}_n \xrightarrow{g^l} \mathbb{C}_n \to 0$ is exactly in $\mathcal{A}^l$. Hence, $j_!$ is an exact functor.

(2) Define $j^!\colon \mathcal{A}^l \to \mathcal{A}$, $\mathbb{A} = (A_i, \phi^i_j, A', \alpha_i) \mapsto A_n$, $f^l = \{f_i, f'\}\colon \mathbb{A} = (A_i, \phi^i_j, A', \alpha_i) \to \mathbb{B} = (B_i, \psi^i_j, B', \beta_i) \mapsto f_n\colon A_n \to B_n$. Naturally $j^!$ is a functor. It is similar to (1) to prove that $j^!$ is exact.

(3) $(j_!, j^!)$ is an adjoint pair. Indeed, for $A \in \operatorname{obj}\mathcal{A}$ and $\mathbb{B} = (B_i, \psi^i_j, B', \beta_i) \in \operatorname{obj}\mathcal{A}^l$, define

$$\begin{aligned} \sigma_{A,\mathbb{B}}\colon \operatorname{Hom}_{\mathcal{A}^l}(j_!A, \mathbb{B}) &\to \operatorname{Hom}_{\mathcal{A}}(A, j^!\mathbb{B}), \\ f^l = \{f_i, f'\}\colon j_!A \to \mathbb{B} &\mapsto f_n\colon A \to B_n. \end{aligned}$$

It is easy to prove that $\sigma_{A,\mathbb{B}}$ is a group homomorphism. And by the definition of $j_!A$, $f^l = \{f_i, f'\} \in \operatorname{Hom}_{\mathcal{A}^l}(j_!A, \mathbb{B})$, that is, $f_i = 0$ (for all $i \neq n$) and $f' = \beta_n f_n$. So $\sigma_{A,\mathbb{B}}$ is a group isomorphism and we assert that σ is natural.

In fact, for $\widetilde{A} \in \operatorname{obj}\mathcal{A}$ and $h\colon A \to \widetilde{A}$, note that

$$\begin{aligned} (j_!h)^* &= \operatorname{Hom}_{\mathcal{A}^l}(-, \mathbb{B})(j_!h)\colon m^l \mapsto m^l(j_!h), \\ (h)^* &= \operatorname{Hom}_{\mathcal{A}}(-, j^!\mathbb{B})(h)\colon u \mapsto uh. \end{aligned}$$

For any $g^l = \{g_i, \beta_n g_n\} \in \operatorname{Hom}_{\mathcal{A}^l}(j_!\widetilde{A}, \mathbb{B})$, where $g_i = 0$ for all $i \neq n$, we have

$$\sigma_{A,\mathbb{B}}(j_!h)^*(g^l) = \sigma_{A,\mathbb{B}}(g^l(j_!h)) = g_n h = (h)^*(g_n) = (h)^*\sigma_{\widetilde{A},\mathbb{B}}(g^l).$$

So $\sigma_{A,\mathbb{B}}(j_!h)^* = (h)^*\sigma_{\widetilde{A},\mathbb{B}}$ with the commutative diagram

$$\begin{tikzcd} A \arrow[d, "h"'] & \operatorname{Hom}_{\mathcal{A}^l}(j_!\widetilde{A}, \mathbb{B}) \arrow[r, "\sigma_{\tilde{A},\mathbb{B}}"] \arrow[d, "(j_!h)^*"'] & \operatorname{Hom}_{\mathcal{A}}(\mathbb{A}, j^!\mathbb{B}) \arrow[d, "(h)^*"] \\ \widetilde{A} & \operatorname{Hom}_{\mathcal{A}^l}(j_!A, \mathbb{B}) \arrow[r, "\sigma_{A,\mathbb{B}}"'] & \operatorname{Hom}_{\mathcal{A}}(A, j^!\mathbb{B}) \end{tikzcd}$$

For $\mathbb{C} = (C_i, \varphi_j^i, C', \gamma_i) \in \operatorname{obj} \mathcal{A}^l$ and $h^l = \{h_i, h'\}\colon\ \mathbb{B} = (B_i, \psi_j^i, B', \beta_i) \to \mathbb{C} = (C_i, \varphi_j^i, C', \gamma_i)$. Put

$$(h^l)_* = \operatorname{Hom}_{\mathcal{A}^l}(j_! A, -)(h^l)\colon\ s^l \mapsto h^l s^l,$$
$$(j^! h^l)_* = \operatorname{Hom}_{\mathcal{A}}(A, -)(j^! h^l)\colon\ t \mapsto (j^! h^l) t.$$

Similarly to the above, we see that

$$\begin{tikzcd}
\mathbb{B} \arrow[d, "h^l"'] & \operatorname{Hom}_{\mathcal{A}^l}(j_! A, \mathbb{B}) \arrow[r, "\sigma_{A,\mathbb{B}}"] \arrow[d, "(h^l)_*"'] & \operatorname{Hom}_{\mathcal{A}}(A, j^! \mathbb{B}) \arrow[d, "(j^! h^l)_*"] \\
\mathbb{C} & \operatorname{Hom}_{\mathcal{A}^l}(j_! A, \mathbb{C}) \arrow[r, "\sigma_{A,\mathbb{C}}"'] & \operatorname{Hom}_{\mathcal{A}}(A, j^! \mathbb{C}).
\end{tikzcd}$$

So $(j_!, j^!)$ is an adjoint pair.

(4) Let $i_*\colon\ \mathcal{A}^c \to \mathcal{A}^l$, $\mathbb{A}^c = (A_i, \phi_j^i, A', \alpha_i) \mapsto \mathbb{A}^c = (A_i, \phi_j^i, A', \alpha_i)$, $f^l \mapsto f^l$. By the definition of $\mathcal{A}^c$, we see that i_* is full, faithfull and exact.

(5) For any $\mathbb{A} = (A_i, \phi_j^i, A', \alpha_i) \in \operatorname{obj} \mathcal{A}^l$, because $\mathcal{A}$ is a cocomplete abelian category, the colimit of the direct system $\{A_i^c, \widetilde{\phi}_j^i\}$, where

$$A_i^c = \begin{cases} A_i, & i \neq n, \\ 0, & i = n, \end{cases} \qquad \widetilde{\phi}_j^i = \begin{cases} \phi_j^i, & j \neq n, \\ 0, & j = n, \end{cases}$$

exists. We denote it by $\{\widetilde{A}', \widetilde{\alpha}_i\}$, and then $\mathbb{A}^c = (A_i^c, \widetilde{\phi}_j^i, \widetilde{A}', \widetilde{\alpha}_i) \in \operatorname{obj} \mathcal{A}^c$.

For any $\mathbb{A} = (A_i, \phi_j^i, A', \alpha_i)$, $\mathbb{B} = (B_i, \psi_j^i, B', \beta_i) \in \operatorname{obj} \mathcal{A}^l$ and $f^l = \{f_i, f'\}\colon\ \mathbb{A} \to \mathbb{B}$, by the definition of colimit for any $i \leqslant j \in I$, $f_j \phi_j^i = \psi_j^i f_i$ $(j \neq n)$, and $0 \cdot 0 = 0 \cdot f_i$, by [10], Proposition 4, there exists a unique $\tilde{f}' \in \operatorname{Hom}_{\mathcal{A}}(\widetilde{A}', \widetilde{B}')$, such that $\tilde{f}^l = \{\tilde{f}_i, \tilde{f}'\} \in \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}^c, \mathbb{B}^c)$, where

$$\tilde{f}_i = \begin{cases} f_i, & i \neq n, \\ 0, & i = n, \end{cases} \qquad \mathbb{B}^c = (B_i^c, \widetilde{\psi}_j^i, \widetilde{B}', \widetilde{\beta}_i),$$
$$B_i^c = \begin{cases} B_i, & i \neq n, \\ 0, & i = n, \end{cases} \qquad \widetilde{\psi}_j^i = \begin{cases} \psi_j^i, & j \neq n, \\ 0, & j = n. \end{cases}$$

From what has been discussed above, we define the functor i^* as follows

$$i^*\colon\ \mathcal{A}^l \to \mathcal{A}^c, \quad \mathbb{A} = (A_i, \phi_j^i, A', \alpha_i) \mapsto \mathbb{A}^c = (A_i^c, \widetilde{\phi}_j^i, \widetilde{A}', \widetilde{\alpha}_i),$$
$$f^l = \{f_i, f'\}\colon\ \mathbb{A} \to \mathbb{B} \mapsto \tilde{f}^l = \{\tilde{f}_i, \tilde{f}'\}\colon\ \mathbb{A}^c \to \mathbb{B}^c.$$

It is easy to prove that i^* is an exact functor.

(6) (i^*, i_*) is an adjoint pair. In fact, for any $\mathbb{A} = (A_i, \phi_j^i, A', \alpha_i) \in \operatorname{obj} \mathcal{A}^l$, $\mathbb{B}^c = (B_i, \psi_j^i, B', \beta_i) \in \operatorname{obj} \mathcal{A}^c$, $f^l = \{f_i, f'\} \in \operatorname{Hom}_{\mathcal{A}^c}(i^*\mathbb{A}, \mathbb{B}^c)$, it holds $f_n = 0$ and $f_j \phi_j^i = \psi_j^i f_i$ (for all $i \leqslant j$). By [20], Proposition 4, there exists a unique $\bar{f}'\colon A' \to B'$ such that

$$\bar{f}^l = \{\bar{f}_i, \bar{f}'\} \in \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}, i_*\mathbb{B}^c), \quad \text{where } \bar{f}_i = \begin{cases} f_i, & i \neq n, \\ 0, & i = n. \end{cases}$$

Let

$$\begin{aligned} &\eta_{\mathbb{A},\mathbb{B}^c}\colon \operatorname{Hom}_{\mathcal{A}^c}(i^*\mathbb{A}, \mathbb{B}^c) \to \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}, i_*\mathbb{B}^c), \\ &f^l = \{f_i, f'\} \mapsto \bar{f}^l = \{\bar{f}_i, \bar{f}'\}. \end{aligned}$$

Naturally, $\eta_{\mathbb{A},\mathbb{B}^c}$ is a group homomorphism. Moreover, $\eta_{\mathbb{A},\mathbb{B}^c}$ is an isomorphism. Next we will prove η to be natural.

For any $\mathbb{C} = (C_i, \varphi_j^i, C', \gamma_i) \in \operatorname{obj} \mathcal{A}^c$, $g^l = \{g_i, g'\}\colon \mathbb{B}^c \to \mathbb{C}$, it holds $g_n = 0$. Put

$$\begin{aligned} (g^l)_* &= \operatorname{Hom}_{\mathcal{A}^c}(i^*\mathbb{A}, -)(g^l)\colon f^l \mapsto g^l f^l, \\ (i_*g^l)_* &= \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}, -)(i_*g^l)\colon h^l \mapsto g^l h^l. \end{aligned}$$

For any $f^l = \{f_i, f'\} \in \operatorname{Hom}_{\mathcal{A}^c}(i^*\mathbb{A}, \mathbb{B}^c)$, we get

$$\eta_{\mathbb{A},\mathbb{C}^c}(g^l)_*(f^l) = \eta_{\mathbb{A},\mathbb{C}^c}(g^l f^l) = \{\overline{g_i f_i}, \overline{g' f'}\}, \quad \text{where } \overline{g_i f_i} = \begin{cases} g_i f_i, & i \neq n, \\ 0, & i = n, \end{cases}$$

$$(i_*g^l)_*\eta_{\mathbb{A},\mathbb{B}^c}(f^l) = (i_*g^l)_*(\bar{f}^l) = \{g_i\bar{f}_i, g'\bar{f}'\}, \quad \text{where } g_i\bar{f}_i = \begin{cases} g_i f_i, & i \neq n, \\ 0, & i = n. \end{cases}$$

Then $\{\overline{g_i f_i}, \overline{g' f'}\} = \{g_i\bar{f}_i, g'\bar{f}'\}$, that is, $\eta_{\mathbb{A},\mathbb{C}^c}(g^l)_* = (i_*g^l)_*\eta_{\mathbb{A},\mathbb{B}^c}$, diagrammatically expressed by

$$\begin{array}{ccc} \mathbb{B}^c & \quad & \operatorname{Hom}_{\mathcal{A}^c}(i^*\mathbb{A}, \mathbb{B}^c) \xrightarrow{\eta_{\mathbb{A},\mathbb{B}^c}} \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}, i_*\mathbb{B}^c) \\ {\scriptstyle g^l}\downarrow & & {\scriptstyle (g^l)_*}\downarrow \qquad\qquad\qquad\qquad \downarrow{\scriptstyle (i_*g^l)_*} \\ \mathbb{C}^c & & \operatorname{Hom}_{\mathcal{A}^c}(\mathbb{A}, i_*\mathbb{C}^c) \xrightarrow[\eta_{\mathbb{A},\mathbb{C}^c}]{} \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}, i_*\mathbb{C}^c). \end{array}$$

For any $\mathbb{D} = (D_i, \varrho_j^i, D', d_i) \in \operatorname{obj} \mathcal{A}^l$ and $h^l = \{h_i, h'\} \in \operatorname{Hom}_{\mathcal{A}^l}(\mathbb{A}, \mathbb{D})$, we put

$$\begin{aligned} (i^*h^l)^* &= \operatorname{Hom}_{\mathcal{A}^c}(-, \mathbb{B}^c)(i^*h^l)\colon s^l \mapsto s^l(i^*h^l), \\ (h^l)^* &= \operatorname{Hom}_{\mathcal{A}^l}(-, i_*\mathbb{B}^c)(h^l)\colon t^l \mapsto t^l h^l. \end{aligned}$$

The commutative diagram follows:

$$\begin{tikzcd}
\mathbb{A} \arrow[d, "h^l"'] & \mathrm{Hom}_{\mathcal{A}^c}(i^*\mathbb{D}, \mathbb{B}^c) \arrow[r, "\eta_{\mathbb{D},\mathbb{B}^c}"] \arrow[d, "(i^*h^l)^*"'] & \mathrm{Hom}_{\mathcal{A}^l}(\mathbb{D}, i_*\mathbb{B}^c) \arrow[d, "(h^l)^*"] \\
\mathbb{D} & \mathrm{Hom}_{\mathcal{A}^c}(i^*\mathbb{A}, \mathbb{B}^c) \arrow[r, "\eta_{\mathbb{A},\mathbb{B}^c}"'] & \mathrm{Hom}_{\mathcal{A}^l}(\mathbb{A}, i_*\mathbb{B}^c).
\end{tikzcd}$$

So (i^*, i_*) is an adjoint pair.

(7) By the definition of $j^!$ and i_*, we can have $\ker j^! = \mathrm{Im}\, i_*$. Combining these parts yields the result. $\square$

If $\mathcal{A}$ is an abelian category, $\mathcal{A}$ is finitely cocomplete. Then the colimit of any finite I-system in $\mathcal{A}$ exists.

Corollary 3.2. *Let $\mathcal{A}$ be an abelian category and $\mathcal{A}^l$ denote the finitely colimit category. Then there is a recollement of $\mathcal{A}^l$ relative to $\mathcal{A}^c$ and $\mathcal{A}^l$,*

$$\begin{tikzcd}
\mathcal{A}^c \arrow[r, "i_*=i_!"] & \mathcal{A}^l \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=j^*"] & \mathcal{A}. \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"]
\end{tikzcd}$$

4. Applications

In this section, we mention three further applications. Furthermore, let K be a field, A be a finite dimensional algebra over K. The symbols $\mathrm{Mod}\, A$ and $\mathrm{Mod}\, B$ denote the right module categories over A and B, respectively. The symbols $\mathrm{mod}\, A$ and $\mathrm{mod}\, B$ denote the finitely right module categories over A and B, respectively.

Firstly, we relate the recollements of module categories to one-point coextension algebras, see [5].

Definition 4.1. Let A be an algebra, $X \in \mathrm{Mod}\, A$, and let each right module X can be viewed as a (K, A)-bimodule. Form the matrix algebra

$$[X]A = \begin{bmatrix} K & 0 \\ DX & A \end{bmatrix},$$

where $DX = \mathrm{Hom}_K({}_K X_A, K)$. It is called a *one-point coextension of A by X*, denoted by $[X]A$.

Let $\mathcal{A}$ be $\mathrm{Mod}\, K$, $\mathcal{A}^l$ be the colimit category in $\mathcal{A}$ over quiver $Q_{[X]A}$, then $\mathcal{A}^l \cong \mathrm{Mod}\, [X]A$, $\mathcal{A}^c \cong \mathrm{Mod}\, A$ (see [9], Theorem 4.3). By applying Theorem 3.1, we get the following result.

Proposition 4.2. *Let A be a K algebra. Then there is a recollement of* $\operatorname{Mod}[X]A$ *relative to* $\operatorname{Mod} A$ *and* $\operatorname{Mod} K$

$$\begin{tikzcd} \operatorname{Mod} A \arrow[r, "i_*=i_!"] & \operatorname{Mod}[X]A \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=j^*"] & \operatorname{Mod} K. \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd}$$

Triangular matrix algebras are generally kinds of one-point coextensions. Next, we show that a special Leavitt path algebra is an isomorphism to a triangular matrix algebra.

We briefly recall some graph-theoretic definitions and properties; more complex explanations and descriptions can be found in [1]. Throughout this section, the directed graph Q is finite and acyclic. $Q = (Q^0, Q^1, r, s)$ consists of two sets Q^0 and Q^1 together with maps $r, s\colon Q^1 \to Q^0$. The elements of Q^0 are called *vertices* and the elements of Q^1 *edges*. We define a relation $\geqslant$ on Q^0 by setting $v \geqslant w$ if there exists a path in Q from v to w. A subset H of Q^0 is called *hereditary* if $v \geqslant w$ and $v \in H$ implies $w \in H$.

For a hereditary saturated subset H of Q^0, the *quotient graph* Q/H is defined as

$$(Q^0 \setminus H,\ \{e \in Q^1\colon\ r(e) \text{ is not in } H,\ r|_{(Q/H)},\ s|_{(Q/H)}\})$$

and the *restriction graph* Q_H is

$$(H,\ \{e \in Q^1\colon\ s(e) \in H,\ r|_{(Q_H)},\ s|_{(Q_H)}\}).$$

We include some notation here. For a graph Q and a field K, $L_K(Q)$ is the Leavitt path algebra (see [1]). Given two algebras B, C and a (C, B)-bimodule M define a triangular matrix algebra, denoted by (B, C, M).

Then we get that the Leavitt path algebra of a special graph is isomorphic to a triangular matrix algebra.

Theorem 4.3. *Let Q be a directed graph, H be a hereditary saturated subset of Q^0. Let Q be decomposed as $Q_H \cup Q/H \cup \{\alpha\colon\ s(\alpha) \in (Q/H)^0,\ r(\alpha) \in (Q_H)^0\}$. Then*

$$L_K(Q) \cong \begin{pmatrix} L_K(Q/H) & M \\ 0 & L_K(Q_H) \end{pmatrix}$$

as algebras.

P r o o f. Firstly, we can construct an $L_K(Q/H)$-$L_K(Q_H)$-bimodule M. Let $M = \langle \alpha\colon\ s(\alpha) \in (Q/H)^0,\ r(\alpha) \in (Q_H)^0 \rangle$. We put

$$\begin{aligned} L_K(Q/H) \times M \to M, &\quad (pq^*, m) \mapsto qp^*pq^*m,\\ M \times L_K(Q/H) \to M, &\quad (m, pq^*) \mapsto mpq^*qp^*. \end{aligned}$$

It is easy to prove that M is an $L_K(Q/H)$-$L_K(Q_H)$-bimodule.

Secondly, we define a map

$$\phi\colon\ L_K(Q) \to \begin{pmatrix} L_K(Q/H) & M \\ 0 & L_K(Q_H) \end{pmatrix}, \quad v_{Q/H} \mapsto \begin{pmatrix} v_{Q/H} & 0 \\ 0 & 0 \end{pmatrix},$$

$$v_{Q_H} \mapsto \begin{pmatrix} 0 & 0 \\ 0 & v_{Q_H} \end{pmatrix}, \quad e_{Q/H} \mapsto \begin{pmatrix} e_{Q/H} & 0 \\ 0 & 0 \end{pmatrix}, \quad e^*_{Q/H} \mapsto \begin{pmatrix} e^*_{Q/H} & 0 \\ 0 & 0 \end{pmatrix},$$

$$e_{Q_H} \mapsto \begin{pmatrix} 0 & 0 \\ 0 & e_{Q_H} \end{pmatrix}, \quad e^*_{Q_H} \mapsto \begin{pmatrix} 0 & 0 \\ 0 & e^*_{Q_H} \end{pmatrix}, \quad \alpha \mapsto \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix},$$

where $\alpha, s(\alpha) \in (Q/H)^0$, $r(\alpha) \in (Q_H)^0$.

It is easy to prove that ϕ is an isomorphism. Since

$$Q = Q_H \cup Q/H \cup \{\alpha\colon\ s(\alpha) \in (Q/H)^0,\ r(\alpha) \in (Q_H)^0\},$$

then

$$L_K(Q) \cong \begin{pmatrix} L_K(Q/H) & M \\ 0 & L_K(Q_H) \end{pmatrix}.$$

□

Using Theorem 4.3 and the characterisation of triangular matrix algebras of Li, see [15], we can deduce the following theorem.

Proposition 4.4. *Let Q be a directed graph, H be a hereditary saturated subset of Q^0. Let Q be decomposed as $Q_H \cup Q/H \cup \{\alpha\colon\ s(\alpha) \in (Q/H)^0, r(\alpha) \in (Q_H)^0\}$. There is a recollement of* $\operatorname{mod} L_K(Q)$ *by* $\operatorname{mod} L_K(Q_H)$ *and* $\operatorname{mod} L_K(Q/H)$,

$$\begin{tikzcd} \operatorname{mod} L_K(Q/H) \arrow[r, "i_*=i_!" description] & \operatorname{mod} L_K(Q) \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=j^*" description] & \operatorname{mod} L_K(Q_H). \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd}$$

Proposition 4.5. *Let Q be a directed graph, H be a hereditary saturated subset of Q^0, and Q can be decomposed as $Q_H \cup Q/H \cup \{\alpha\colon s(\alpha) \in (Q/H)^0, r(\alpha) \in (Q_H)^0\}$. Then there is a recollement of $\mathrm{D}^b(L_K(Q))$ by $\mathrm{D}^b(L_K(Q_H))$ and $\mathrm{D}^b(L_K(Q/H))$ which is diagrammatically expressed as*

$$\begin{tikzcd}[column sep=huge] \mathrm{D}^b(L_K(Q/H)) \arrow[r, "i_*=i_!" description] & \mathrm{D}^b(L_K(Q)) \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!=j^*" description] & \mathrm{D}^b(L_K(Q_H)). \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd}$$

where $\mathbb{D}^b(L_K(Q))$, $\mathbb{D}^b(L_K(Q_H))$ and $\mathbb{D}^b(L_K(Q/H))$ are derived categories of Leavitt path algebras.

P r o o f. The global dimension of the Leavitt path algebra is less than or equal to 1 (see [3], Theorem 3.5), with respect to Theorem 2 of Chen (see [8]) and Theorem 4.3, we get this statement immediately. □

Secondly, using Theorem 3.1, we can get the sufficient conditions for two finite colimit categories to be a derived equivalence.

As we know, when the directed graph Q is finite and A is a finite dimensional K algebra, then AQ/J_A can be constructed by a one point coextension applied finite many times (where J_A is an admissible ideal of AQ).

Theorem 4.6. *Let A and B be two finite dimensional K algebras. If A and B are a derived equivalence, then finite colimit categories $(\mathrm{mod}\ A)^l$ and $(\mathrm{mod}\ B)^l$ are a derived equivalence.*

P r o o f. Barot and Lenzing in [5] have proved a similar theorem for one point extensions. As we know, the extension and coextension are dual definitions. So it stands for one point coextension still. By [9], Theorem 4.3, $\mathrm{Mod}\,(AQ/J_Q) = (\mathrm{Mod}\,A)^l$, the result is true. □

Thirdly, we apply the main theorem to K_i groups (refer to [19]). We recall some propositions about recollements, see [10].

Proposition 4.7. *Let $\mathcal{A}$, $\mathcal{A}^c$ and $\mathcal{A}^l$ be defined as above. There is a recollement of $\mathcal{A}^l$ relative to $\mathcal{A}$ and $\mathcal{A}^c$. Then*

(1) $i^*j_! = 0$, $i^!j_* = 0$, $i^*i_* \cong \mathrm{id}_{\mathcal{A}^l} \cong i^!i_*$, $j^!j_* \cong \mathrm{id}_{\mathcal{A}} \cong j^*j_!$;

(2) i^*, $j_!$ *are right exact functors,* $i^!$ *and* j_* *are left exact functors, and* i_*, $j^!$ *are exact functors.*

Proposition 4.8. *Let $\mathcal{A}$, $\mathcal{A}^c$ and $\mathcal{A}^l$ by defined as above. There is a recollement of $\mathcal{A}^l$ relative to $\mathcal{A}$ and $\mathcal{A}^c$. Then $K_i(\mathcal{A}^l) \cong K_i(\mathcal{A}^c) \oplus K_i(\mathcal{A})$.*

P r o o f. $K_i(\mathcal{A}^c)\oplus K_i(\mathcal{A})$ is a summand of $K_i(\mathcal{A}^l)$. Indeed, i^* and j^* are exact functors and they induce two abelian group homomorphisms

$$\varphi_1 = K_i(i^*)\colon\ K_i(\mathcal{A}^l)\to K_i(\mathcal{A}^c),\quad \varphi_2 = K_i(j^*)\colon\ K_i(\mathcal{A}^l)\to K_i(\mathcal{A}).$$

By the universal property of direct sum,

$$\overline{\varphi} = \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}\colon\ K_i(\mathcal{A}^l)\to K_i(\mathcal{A}^c)\oplus K_i(\mathcal{A}).$$

Similarly, we get an abelian homomorphism

$$\overline{\psi} = (\,\psi_1\quad \psi_2\,)\colon\ K_i(\mathcal{A}^c)\oplus K_i(\mathcal{A})\to K_i(\mathcal{A}^l),$$

and combining it with $j^*j_!\cong \mathrm{id}_{\mathcal{A}}$, $i^*i_!\cong \mathrm{id}_{\mathcal{A}^l}$ and $j^*i_! = 0 = i^*j_!$, we have

$$\overline{\varphi}\overline{\psi} = \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}(\psi_1\psi_2) = \begin{pmatrix}\mathrm{id}_{K_i(\mathcal{A}^c)} & 0\\ 0 & \mathrm{id}_{K_i(\mathcal{A})}\end{pmatrix}.$$

Hence, $K_i(\mathcal{A}^c)\oplus K_i(\mathcal{A})$ is a summand of $K_i(\mathcal{A}^l)$.

For $\mathbb{A} = (A_i,\phi_j^i,A',\alpha_i)\in \operatorname{obj}\mathcal{A}^l$, by the definition of i_*, i^*, $j^!, j_!$,

$$j_!j^!\mathbb{A} = \widehat{\mathbb{A}} = (\widehat{A}_i,\widehat{\phi}_j^i,A_n,\widehat{\alpha}_i),\quad \text{where } \widehat{A}_i = \begin{cases}0, & i\neq n,\\ A_i, & i=n,\end{cases}$$

$$i_*i^*\mathbb{A} = (A_i^c,\widetilde{\phi}_j^i,\widetilde{A}',\widetilde{\alpha}_i),\quad \text{where } A_i^c = \begin{cases}A_i, & i\neq n,\\ 0, & i=n,\end{cases}\quad\text{and}\quad \widetilde{\phi}_j^i = \begin{cases}\phi_j^i, & j\neq n,\\ 0, & j=n.\end{cases}$$

Then we get a natural monomorphism in $\mathcal{A}^l$, that is,

$$f^l = \{f_i,f'\}\colon\ j_!j^!\mathbb{A}\to\mathbb{A},\quad \text{where } f_i = \begin{cases}0, & i\neq n,\\ 1_{A_n}, & i=n,\end{cases}$$

a natural epimorphism

$$g^l = \{g_i,g'\}\colon\ \mathbb{A}\to i_*i^*\mathbb{A},\quad \text{where } g_i = \begin{cases}1_{A_i}, & i\neq n,\\ 0, & i=n,\end{cases}$$

and a coker $f^l = g^l$ (see [9], Theorem 1.6).

Above all, we have an exact sequence in colimit-category $\mathcal{A}^l$,

$$0\longrightarrow j_!j^!\mathbb{A}\xrightarrow{f^l}\mathbb{A}\xrightarrow{g^l} i_*i^*\mathbb{A}\longrightarrow 0.$$

Moreover,

$$0\longrightarrow j_!j^!\longrightarrow \mathrm{id}_{\mathcal{A}^l}\longrightarrow i_*i^*\longrightarrow 0.$$

By Theorem 3.1, $K_i(\mathcal{A}^l)\cong K_i(\mathcal{A}^c)\oplus K_i(\mathcal{A})$. $\square$

Corollary 4.9. *Let $\mathcal{A}$ be an abelian category. Then $K_i(\mathcal{A}^l)\cong K_i(\mathcal{A}^c)\oplus K_i(\mathcal{A})$.*

Note that when I is a finite poset, there exists a maximal element n and in $I_1 = I \setminus \{n\}$ there also exists a maximal element n_1. Similarly, define $\mathcal{A}^c$, $\mathcal{A}_1^l$ the full subcategory of $\mathcal{A}_1^c$, then by Proposition 4.5, $K_i(\mathcal{A}_1^l) \cong K_i(\mathcal{A}_1^c) \oplus K_i(\mathcal{A})$. Since $|I|$ is finite, denoted by $|I| = m$, we can repeat the above-mentioned process and get the following results.

Corollary 4.10. *Let $\mathcal{A}$ be an abelian category. Let $\mathcal{A}^l$ be a colimit category indexed by an index set I and $|I| = m$. Then $K_i(\mathcal{A}^l) \cong K_i^m(\mathcal{A})$.*

Example 4.11. Let $\mathcal{A}$ be an abelian group category. Consider the direct system $A_i = \mathbb{Z}$, $\phi_j^{j+1}(1) = j$. Then $\mathcal{A}^l$ is an isomorphism to $\mathbb{Q}$, therefore $K_0(\varinjlim A_i) = \mathbb{Z}$.

Combining this with Proposition 4.2, we get the following result.

Proposition 4.12. *Let A be a K algebra and $[X]A$ a one point coextension of A. Then $K_i(\operatorname{Mod}[X]A) \cong K_i(\operatorname{Mod} A) \oplus K_i(\operatorname{Mod} K)$.*

Example 4.13. Let Q be $A_n\colon 1 \to 2 \to 3 \to \ldots \to n-1 \to n$, where $\mathcal{A}$ is a module category over a field K, the element in poset I coincide with the vertices set of the quiver Q, $\mathcal{A}^l$ be the colimit category in $\mathcal{A}$, $K_0(KQ) = \mathbb{Z}^n$, hence $K_0(\mathcal{A}^l) = \mathbb{Z}^n$.

Example 4.14. Suppose KQ is a finite dimensional path algebra, $\mathcal{A}$ is a module category over K, and I is a poset associated with Q. Then

$$K_1(\mathcal{A}^l) = K_1(KQ) = \begin{cases} (K^\times)^n, & K \neq \{0,1\}, \\ G_2^m, & K = \{0,1\}, \end{cases}$$

where $n = |Q^0|$, $m = |Q^1|$, $K^\times$ is the multiplication group of K, G_2 is the multiplication group $\{-1, 1\}$, see [13].

Acknowledgement. The authors would like to thank the referee for careful reading the manuscript and many useful hints and comments that improved the exposition.

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