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EXPONENTIAL STABILITY VIA APERIODICALLY INTERMITTENT CONTROL OF COMPLEX-VARIABLE TIME DELAYED CHAOTIC SYSTEMS

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This paper focuses on the problem of exponential stability analysis of uncertain complex-variable time delayed chaotic systems, where the parameters perturbation are bounded assumed. The aperiodically intermittent control strategy is proposed to stabilize the complex-variable delayed systems. By taking the advantage of Lyapunov method in complex field and utilizing inequality technology, some sufficient conditions are derived to ensure the stability of uncertain complex-variable delayed systems, where the constrained time delay are considered in the conditions obtained. To protrude the availability of the devised stability scheme, simulation examples are ultimately demonstrated.

Keywords: complex-variable system, delayed, uncertain, stability, aperiodically intermittent control

Classification: 34D06, 34D35, 34C15

1. INTRODUCTION

Stability of chaotic systems has been extensively investigated for the past decades because of its importance in various fields([2, 20, 21, 23]). At the same time, many control strategies have been developed to regulate the stability or synchronization behavior, such as active control ([16]), adaptive control ([31]), pinning control ([3]), sliding-mode control ([15]), impulsive control([29]) and intermittent control ([32]). Different control schemes have different advantages. Especially, intermittent control with work time and rest time emerging alternately has been extensively used in realizing stability and synchronization of chaotic systems, for example, see ([1, 4, 8, 10, 11, 12, 22, 24]) and references therein. In this type of control strategy, each period usually contains two types of time, one is work time where the controller is activated, and the other one is rest time where the controller is off. When rest time is zero, the intermittent control reduces to continuous control, while work time is zero means that the intermittent control becomes the impulsive control. Obviously, compared with the continuous control methods, intermittent control is more economical and efficient because the system output is measured intermittently rather than continuously.

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From the literature, there exist two kinds of intermittent control schemes: periodically intermittent control([1, 8, 10, 11, 24, 16]) and aperiodically intermittent control ([13, 14]). In periodically intermittent control ([1, 8, 16, 10, 11, 24]), the control period, the control time as well as the control rate was assumed to be constants. This requirement of periodicity in intermittent control is unreasonable, which can unavoidably lead to some conservatism in practical applications. In many practical applications, each work time in intermittent control is needed to be changeable and therefore adjusted in accordance with actual situations. Aperiodically intermittent control is less conservative and more practical in the real world. In the structure of aperiodically intermittent control, work time and rest time are aperiodic. Thus, periodically intermittent control is a special case of aperiodically intermittent control. From these points of view, exploitation of aperiodically intermittent control is more reasonable and necessary. There are few results about synchronization via aperiodically intermittent control. Synchronization for the linearly coupled network was investigated by pinning a simple aperiodically intermittent controller in ([13]). The work ([14]) studied quasi-synchronization for nonlinear coupled chaotic systems via aperiodically intermittent pinning control.

It is worthwhile to note that most of the existing results in relation to stability and synchronization problems under intermittent control focus on chaotic systems and complex dynamical networks of coupled dynamical systems with real-variables ([1, 8, 10, 11, 13, 14, 16, 24]), and ignore the study of complex-variable systems. However, as indicated in ([5, 6, 7, 9, 17, 18, 25, 26, 27, 28, 30]), complex- variable systems have played an important role in the description of various physical phenomena, such as detuned laser systems and thermal convection of liquid flows ([7, 19])since the complex Lorenz system was found in 1982. Afterwards, some works have been done on the stability and synchronization problem for complex-variable chaotic systems and coupled complex-variable dynamical systems, see Refs. ([5, 6, 9, 17, 18, 25, 26, 27, 28, 30]) and the references therein. The stabilization of uncertain complex-variable delayed systems ([30]) by using intermittent control methods with multiple switched periods was investigated. Different forms of modified time delay hyperchaotic complex Lü system were constructed by including the delay. And the active control method based on Lyapunov–Krasovskii function was used to synchronize the hyperchaotic attractors ([17]).Furthermore, taking the control cost and practical implementation into account, it is of great importance to study the stability analysis of time delay complex system by using aperiodically intermittent control. However, to the best of our knowledge, most of existing works are dealing with the problem of synchronization and stability in complex-variable delayed systems with continuous control ([9, 17, 18]). There is little analytical results about the stability of complex-variable systems in the form of aperiodically intermittent control scheme. Therefore, how to design effective aperiodically intermittent controller for this class of complex-variable systems is an important topic and deserves further studies. This is the main motivation of this paper.

Based on the above discussions, the aperiodically intermittent control problem has been primary issue, there are still more rooms for improvement, which motivates our research. This paper further investigates the stability of uncertain complex-variable delayed systems by using aperiodically intermittent control. By taking the advantage of Lyapunov method in complex field and utilizing inequality technology, some sufficient conditions are derived analytically. Numerical simulations are also given to show the effectiveness of the theoretical results. The main contributions in this paper are given as follows: (1) We overcome the difficulty of uncertain factor with parameters perturbation; (2) The stability of uncertain complex-variable delayed system is studied under aperiodically intermittent control, which extends the scope of practical applications of intermittent control strategy. (3) The derived theoretical results indicate that the globally exponential stability criteria depend on the constrained time delay $\overline{\tau}$.

The rest of this paper is organized as follows. Section 2 gives some necessary preliminaries. In section 3, stability criteria for stability are derived analytically via the aperiodically intermittent control designed. In Section 4, numerical examples are given to illustrate the theoretical results. Finally some conclusions are given in Section 5.

The notations used throughout this paper are fairly standard. For any complex number (or complex vector) x, the notations x^r and x^i denote its real and imaginary parts, respective, and $\overline{x}$ denotes the complex conjugate of x. The norm of any complex vector x is $||x|| = \sqrt{x^T\overline{x}}$, sup denotes the upper bound. Denote $\lambda_{max}(A)$ and $\lambda_{min}(A)$ as the maximal and minimal eigenvalues of matrix A respectively. A^T and $\overline{A}$ denote its transpose and its conjugate, respectively. $||A|| = \sqrt{A^T\overline{A}}$. $H \in \mathcal{H}^{n\times n}$ denotes the set of $n \times n$ Hermite matrices. Let $C([-\overline{\tau}, 0], \mathcal{C}^n)$ be a Banach space of continuous with the norm $||\varphi|| = \sup_{-\overline{\tau}\leq\sigma\leq 0} ||\varphi(\sigma)||$. Denote $[\varphi(t)]_{\overline{\tau}} = ([\varphi_1(t)]_{\overline{\tau}}, [\varphi_2(t)]_{\overline{\tau}}, \cdots, [\varphi_n(t)]_{\overline{\tau}})$, $[\varphi_k(t)]_{\overline{\tau}} = \sup_{-\overline{\tau}\leq\sigma\leq 0} ||\varphi_k(t+\sigma)||$. I is a unit matrix with appropriate dimension.

2. MODEL DESCRIPTION AND PRELIMINARIES

We consider the following form of the complex-variable delayed dynamical system with parameters perturbation

$$\begin{cases} \dot{x} = (A + \Delta A)x + (B + \Delta B)x(t - \tau(t)) \\ \qquad + f(t, x) + g(t, x(t - \tau(t))), \\ x(t_0 + s) = \phi(s) \in C([-\overline{\tau}, 0], C^n), \end{cases} \tag{1}$$

where $x(t) = (x_1, x_2, \cdots, x_n)^T \in \mathcal{C}^n$ is a n-dimensional state complex vector with $x_l = x_l^r + jx_l^i, l = 1, 2, \cdots, n$ and $j = \sqrt{-1}$, superscripts r and i stand for the real and imaginary parts of the state complex vector x, respectively. $A, B \in \mathcal{C}^{n\times n}$, $\Delta A \in \mathcal{C}^{n\times n}$ and $\Delta B \in \mathcal{C}^{n\times n}$ parameters perturbation matrices bounded by $\Delta A^T\overline{A} \leq \alpha I$ and $\Delta B^T\overline{B} \leq \beta I$, and α, $\beta > 0$. $f(\cdot,\cdot), g(\cdot,\cdot)$:$[0, +\infty) \times \mathcal{C}^n \to \mathcal{C}^n$ are continuous nonlinear function vectors, $\tau(t)$ is continuous functions with $0 \leq \tau(t) \leq \overline{\tau}$ ($\overline{\tau}$ is a constant).

Before stating our main results, we give some necessary assumption, definition and lemmas, which are useful in deriving stabilization criteria. Throughout this paper, we always assume that the complex vector-variable functions $f(t, x)$ and $g(t, x(t - \tau(t)))$ satisfy the following Assumption 2.1, that is,

Assumption 2.1. Suppose that there exist two positive constants l_f and l_g such that the complex-variable vector functions $f(t, x)$ and $g(t, x(t - \tau(t)))$ satisfy

$$||f(t, x) - f(t, y)|| \leq l_f||x - y||,$$

$$||g(t, x(t - \tau(t))) - g(t, y(t - \tau(t)))|| \leq l_g||x(t - \tau(t)) - y(t - \tau(t))||.$$

Definition 2.2. If there exist $\varepsilon > 0$ and $M > 0$ such that for any $t \geq 0$, $||x(t)|| \leq M \sup_{-\overline{\tau} \leq \theta \leq 0} ||x(\theta)|| \exp(-\varepsilon t)$, then the system (1) is said to be exponentially stable.

Lemma 2.3. (Fang and Sun [5]) For any $X, Y \in \mathcal{C}^n$ and constant $\zeta > 0$ if $H \in \mathcal{H}^{n\times n}$ is a positive definite matrix, then $X^T H\overline{Y} + Y^T H\overline{X} \leq \zeta X^T H\overline{X} + \zeta^{-1} Y^T H\overline{Y}$.

Lemma 2.4. (Li et al. [10]) Let $x : [t_0 - \overline{\tau}, +\infty) \to [0, +\infty)$ be a continuous function such that $\dot{x}(t) \leq -ax(t) + b \max x_t$ is satisfied for $\forall t \geq t_0$. If $a > b > 0$ then $x(t) \leq \max x_{t_0} \exp(-\gamma(t-t_0)), t \geq 0$, where $\max x_t = \sup_{t-\overline{\tau}\leq\xi\leq t} x(\xi)$ and $\gamma > 0$ is the smallest real root of equation $a - \gamma - b\exp(\gamma\overline{\tau}) = 0$.

Lemma 2.5. (Li et al. [10]) Let $x : [t_0 - \overline{\tau}, +\infty) \to [0, +\infty)$ be a continuous function such that $\dot{x}(t) \leq ax(t) + b \max x_t$ is satisfied for $\forall t \geq t_0$. If $a > 0, b > 0$ then $x(t) \leq \max x_t \leq \max x_{t_0} \exp((a + b)(t - t_0)), t \geq 0$, where $\max x_t = \sup_{t-\overline{\tau}\leq\xi\leq t} x(\xi)$.

3. MAIN RESULTS

To achieve the stability, the controlled system of (1) can be described by the following form

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)x(t - \tau(t)) + f(t, x) + g(t, x(t - \tau(t))) + u(t) \tag{2}$$

where $u(t)$ is a linear state feedback controller with aperiodically intermittent control designed later.

For $m = 0, 1, 2, \cdots$, the aperiodically intermittent controller is defined as follows

$$u(t) = \begin{cases} Kx, & t \in [t_m, s_m), \\ 0, & t \in [s_m, t_{m+1}). \end{cases} \tag{3}$$

Here, $K \in \mathcal{C}^{n\times n}$ is a control gain matrix.

Theorem 3.1. Suppose that Assumption 2.1 holds and the following conditions are satisfied

(i) $\Omega_1 = A^T H + H\overline{A} + \zeta_1 l_f^2 I + \zeta_4\alpha I + \zeta_1^{-1}H^2 + \zeta_2^{-1}H^2 + \zeta_3^{-1}H^2$
$+\zeta_4^{-1}H^2 + \zeta_5^{-1}H^2 - H\overline{K} - K^T H + g_1 H \leq 0,$

(ii) $\Omega_2 = A^T H + H\overline{A} + \zeta_1 l_f^2 I + \zeta_4\alpha I + \zeta_1^{-1}H^2 + \zeta_2^{-1}H^2 + \zeta_3^{-1}H^2$
$+\zeta_4^{-1}H^2 + \zeta_5^{-1}H^2 - g_3 H \leq 0,$

(iii) $\Omega_3 = \zeta_3 B^T\overline{B} + \zeta_2 l_g^2 I + \zeta_5\beta I - g_2 H \leq 0,$

(iv) $(g_3 + g_2)(t_{m+1} - s_m) - \gamma(s_m - \overline{\tau} - t_m) \leq \Delta, m = 0, 1, 2, \cdots,$

where $g_1 > g_2 > 0, g_3 > 0$. $\gamma > 0$ is the smallest real root of equation $g_1 - \gamma - g_2 \exp(\gamma\overline{\tau}) = 0$. Δ is an arbitrarily small positive number. Then the aperiodically intermittent controlled complex-variable delayed chaotic system is globally exponentially stable at origin.

Proof. Select the following Lyapunov type function defined as

$$V(t,x) = x^T H\overline{x}. \tag{4}$$

When $t \in [t_m, s_m)$, then, the derivative of (4) along the trajectories of (2) and Assumption 2.1, we can obtain

$$\begin{aligned}
\dot{V}(t) &= \dot{x}^T H\overline{x} + x^T H\dot{\overline{x}} \\
&= [(A+\Delta A)x + (B+\Delta B)x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau(t))) + u(t)]^T H\overline{x} \\
&\quad + x^T H\overline{[(A+\Delta A)x + (B+\Delta B)x(t-\tau(t)) + f(t,x) + g(t,x(t-\tau(t))) + u(t)]} \\
&= x^T(A^T H + H\overline{A})\overline{x} + x^T(t-\tau(t))B^T H\overline{x} + x^T H\overline{Bx(t-\tau(t))} \\
&\quad + (\Delta Ax)^T H\overline{x} + x^T H\overline{(\Delta Ax)} \\
&\quad + (\Delta Bx(t-\tau(t)))^T H\overline{x} + x^T H\overline{(\Delta Bx(t-\tau(t)))} \\
&\quad + g^T(t,x(t-\tau(t)))H\overline{x} + x^T H\overline{g(t,x(t-\tau(t)))} \\
&\quad + f^T(t,x)H\overline{x} + x^T H\overline{f(t,x)} + u(t)^T H\overline{x} + x^T H\overline{u(t)}.
\end{aligned}$$

From Assumption 2.1 and Lemma 2.3, we have

$$\begin{aligned}
&f(t,x)^T H\overline{x} + x^T H\overline{f(t,x)} \\
&\qquad \le \zeta_1 f(t,x)^T\overline{f(t,x)} + \zeta_1^{-1}x^T H^2\overline{x} \\
&\qquad \le \zeta_1 x^T l_f^2 I\overline{x} + \zeta_1^{-1}x^T H^2\overline{x},
\end{aligned} \tag{5}$$

$$\begin{aligned}
&g^T(t,x(t-\tau(t)))H\overline{x} + x^T H\overline{g(t,x(t-\tau(t)))} \\
&\qquad \le \zeta_2 g^T(t,x(t-\tau(t)))\overline{g(t,x(t-\tau(t)))} + \zeta_2^{-1}x^T H^2\overline{x} \\
&\qquad \le \zeta_2 x(t-\tau(t))^T l_g^2 I\overline{x(t-\tau(t))} + \zeta_2^{-1}x^T H^2\overline{x},
\end{aligned} \tag{6}$$

$$\begin{aligned}
&x^T(t-\tau)B^T H\overline{x} + x^T H\overline{Bx(t-\tau(t))} \\
&\qquad \le \zeta_3 x(t-\tau(t))^T B^T\overline{Bx(t-\tau(t))} + \zeta_3^{-1}x^T H^2\overline{x},
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\Delta A^T H + H\overline{\Delta A} \\
&\qquad \le \zeta_4(\Delta A)^T\overline{(\Delta A)} + \zeta_4^{-1}H^T\overline{H} \\
&\qquad \le \zeta_4\alpha I + \zeta_4^{-1}H^2,
\end{aligned} \tag{8}$$

$$\begin{aligned}
&(\Delta Bx(t-\tau(t)))^T H\overline{x} + x^T H\overline{(\Delta Bx(t-\tau(t)))} \\
&\qquad \le \zeta_5 x^T(t-\tau(t))\beta I\overline{x(t-\tau(t))} + \zeta_5^{-1}x^T H^2\overline{x}.
\end{aligned} \tag{9}$$

Thus, we obtain

$$
\begin{aligned}
\dot{V}(t) \le{}& x^T\Big[A^TH + H\overline{A} + \zeta_1 l_f^2 I + \zeta_4\alpha I + \zeta_1^{-1}H^2 + \zeta_2^{-1}H^2 + \zeta_3^{-1}H^2 \\
& + \zeta_4^{-1}H^2 + \zeta_5^{-1}H^2 - H\overline{K} - K^TH + g_1H\Big]\overline{x} - g_1x^TH\overline{x} \\
& + x^T(t-\tau(t))\Big[\zeta_3B^T\overline{B} + \zeta_2 l_g^2 I + \zeta_5\beta I - g_2H\Big]\overline{x(t-\tau(t))} \\
& + g_2x^T(t-\tau(t))H\overline{x(t-\tau(t))} \\
={}& x^T\Omega_1\overline{x} - g_1V(t) + x^T\Omega_3\overline{x} + g_2V(t-\tau(t)) \\
\le{}& -g_1V(t) + g_2V(t-\tau(t))
\end{aligned}
\tag{10}
$$

where $\Omega_1 \le 0$ and $\Omega_3 \le 0$.

On the other hand, when $t \in [s_m, t_{m+1})$, similarly, we can derive

$$
\begin{aligned}
\dot{V}(t) &= x^T\Omega_2\overline{x} + g_3V(t) + x^T\Omega_3\overline{x} + g_2V(t-\tau(t)) \\
&\le g_3V(t) + g_2V(t-\tau(t))
\end{aligned}
\tag{11}
$$

where $\Omega_2 \le 0$ and $\Omega_3 \le 0$.

Thus, we have

$$
\begin{cases}
\dot{V}(t) \le -g_1V(t) + g_2V(t-\tau(t)), & t \in [t_m, s_m), \\
\dot{V}(t) \le g_3V(t) + g_2V(t-\tau(t)), & t \in [s_m, t_{m+1}).
\end{cases}
\tag{12}
$$

When $t \in [t_m, s_m)$ from (12), we have

$$
\begin{aligned}
\dot{V}(t) &\le -g_1V(t) + g_2V(t-\tau(t)) \\
&\le -g_1V(t) + g_2\overline{V}(t_m)
\end{aligned}
$$

where $\overline{V}(t_m) = \max_{t_m-\overline{\tau}\le\xi\le t_m} V(\xi)$.

From lemma 2.4, $V(t)$ satisfies

$$
V(t) \le \overline{V}(t_m)\exp(-\gamma(t-t_m)),
\tag{13}
$$

here $\gamma > 0$ is the smallest real root of equation $g_1 - \gamma - g_2\exp(\gamma\tau) = 0$.

When $t \in [s_m, t_{m+1})$ from (12), we have

$$
\begin{aligned}
\dot{V}(t) &\le g_3V(t) + g_2V(t-\tau(t)) \\
&\le g_3V(t) + g_2\overline{V}(s_m)
\end{aligned}
$$

where $\overline{V}(s_m) = \max_{s_m-\overline{\tau}\le\xi\le s_m} V(\xi)$.

From lemma 2.5, $V(t)$ satisfies

$$
V(t) \le \overline{V}(s_m)\exp\Big[(g_3+g_2)(t-s_m)\Big].
\tag{14}
$$

Now, based on above Eqs.(12)-(14), we estimate $V(t)$.

When $m = 0$, the following inequalities can be derived.
For $t \in [t_0, s_0)$, using (13)

$$V(t) \leq \overline{V}(t_0)\exp(-\gamma(t-t_0)). \tag{15}$$

For $t \in [s_0, t_1)$, using (14)

$$\begin{aligned} V(t) &\leq \overline{V}(s_0)\exp[(g_3+g_2)(t-s_0)] \\ &= \Big[\max_{s_0-\overline{\tau}\leq\xi\leq s_0} V(\xi)\Big]\exp[(g_3+g_2)(t-s_0)] \\ &\leq \overline{V}(t_0)\exp[-\gamma(s_0-\overline{\tau}-t_0)]\exp[(g_3+g_2)(t-s_0)] \\ &= \overline{V}(t_0)\exp[(g_3+g_2)(t-s_0)-\gamma(s_0-\overline{\tau}-t_0)] \\ &\leq \overline{V}(t_0)\exp[(g_3+g_2)(t_1-s_0)-\gamma(s_0-\overline{\tau}-t_0)]. \end{aligned} \tag{16}$$

For $t \in [t_1, s_1)$,

$$\begin{aligned} V(t) &\leq \Big[\max_{t_1-\overline{\tau}\leq\xi\leq t_1} V(\xi)\Big]\exp[-\gamma(t-t_1)] \\ &\leq \overline{V}(t_0)\exp[(g_3+g_2)(t_1-s_0)-\gamma(t-t_1+s_0-\overline{\tau}-t_0)]. \end{aligned} \tag{17}$$

For $t \in [s_1, t_2)$,

$$\begin{aligned} V(t) &\leq \Big[\max_{s_1-\overline{\tau}\leq\xi\leq s_1} V(\xi)\Big]\exp[(g_3+g_2)(t-s_1)] \\ &\leq \overline{V}(t_0)\exp[(g_3+g_2)(t_2-s_1+t_1-s_0) \\ &\quad -\gamma(s_1-\overline{\tau}-t_1+s_0-\overline{\tau}-t_0)]. \end{aligned} \tag{18}$$

For $t \in [t_2, s_2)$,

$$\begin{aligned} V(t) &\leq \Big[\max_{t_2-\overline{\tau}\leq\xi\leq t_2} V(\xi)\Big]\exp[-\gamma(t-t_2)] \\ &\leq \overline{V}(t_0)\exp[(g_3+g_2)(t_2-s_1+t_1-s_0) \\ &\quad -\gamma(s_1-\overline{\tau}-t_1+s_0-\overline{\tau}-t_0)]\exp[-\gamma(t-t_2)] \\ &= \overline{V}(t_0)\exp[(g_3+g_2)(t_2-s_1+t_1-s_0) \\ &\quad -\gamma(t-t_2+s_1-\overline{\tau}-t_1+s_0-\overline{\tau}-t_0)]. \end{aligned} \tag{19}$$

For $t \in [s_2, t_3)$,

$$\begin{aligned} V(t) &\leq \Big[\max_{s_2-\overline{\tau}\leq\xi\leq s_2} V(\xi)\Big]\exp[(g_3+g_2)(t-s_2)] \\ &\leq \overline{V}(t_0)\exp[(g_3+g_2)(t_3-s_2+t_2-s_1+t_1-s_0) \\ &\quad -\gamma(s_2-\overline{\tau}-t_2+s_1-\overline{\tau}-t_1+s_0-\overline{\tau}-t_0)]. \end{aligned} \tag{20}$$

Similarly, we have

For $t \in [t_m, s_m)$,

$$V(t) \leq \overline{V}(t_0)\exp[(g_3+g_2)(t_m - s_{m-1} + \cdots + t_2 - s_1 + t_1 - s_0) \\ - \gamma(s_{m-1} - \overline{\tau} - t_{m-1} + \cdots + s_1 - \overline{\tau} - t_1 + s_0 - \overline{\tau} - t_0)]. \tag{21}$$

For $t \in [s_m, t_{m+1})$,

$$V(t) \leq \overline{V}(t_0)\exp[(g_3+g_2)(t_{m+1} - s_m + \cdots + t_2 - s_1 + t_1 - s_0) \\ - \gamma(s_m - \overline{\tau} - t_m + \cdots + s_1 - \overline{\tau} - t_1 + s_0 - \overline{\tau} - t_0)]. \tag{22}$$

From the condition in the Thereom 3.1, we obtain

$$\begin{aligned}&(g_3+g_2)(t_{m+1} - s_m + \cdots + t_2 - s_1 + t_1 - s_0)\\ &\quad - \gamma(s_m - \overline{\tau} - t_m + \cdots + s_1 - \overline{\tau} - t_1 + s_0 - \overline{\tau} - t_0)\\ &\leq -(m+1)\Delta,\end{aligned}$$

then, we have

$$V(t) \leq \overline{V}(t_0)\exp[-(m+1)\Delta]. \tag{23}$$

Then, one has

$$||x|| \leq \sqrt{\frac{\overline{V}(t_0)}{\lambda_{\min}(H)}}\exp(-(m+1)\frac{\Delta}{2}t). \tag{24}$$

Therefore, the stabilization of the complex-variable chaotic delayed system (1) is realized. The proof is completed. □

Furthermore, we let $H = I, K = kI$ and $\zeta_i = 1 (i = 1, 2, \cdots, 5)$, so $g_1 = 2k - \lambda_{\max}(A^T + \overline{A}) - l_f^2 - \alpha - 5$, $g_2 = l_g^2 + \lambda_{\max}(B^T\overline{B}) + \beta$, $g_3 = 2k - g_1$. Then, from the Theorem 3.1, we have the following corollary.

Corollary 3.2. Given Δ and $\overline{\tau}$, if there exist control gain k and positive γ such that $g_1 = \gamma + g_2\exp(\gamma\overline{\tau})$ and $(g_3+g_2)(t_{m+1} - s_m) - \gamma(s_m - \overline{\tau} - t_m) \leq -\Delta,\ m = 0, 1, 2, \cdots$, then the aperiodically intermittent controlled complex-variable chaotic delayed system is exponentially stable at origin.

Remark 3.3. From the Theorem 3.1 and Corollary 3.2, we can find that the condition $(g_3+g_2)(t_{m+1} - s_m) - \gamma(s_m - \overline{\tau} - t_m) \leq -\Delta$ could be replaced by the following equality

$$\lim_{m\to\infty}[(g_3+g_2)(t_{m+1} - s_m) - \gamma(s_m - \overline{\tau} - t_m)] = -\infty.$$

Remark 3.4. In ([5, 6, 7, 9, 16, 17, 19, 25, 26, 27, 28, 29, 30]), the stability and synchronization problem for complex-variable systems was addressed. Compared with them, this paper has studied stability of complex-variable delayed systems with parameters perturbation. Meanwhile, the designed intermittent controllers can be aperiodic. Moreover, the constrained time delay is considered in the obtained conditions, that is, we take the time-delay influence into account. Obviously, our results contain the works of stability of complex chaotic systems which studied by using periodically intermittent control. In addition, it should be pointed out that these results ([5, 6, 7, 9, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30])

were obtained based on some novel research methods, which inspires us to do some further investigations, such as stability of complex-variable systems with stochastic perturbation.

4. NUMERICAL SIMULATIONS

In this subsection, the illustrative examples are provided to show the effectiveness of the stability of delayed complex systems with parameters perturbation. We consider the following delayed complex Lorenz system

$$\begin{cases} \dot{x}_1 = a_1(x_2 - x_1) + x_1(t - \overline{\tau}) \\ \dot{x}_2 = a_2 x_2 + x_1 x_3 + x_2(t - \overline{\tau}) \\ \dot{x}_3 = \dfrac{1}{2}(\overline{x_1} x_2 + x_1 \overline{x_2}) - a_3 x_3 + x_3(t - \overline{\tau}). \end{cases} \tag{25}$$

When the parameters $a_1 = 20, a_2 = 12, a_3 = 5, \overline{\tau} = 0.01$, the delayed complex Lorenz system exhibits chaotic behavior (see Figure 1). $f(x) = (0, -x_1 x_3, \frac{1}{2}(\overline{x_1} x_2 + x_1 \overline{x_2}))^T$, $g(x(t-\tau(t))) = 0$,$B = I_3$ and $A = \begin{pmatrix} -a_1 & a_1 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & -a_3 \end{pmatrix}$.

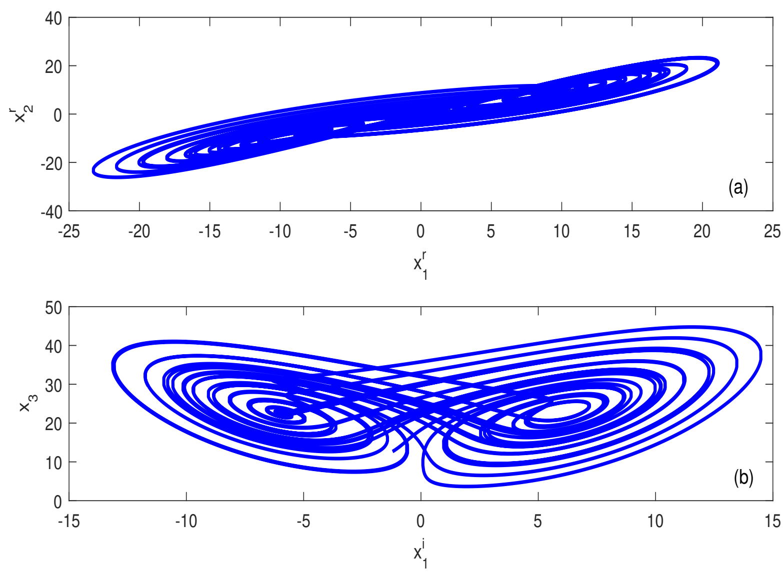

Fig. 1. Phase trajectory of time delayed chaotic system (25).

According to Ref. ([5]) one has $l_f = 0$ and $l_g = 0$, so that Assumption 2.1 holds. Take the parameters perturbation matrices as the following

$$\Delta A = \begin{pmatrix} -0.4 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -0.6 \end{pmatrix}, \ \Delta B = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0.5 \end{pmatrix},$$

it is obviously that $\Delta A^T \Delta A \leq 0.36I, \Delta B^T \Delta B \leq 1.5931I$. For simplicity, in the numerical simulations, the initial states of the complex-variable delayed system is $(2, 1-5j, 3-4j)$. Trajectories of real and imaginary parts of $x_i(t)$ without control are shown in Figure 2.

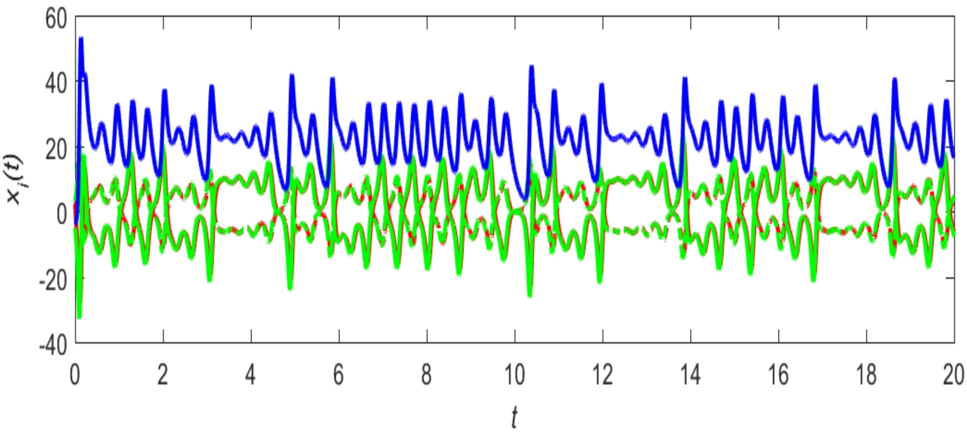

Fig. 2. Trajectories of real and imaginary parts of $x_i(t)$ without control.

Let $\alpha = 0.36$, $\beta = 1.5931$,$k = 20$, $\Delta = 0.01$. By solving conditions in Corollary 3.2, we can obtain $g_1 = 2k - \lambda_{\max}(A^T + \overline{A}) - l_f^2 - \alpha - 5 = 4.9041$, $g_2 = l_g^2 + \lambda_{\max}(B^T\overline{B}) + \beta = 2.5931$, $g_3 = 2k - g_1 = 35.0959$. The $\gamma = 2.2519$ is the unique positive solution of the equation $g_1 - \gamma - g_2 \exp(\gamma\overline{\tau}) = 0$, then we can set the work time interval with the following inequality

$$37.689(t_{m+1} - s_m) - 2.2519(s_m - t_m - 0.01) \leq -0.01, \forall m = 0, 1, 2, \cdots.$$

For example, when $s_m - t_m = 0.21$, one gets $t_{m+1} - s_m \leq 0.1168$; when $s_m - t_m = 1.01$, one gets $t_{m+1} - s_m \leq 0.5948$; when $s_m - t_m = 2.01$, one gets $t_{m+1} - s_m \leq 0.1192$; when $s_m - t_m = 3.01$, one gets $t_{m+1} - s_m \leq 0.1789$; when $s_m - t_m = 4.01$, one gets $t_{m+1} - s_m \leq 0.2381$; when $s_m - t_m = 5.11$, one gets $t_{m+1} - s_m \leq 0.3044$; ....

Based on the above analysis, the work time intervals are defined as
$[0, 0.21] \cup [0.31, 0.52] \cup [0.62, 1.63] \cup [2.22, 4.23] \cup [4.34, 7.35] \cup [7.52, 11.53] \cup [11.76, 16.87] \cup [17.17, 20.18] \cup \cdots$.

According to Corollary 3.2, all conditions are satisfied, so this system is robustly exponentially stable. This is verified by the simulation results shown in Figure 3. There results show that the stabilization has been achieved according to the intermittent control.

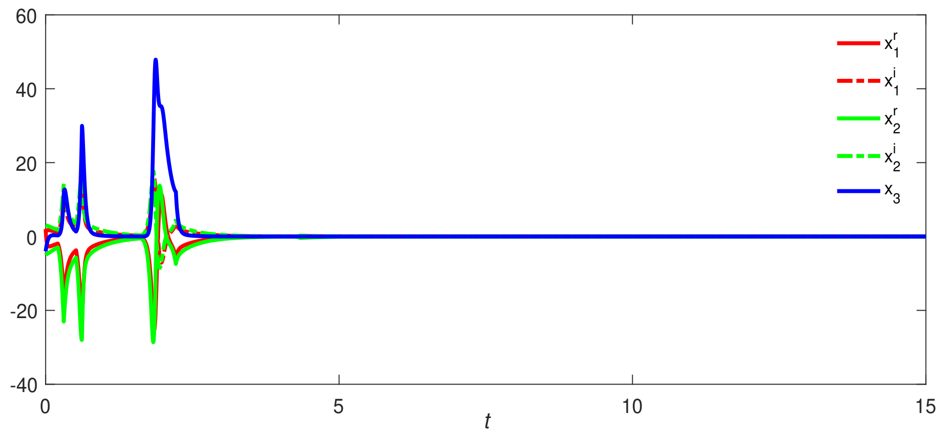

Fig. 3. Trajectories of real and imaginary parts of $x_i(t)$ with aperiodically intermittent control.

Remark 4.1. In this paper, we have investigated the stability problem of a class of complex -variable delayed systems with parameters perturbation via aperiodically intermittent control method. The robustness of the stability is mainly reflected in the resistance to external parameters perturbation. That is, for any unknown parameters perturbation satisfied with the bounded conditions, the proposed method remain valid and robustness. Additionally, our results can be also applied on the complex-variable delayed systems without parameters perturbation.

Remark 4.2. Recently, a common approach ([9, 16, 17, 18]) studying the stability and synchronization of complex-variable systems is to separate them into real parts and imaginary parts, and rewrite them as two equivalent real variable systems, then discuss the synchronization problem by use of the stability criteria of real systems, but it is very lengthy and complicated. However, in this paper, we directly discuss the stability problem of delayed complex-variable systems by constructing a positive definite function in the complex fields. That is, we study the stability of complex-variable systems, which does not require us to split the real and imaginary parts.

5. CONCLUSION

In this paper, the stability problem of uncertain complex-variable time delayed chaotic system has been investigated by using aperiodically intermittent. Sufficient conditions for stability are obtained based on the stability theory and inequality technique. Different from previous work, the time delay constrained was considered in our results. Moreover, we can design the work width and the rest width to achieve the stability of the uncertain complex-variable time delayed chaotic system. Finally, the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the suggested stability methodology. Note that, in this paper, the robustness of the stability is mainly reflected in the resistance to external parameters perturbation. To our knowledge, a real world system can be affected by external perturbation, like stochastic perturbation, theoretically, stability is not easy to achieve. Therefore, exponential stability of complex-variable delayed chaotic systems with stochastic perturbation via aperiodically intermittent control presents an important topic for our future research.

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