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PARAMETRIC CONTROL TO QUASI-LINEAR SYSTEMS BASED ON DYNAMIC COMPENSATOR AND MULTI-OBJECTIVE OPTIMIZATION

Da-Ke Gu and Da-Wei Zhang

This paper considers a parametric approach for quasi-linear systems by using dynamic compensator and multi-objective optimization. Based on the solutions of generalized Sylvester equations, we establish the more general parametric forms of dynamic compensator and the left and right closed-loop eigenvector matrices, and give two groups of arbitrary parameters. By using the parametric approach, the closed-loop system is converted into a linear constant one with a desired eigenstructure. Meanwhile, it also proposes a novel method to realize multi-objective design and optimization. Multiple performance objectives, containing overall eigenvalue sensitivity, H_2 norm, H_∞ norm and low compensation gain, are formulated by arbitrary parameters, then robustness and low compensation gain criteria are expressed by a comprehensive objective function which contains each performance index weighted. By utilizing degrees of freedom (DOFs) in arbitrary parameters, we can optimize the comprehensive objective function such that an optimized dynamic compensator is found to satisfy the robustness and low compensation gain criteria. Finally, an example of attitude control of combined spacecrafts is presented which proves the effectiveness and feasibility of the parametric approach.

Keywords: quasi-linear systems, parametric control, dynamic compensator, multi-objective design and optimization, utilize DOFs in parameter matrices

Classification: 93B60, 93B52, 93B51

1. INTRODUCTION

In the real-world, many practical nonlinear dynamical systems are quasi-linear, such as attitude control of combined spacecrafts [16], spacecraft rendezvous [7], chaotic systems synchronization [8], robotics [26] and other applications [20, 25], such that there has been attracting considerable research attention. For example, Knüppel and Woittennek present a class of feed-forward control for the model of heavy rope described by quasi-linear hyperbolic equations, through this method, the control problem can be transformed into a Cauchy problem *w.r.t.* space [14]. Rotondo *et al.* combine reference model approach with linear matrix inequalities to design a linear parameter-varying controller which is suitable for quasi-linear time-varying form, therefore, a trajectory tracking problem of a four wheeled omni-bearing robot is solved based on this technique

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[23]. Jadachowski *et al.* consider a design problem of observers for a type of quasi-linear parabolic partial differential equations, they propose an extended Luenberger observer based on backstepping method such that ensuring the boundary conditions of dynamic exponential decay when linearizing observer error [12]. Additionally, there are also other approaches to deal with quasi-linear systems [19, 21].

Unfortunately, there are several disadvantages for the above methods. Firstly, these above methods are invalid in time-varying nonlinear systems. Secondly, the closed-loop systems resulted in by these methods are generally nonlinear. Thirdly, the common technique of these methods is to design static output feedback on the basis of different control strategies, however static output feedback cannot arbitrarily configure all closed-loop poles. In the study, we design a dynamic compensator to solve these above mentioned drawbacks by a parametric approach, further, the DOFs can be increased because of the higher order offered by dynamic compensator such that it is benefit to realize multi-objective design and optimization.

Dynamic compensator, a class of dynamical output feedback, has attracted the great attention of scholars. For instance, Tang *et al.* utilize robust sliding control and linear quadratic optimal regulation to design dynamic compensator for an unmanned aerial vehicle quad-rotor [27]. For polynomial systems, Yuno and Ohtsuka provide a sufficient condition for existence of dynamic compensator and propose an exact algorithm to compute such a compensator [30]. Chen *et al.* realize the master-slave chaotic synchronization through dynamic compensator and give a sufficient condition to maintain the global synchronization such that the strictly positively real constraint is ignored [2]. Tsuzuki and Yamashita implement a global asymptotic stabilization on a Riemannian manifold by using a dynamic compensator and a global Lyapunov function for input-affine systems [28]. Moreover, some others researches also give effective approaches to design dynamic compensator [1, 13, 24]. However, on the one hand, the performances of dynamic compensator of these above methods are limited because of utilizing the original state vector rather than the additional compensation vector. On the other hand, these methods lead into a large and complicated computation load such that the design process is inflexible and it is difficult to realize performances optimization. Noteworthy, the proposed parametric approach can deal with these drawbacks effectively and has been successfully used to design dynamic compensator and implement multi-objective optimization for linear time-varying systems [9, 10].

Note that optimization has become an essential problem to be urgently solved when implementing the basic requirements of control systems. In practice, multi-objective control problems are difficult and remain mostly open to this date. Therefore, a large number of works have been devoted to multi-objective optimization methods. For example, Lim et al. consider a novel surrogate-assisted multi-objective optimization algorithm to optimize the torque amplitude, torque ripple, and magnet usage simultaneously for interior permanent magnet synchronous motor, which can reduce the noise, vibration, and cost [17]. Hashem *et al.* present a divide and conquer technique to realize multi-objective optimization based on the solution to the Pareto front, under this method, a more intuitive and more high-speed procedure can be obtained to handle conflict design objectives for a plug flow reactor [11]. Zhou *et al.* transform multi-task multi-view problem into a multi-objective optimization problem and present a cooperative multi-objective quantum-behaved particle swarm optimization algorithm, which is better than other machine-learning algorithms, to solve the multi-objective optimization problem [32]. For more see [29, 33], the most notable fact is that variables to be optimized in the above methods possess physical meanings such that it can be only optimized in a given region, that is, local optimal solution. However, in this paper, these variables to be optimized are arbitrary parameters provided by the proposed parametric approach, which have no physical meanings, therefore, the optimized interval is greatly expanded such that it is a benefit to find a globally optimal solution.

This research investigates a parametric approach for quasi-linear systems by using dynamic compensator and multi-objective optimization. Inspiring by the solutions of generalized Sylvester equations [4, 31], parametric approach transforms system design into determining a matrix Λ which contains closed-loop eigenvalues. Moreover, this approach gives a more complete parametric expressions of the left and right eigenvector matrices with a group of arbitrary parameters. Then, the generally parametric form of dynamic compensator is developed which consists of the left and right eigenvector matrices and the matrix Λ. Further, it also considers a multi-objective optimization problem. By utilizing the DOFs in arbitrary parameters to optimize a synthetic objective function, an approximately global optimal solution can be obtained to deal with conflicts among design objectives and to reduce the difficulties when implementing dynamic compensator.

The main contributions of the proposed work focus on three aspects. Firstly, the presented work proposes a parametric approach which can convert the closed-loop system into a linear constant one with expected eigenstructure and provide the flexibility and DOFs in arbitrary parameters when designing controller. Secondly, we design a class of dynamic output feedback controller, called dynamic compensator, which can effectively deal with the flaws of state and static output feedback. Thirdly, we also consider a novel technique to multi-objective optimization which can implement a string of practical requirements of control systems.

The remainder of this paper is divided into 5 sections. In Section 2, the problem formulation of dynamic compensator for quasi-linear systems is presented, and some preliminary preparations are given. Section 3 proposes the generally parametric form of dynamic compensator in two cases, and gives a general design procedure of dynamic compensator. Further, a multi-objective optimization problem is considered in Section 4. In Section 5, attitude control of combined spacecrafts is presented to prove the effectiveness and feasibility of the parametric approach. Section 6 summarizes the proposed work and prospects the future work.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem statement

In this study, we investigate a class of quasi-linear systems as follows

$$\begin{cases} \dot{x} = A(\theta, x)x + B(\theta, x)u, \\ y = C(\theta, x)x, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^r$, $y \in \mathbb{R}^m$ are state vector, control input and measured output, $A(\theta,x) \in \mathbb{R}^{n\times n}$, $B(\theta,x) \in \mathbb{R}^{n\times r}$, $C(\theta,x) \in \mathbb{R}^{m\times n}$ are the system coefficient matrices, which are piecewise continuous functions of x and θ, θ is a time-variant parameter as

$$\theta(t) = \begin{bmatrix} \theta_1(t) & \theta_2(t) & \cdots & \theta_k(t) \end{bmatrix}^{\mathrm{T}} \in \Omega \subset \mathbb{R}^k, t \geq 0, \tag{2}$$

where Ω is a compact set.

Assumption 2.1. $B(\theta,x)$ and $C(\theta,x)$ are uniformly bounded in relation to θ and x.

Assumption 2.2.

$$\operatorname{rank}\begin{bmatrix} sI - A(\theta,x) & B(\theta,x) \end{bmatrix} = \operatorname{rank}\begin{bmatrix} sI - A^{\mathrm{T}}(\theta,x) & C^{\mathrm{T}}(\theta,x) \end{bmatrix} = n,\ \forall s \in \mathbb{C}.$$

For system (1), we design a dynamic compensator as

$$\begin{cases} \dot{\xi} = F(\theta,x)\xi + M(\theta,x)y, \\ u = P(\theta,x)\xi + Q(\theta,x)y, \end{cases} \tag{3}$$

where $\xi \in \mathbb{R}^p$ is compensation vector, $F(\theta,x) \in \mathbb{R}^{p\times p}$, $M(\theta,x) \in \mathbb{R}^{p\times m}$ and $P(\theta,x) \in \mathbb{R}^{r\times p}$, $Q(\theta,x) \in \mathbb{R}^{r\times m}$ are the coefficient matrices of dynamic compensator (3) to be determined. With dynamic compensator (3), we obtain the closed-loop system as

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = A_c(\theta,x)\begin{bmatrix} x \\ \xi \end{bmatrix}, \tag{4}$$

where

$$A_c(\theta,x) = \begin{bmatrix} A(\theta,x) + B(\theta,x)Q(\theta,x)C(\theta,x) & B(\theta,x)P(\theta,x) \\ M(\theta,x)C(\theta,x) & F(\theta,x) \end{bmatrix}.$$

This paper considers the design problem of dynamic compensator (3) for quasi-linear systems (1) such that closed-loop system (4) has a linear constant form with an expected eigenstructure, that is the aim of this design is to let $A_c(\theta,x)$ be similar to an arbitrarily constant matrix $\Lambda \in \mathbb{C}^{(n+p)\times(n+p)}$.

Lemma 2.1. Let $T(\theta,x)$ and $V(\theta,x) \in \mathbb{C}^{(n+p)\times(n+p)}$ be the left and right closed-loop eigenvector matrices satisfied

$$\begin{aligned} T^{\mathrm{T}}(\theta,x)A_c(\theta,x) &= \Lambda T^{\mathrm{T}}(\theta,x), \\ A_c(\theta,x)V(\theta,x) &= V(\theta,x)\Lambda. \end{aligned} \tag{5}$$

Based on the above discussion, the problem of parametric design for quasi-linear systems (1) with a dynamic compensator (3) can be stated as follows.

Problem 2.1. (DC—I) Given the quasi-linear systems (1) satisfied Assumptions 2.1 and 2.2, and an arbitrarily constant matrix Λ, exist the left and right closed-loop eigenvector matrices $T(\theta,x)$ and $V(\theta,x)$, and obtain the coefficient matrices $F(\theta,x)$, $M(\theta,x)$ and $P(\theta,x)$, $Q(\theta,x)$ satisfying

$$T^{\mathrm{T}}(\theta,x)V(\theta,x) = I, \tag{6}$$

and

$$T^{\mathrm{T}}(\theta,x)A_c(\theta,x)V(\theta,x) = \Lambda. \tag{7}$$

Generally, quasi-linear systems (1) under dynamic compensator (3) is equivalent to that

$$\begin{cases} \dot{X} = \bar{A}(\theta, x)X + \bar{B}(\theta, x)U, \\ Y = \bar{C}(\theta, x)X, \end{cases} \tag{8}$$

where

$$U = K(\theta, x)Y, \tag{9}$$

meanwhile, $X^{\mathrm{T}} = \begin{bmatrix} x^{\mathrm{T}} & \xi^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, and

$$\bar{A}(\theta, x) = \begin{bmatrix} A(\theta, x) & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{B}(\theta, x) = \begin{bmatrix} B(\theta, x) & 0 \\ 0 & I_p \end{bmatrix},$$
$$\bar{C}(\theta, x) = \begin{bmatrix} C(\theta, x) & 0 \\ 0 & I_p \end{bmatrix}, \ K(\theta, x) = \begin{bmatrix} Q(\theta, x) & P(\theta, x) \\ M(\theta, x) & F(\theta, x) \end{bmatrix}.$$

Thus the closed-loop system is obtained as

$$\dot{X} = \bar{A}_c(\theta, x)X, \tag{10}$$

where

$$\bar{A}_c(\theta, x) = \bar{A}(\theta, x) + \bar{B}(\theta, x)K(\theta, x)\bar{C}(\theta, x).$$

Based on the above deduction, Problem 2.1 is converted into the following Problem 2.2.

Problem 2.2. (DC—II) Given the quasi-linear systems (8) and an arbitrarily constant matrix Λ, exist the left and right closed-loop eigenvector matrices $T(\theta, x)$ and $V(\theta, x)$, and obtain the static output feedback gain matrix $K(\theta, x) \in \mathbb{R}^{(r+p)\times(m+p)}$ satisfying

$$T^{\mathrm{T}}(\theta, x)V(\theta, x) = I, \tag{11}$$

and

$$T^{\mathrm{T}}(\theta, x)\bar{A}_c(\theta, x)V(\theta, x) = \Lambda. \tag{12}$$

2.2. Preliminary results

For the quasi-linear systems (1), a pair of right coprime factorization (RCF) is given as

$$(sI - A(\theta, x))N(\theta, x, s) = B(\theta, x)D(\theta, x, s), \tag{13}$$

where $N(\theta, x, s) \in \mathbb{R}^{n\times r}[s]$ and $D(\theta, x, s) \in \mathbb{R}^{r\times r}[s]$ are a pair of polynomial matrices. Denote $N(\theta, x, s) = [n_{ij}(\theta, x, s)]_{n\times r}$ and $D(\theta, x, s) = [d_{ij}(\theta, x, s)]_{r\times r}$, and

$$\begin{cases} \omega_1 = \max\left\{\deg(n_{ij}(\theta, x, s)), i = 1, 2, \cdots, n, j = 1, 2, \cdots, r\right\}, \\ \omega_2 = \max\left\{\deg(d_{ij}(\theta, x, s)), i = 1, 2, \cdots, r, j = 1, 2, \cdots, r\right\}, \\ \omega = \max\left\{\omega_1, \omega_2\right\}. \end{cases} \tag{14}$$

Then, $N(\theta,x,s)$ and $D(\theta,x,s)$ are written as

$$\begin{cases} N(\theta,x,s) = \sum_{i=0}^{\omega} N_i(\theta,x)s^i, \\ D(\theta,x,s) = \sum_{i=0}^{\omega} D_i(\theta,x)s^i. \end{cases} \tag{15}$$

Another pair of RCF is also given as

$$(sI - A^{\mathrm{T}}(\theta,x))H(\theta,x,s) = C^{\mathrm{T}}(\theta,x)L(\theta,x,s), \tag{16}$$

where $H(\theta,x,s) \in \mathbb{R}^{n\times m}[s]$ and $L(\theta,x,s) \in \mathbb{R}^{m\times m}[s]$ are a pair of polynomial matrices. Denote $H(s) = [h_{ij}(\theta,x,s)]_{n\times m}$ and $L(s) = [l_{ij}(\theta,x,s)]_{m\times m}$, and

$$\begin{cases} \tau_1 = \max\left\{\deg(h_{ij}(\theta,x,s)), i = 1,2,\cdots,n, j = 1,2,\cdots,m\right\}, \\ \tau_2 = \max\left\{\deg(l_{ij}(\theta,x,s)), i = 1,2,\cdots,m, j = 1,2,\cdots,m\right\}, \\ \tau = \max\left\{\tau_1,\tau_2\right\}. \end{cases} \tag{17}$$

Then, $H(\theta,x,s)$ and $L(\theta,x,s)$ are written as

$$\begin{cases} H(\theta,x,s) = \sum_{i=0}^{\tau} H_i(\theta,x)s^i, \\ L(\theta,x,s) = \sum_{i=0}^{\tau} L_i(\theta,x)s^i. \end{cases} \tag{18}$$

Lemma 2.2. Given the quasi-linear systems (1) satisfied Assumptions 2.1 and 2.2, let

$$\begin{aligned} \bar{N}(\theta,x,s) &= \begin{bmatrix} 0 & N(\theta,x,s) \\ I_p & 0 \end{bmatrix}, \ \bar{D}(\theta,x,s) = \begin{bmatrix} 0 & D(\theta,x,s) \\ sI_p & 0 \end{bmatrix}, \\ \bar{H}(\theta,x,s) &= \begin{bmatrix} 0 & H(\theta,x,s) \\ I_p & 0 \end{bmatrix}, \ \bar{L}(\theta,x,s) = \begin{bmatrix} 0 & L(\theta,x,s) \\ sI_p & 0 \end{bmatrix}, \end{aligned} \tag{19}$$

thereby, $\bar{N}(\theta,x,s)$, $\bar{D}(\theta,x,s)$ and $\bar{H}(\theta,x,s)$, $\bar{L}(\theta,x,s)$ in Equation (19) satisfy the following RCF

$$\begin{aligned} (sI - \bar{A}(\theta,x))\bar{N}(\theta,x,s) &= \bar{B}(\theta,x)\bar{D}(\theta,x,s), \\ (sI - \bar{A}^{\mathrm{T}}(\theta,x))\bar{H}(\theta,x,s) &= \bar{C}^{\mathrm{T}}(\theta,x)\bar{L}(\theta,x,s). \end{aligned} \tag{20}$$

3. SOLUTION TO PROBLEM DC

3.1. Case of Λ arbitrary

Theorem 3.1. Let $N(\theta,x,s)$, $D(\theta,x,s)$ and $H(\theta,x,s)$, $L(\theta,x,s)$ satisfy RCFs (13) and (16), meanwhile, $\bar{N}(\theta,x,s)$, $\bar{D}(\theta,x,s)$ and $\bar{H}(\theta,x,s)$, $\bar{L}(\theta,x,s)$ satisfy RCF (20), then,

1. The Problem 2.2 has a solution if and only if there exist two arbitrary parameter matrices $Z_b \in \mathbb{C}^{(m+p)\times(n+p)}$ and $Z_c \in \mathbb{C}^{(r+p)\times(n+p)}$ satisfying

$$\begin{bmatrix} I & \Lambda & \cdots & \Lambda^{\tau} \end{bmatrix} \Phi(Z_b, Z_c) \begin{bmatrix} I \\ \Lambda \\ \vdots \\ \Lambda^{\omega} \end{bmatrix} = I, \tag{21}$$

where

$$\Phi(Z_b, Z_c) = \Phi_H^{\mathrm{T}}(Z_b)\Phi_N(Z_c), \tag{22}$$

and

$$\begin{cases} \Phi_H(Z_b) = \begin{bmatrix} \bar{H}_0(\theta,x)Z_b & \bar{H}_1(\theta,x)Z_b & \cdots & \bar{H}_{\tau}(\theta,x)Z_b \end{bmatrix}, \\ \Phi_N(Z_c) = \begin{bmatrix} \bar{N}_0(\theta,x)Z_c & \bar{N}_1(\theta,x)Z_c & \cdots & \bar{N}_{\omega}(\theta,x)Z_c \end{bmatrix}. \end{cases} \tag{23}$$

2. When satisfying the above condition, the generally parametric forms of left and right eigenvector matrices are obtained as

$$\begin{aligned} V(\theta,x) = \begin{bmatrix} V_0(\theta,x) \\ V_1(\theta,x) \end{bmatrix} &= \sum_{i=0}^{\omega} \bar{N}_i(\theta,x) Z_c \Lambda^i \\ &= \sum_{i=0}^{\omega} \bar{N}_i(\theta,x) \begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix} \Lambda^i, \end{aligned} \tag{24}$$

with

$$V_0(\theta,x) = \sum_{i=0}^{\omega} N_i(\theta,x) Z_{c0} \Lambda^i,\ V_1(\theta,x) = Z_{c1}, \tag{25}$$

and

$$\begin{aligned} T(\theta,x) = \begin{bmatrix} T_0(\theta,x) \\ T_1(\theta,x) \end{bmatrix} &= \sum_{i=0}^{\tau} \bar{H}_i(\theta,x) Z_b (\Lambda^{\mathrm{T}})^i \\ &= \sum_{i=0}^{\tau} \bar{H}_i(\theta,x) \begin{bmatrix} Z_{b1} \\ Z_{b0} \end{bmatrix} (\Lambda^{\mathrm{T}})^i, \end{aligned} \tag{26}$$

with

$$T_0(\theta,x) = \sum_{i=0}^{\tau} H_i(\theta,x) Z_{b0} (\Lambda^{\mathrm{T}})^i,\ T_1(\theta,x) = Z_{b1}, \tag{27}$$

where $Z_{b0} \in \mathbb{C}^{m\times(n+p)}$, $Z_{b1} \in \mathbb{C}^{p\times(n+p)}$ and $Z_{c0} \in \mathbb{C}^{r\times(n+p)}$, $Z_{c1} \in \mathbb{C}^{p\times(n+p)}$ satisfy the following Constraints.

Constraint 3.1. $\det V(\theta,x) \neq 0$.

Constraint 3.2. $T_0^{\mathrm{T}}(\theta,x)V_0(\theta,x) + T_1^{\mathrm{T}}(\theta,x)V_1(\theta,x) = I$.

3. Based on the above deduction, the static output feedback gain matrix $K(\theta,x)$ is solved by

$$\begin{aligned} K(\theta,x) = {} & W_c(\theta,x)\left(\bar{C}(\theta,x)V(\theta,x)\right)^{\mathrm{T}} \\ & \times\left(\left(\bar{C}(\theta,x)V(\theta,x)\right)\left(\bar{C}(\theta,x)V(\theta,x)\right)^{\mathrm{T}}\right)^{-1}, \end{aligned} \tag{28}$$

or

$$\begin{aligned} K(\theta,x) = {} & \left((T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))^{\mathrm{T}}(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))\right)^{-1} \\ & \times(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))^{\mathrm{T}}W_b^{\mathrm{T}}(\theta,x), \end{aligned} \tag{29}$$

where

$$\begin{aligned} W_c(\theta,x) = \begin{bmatrix} W_{c0}(\theta,x) \\ W_{c1}(\theta,x) \end{bmatrix} &= \sum_{i=0}^{\omega}\bar{D}_i(\theta,x)Z_c\Lambda^i \\ &= \sum_{i=0}^{\omega}\bar{D}_i(\theta,x)\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix}\Lambda^i, \end{aligned} \tag{30}$$

with

$$W_{c0}(\theta,x) = \sum_{i=0}^{\omega}D_i(\theta,x)Z_{c0}\Lambda^i,\ W_{c1}(\theta,x) = V_1(\theta,x)\Lambda, \tag{31}$$

and

$$\begin{aligned} W_b(\theta,x) = \begin{bmatrix} W_{b0}(\theta,x) \\ W_{b1}(\theta,x) \end{bmatrix} &= \sum_{i=0}^{\tau}\bar{L}_i(\theta,x)Z_b(\Lambda^{\mathrm{T}})^i \\ &= \sum_{i=0}^{\tau}\bar{L}_i(\theta,x)\begin{bmatrix} Z_{b1} \\ Z_{b0} \end{bmatrix}(\Lambda^{\mathrm{T}})^i, \end{aligned} \tag{32}$$

with

$$W_{b0}(\theta,x) = \sum_{i=0}^{\tau}L_i(\theta,x)Z_{b0}(\Lambda^{\mathrm{T}})^i,\ W_{b1}(\theta,x) = T_1(\theta,x)\Lambda^{\mathrm{T}}. \tag{33}$$

Proof. Firstly, let us derive Equation (21).

Considering $V(\theta,x)$ in Equation (24) and $T(\theta,x)$ in Equation (26), we have

$$V(\theta,x) = \sum_{i=0}^{\omega}\bar{N}_iZ_c\Lambda^i = \Phi_N(Z_c)\begin{bmatrix} I \\ \Lambda \\ \vdots \\ \Lambda^{\omega} \end{bmatrix}, \tag{34}$$

and

$$T^{\mathrm{T}}(\theta,x) = \sum_{i=0}^{\tau}\Lambda^iZ_b^{\mathrm{T}}\bar{H}_i^{\mathrm{T}} = \begin{bmatrix} I & \Lambda & \cdots & \Lambda^{\tau} \end{bmatrix}\Phi_H^{\mathrm{T}}(Z_b). \tag{35}$$

According to Equation (11)

$$T^{\mathrm{T}}(\theta,x)V(\theta,x) = \begin{bmatrix} I & \Lambda & \cdots & \Lambda^{\tau} \end{bmatrix}\Phi_H^{\mathrm{T}}(Z_b)\Phi_N(Z_c)\begin{bmatrix} I \\ \Lambda \\ \vdots \\ \Lambda^{\omega} \end{bmatrix} = I, \tag{36}$$

then, the Equation (21) is proved. Therefore, the Equation (11) is equivalent to Equation (21). Furthermore, by Equation (23), it can be clearly shown that $\Phi(Z_b, Z_c)$ is given by Equation (22).

We secondly show that $V(\theta,x)$ and $T(\theta,x)$ can be expressed in the forms of (24) and (26). Combining Equations (8)–(12), we obtain the following generalized Sylvester equations as

$$\begin{aligned} T^{\mathrm{T}}(\theta,x)\bar{A}(\theta,x)+W_b^{\mathrm{T}}(\theta,x)\bar{C}(\theta,x)=\Lambda T^{\mathrm{T}}(\theta,x),\\ \bar{A}(\theta,x)V(\theta,x)+\bar{B}(\theta,x)W_c(\theta,x)=V(\theta,x)\Lambda, \end{aligned} \tag{37}$$

where

$$\begin{aligned} W_b^{\mathrm{T}}(\theta,x)=T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x)K(\theta,x),\\ W_c(\theta,x)=K(\theta,x)\bar{C}(\theta,x)V(\theta,x). \end{aligned} \tag{38}$$

Therefore, using the general solution to the generalized Sylvester equations [4, 31], the parametric solutions are given in Equations (24), (26) and (30), (32).

Considering Equations (19) and (24), $V(\theta,x)$ can be written as

$$\begin{aligned} V(\theta,x) &= \sum_{i=0}^{\omega}\bar{N}_i Z_c\Lambda^i \\ &= \begin{bmatrix} 0 & N_0 \\ I_p & 0 \end{bmatrix} Z_c + \begin{bmatrix} 0 & N_1 \\ 0 & 0 \end{bmatrix} Z_c\Lambda + \cdots + \begin{bmatrix} 0 & N_\omega \\ 0 & 0 \end{bmatrix} Z_c\Lambda^\omega \\ &= \begin{bmatrix} 0 & N_0 \\ I_p & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix} + \begin{bmatrix} 0 & N_1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix}\Lambda + \cdots + \begin{bmatrix} 0 & N_\omega \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix}\Lambda^\omega \\ &= \begin{bmatrix} \sum_{i=0}^{\omega} N_i Z_{c0}\Lambda^i \\ Z_{c1} \end{bmatrix} = \begin{bmatrix} V_0(\theta,x) \\ V_1(\theta,x) \end{bmatrix}, \end{aligned} \tag{39}$$

thus, Equation (25) is proved. The Equation (27) can be proved in the similar way as shown above.

Now, we derive the parametric solutions of output feedback matrix $K(\theta,x)$ in (28) or (29). Considering Equations (19) and (30), $W_c(\theta,x)$ can be written as

$$\begin{aligned} W_c(\theta,x) &= \sum_{i=0}^{\omega}\bar{D}_i Z_c\Lambda^i \\ &= \begin{bmatrix} 0 & D_0 \\ 0 & 0 \end{bmatrix} Z_c + \begin{bmatrix} 0 & D_1 \\ I_p & 0 \end{bmatrix} Z_c\Lambda + \begin{bmatrix} 0 & D_2 \\ 0 & 0 \end{bmatrix} Z_c\Lambda^2 + \cdots + \begin{bmatrix} 0 & D_\omega \\ 0 & 0 \end{bmatrix} Z_c\Lambda^\omega \\ &= \begin{bmatrix} 0 & D_0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix} + \begin{bmatrix} 0 & D_1 \\ I_p & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix}\Lambda \\ &\quad + \begin{bmatrix} 0 & D_2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix}\Lambda^2 + \cdots + \begin{bmatrix} 0 & D_\omega \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Z_{c1} \\ Z_{c0} \end{bmatrix}\Lambda^\omega \\ &= \begin{bmatrix} \sum_{i=0}^{\omega} D_i Z_{c0}\Lambda^i \\ Z_{c1}\Lambda \end{bmatrix} = \begin{bmatrix} W_{c0}(\theta,x) \\ V_1(\theta,x)\Lambda \end{bmatrix} \\ &= \begin{bmatrix} W_{c0}(\theta,x) \\ W_{c1}(\theta,x) \end{bmatrix}, \end{aligned} \tag{40}$$

thus, Equation (31) is proved. Similarly, we can also obtain Equation (33).

Meanwhile, considering Equation (37)

$$\begin{aligned} T^{\mathrm{T}}(\theta,x)\bar{A}(\theta,x)V(\theta,x)+W_b^{\mathrm{T}}(\theta,x)\bar{C}(\theta,x)V(\theta,x)=\Lambda, \\ T^{\mathrm{T}}(\theta,x)\bar{A}(\theta,x)V(\theta,x)+T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x)W_c(\theta,x)=\Lambda, \end{aligned} \tag{41}$$

we can obtain

$$W_b^{\mathrm{T}}(\theta,x)\bar{C}(\theta,x)V(\theta,x)=T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x)W_c(\theta,x), \tag{42}$$

then

$$\begin{aligned} &(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))^{\mathrm{T}}T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x)W_c(\theta,x)(\bar{C}(\theta,x)V(\theta,x))^{\mathrm{T}} \\ &=(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))^{\mathrm{T}}W_b^{\mathrm{T}}(\theta,x)\bar{C}(\theta,x)V(\theta,x)(\bar{C}(\theta,x)V(\theta,x))^{\mathrm{T}}, \end{aligned} \tag{43}$$

based on Constraint 3.1, $V(\theta,x)$ and $T(\theta,x)$ are non-singular, under Assumption 2.1, we can clearly know that $(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))^{\mathrm{T}}(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))$ and $(\bar{C}(\theta,x)V(\theta,x))(\bar{C}(\theta,x)$ $V(\theta,x))^{\mathrm{T}}$ are also non-singular (see [5]), pre-multiply both sides of the above Equation (43) by the inverse of $(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))^{\mathrm{T}}(T^{\mathrm{T}}(\theta,x)\bar{B}(\theta,x))$, post-multiply by the inverse of $(\bar{C}(\theta,x)V(\theta,x))(\bar{C}(\theta,x)V(\theta,x))^{\mathrm{T}}$, clearly shows that two expressions of the gain matrix $K(\theta,x)$ are equivalent to each other, hence, Equations (28) and (29) are be derived.

In summary, we have proved Theorem 3.1 completely.

□

3.2. Case of Λ diagonal

In general, Λ is required to be a diagonal one

$$\Lambda=\operatorname{diag}\left\{\lambda_1,\lambda_2,\cdots,\lambda_{n+p}\right\}, \tag{44}$$

where $\lambda_i\in\mathbb{C}^-, i=1,2,\cdots,n+p$ are a set of self-conjugate complex poles. Hence, $V(\theta,x)$, $W_c(\theta,x)$ and $T(\theta,x)$, $W_b(\theta,x)$ are replaced by

$$\begin{cases} V(\theta,x)=\left[\begin{array}{cccc} v_1(\theta,x) & v_2(\theta,x) & \cdots & v_{n+p}(\theta,x)\end{array}\right], \\ v_i(\theta,x)=\left[\begin{array}{c} v_{0i}(\theta,x) \\ v_{1i}(\theta,x)\end{array}\right]=\bar{N}(\theta,x,\lambda_i)z_i^c \\ \qquad\quad =\bar{N}(\theta,x,\lambda_i)\left[\begin{array}{c} z_i^{c1} \\ z_i^{c0}\end{array}\right], \\ v_{0i}(\theta,x)=N(\theta,x,\lambda_i)z_i^{c0}, v_{1i}(\theta,x)=z_i^{c1}, \\ \qquad i=1,2,\cdots,n+p, \end{cases} \tag{45}$$

$$\begin{cases} W_c(\theta,x) = \left[\begin{array}{cccc} w_1^c(\theta,x) & w_2^c(\theta,x) & \cdots & w_{n+p}^c(\theta,x) \end{array}\right], \\ w_i^c(\theta,x) = \left[\begin{array}{c} w_i^{c0}(\theta,x) \\ w_i^{c1}(\theta,x) \end{array}\right] = \bar{D}(\theta,x,\lambda_i) z_i^c \\ \qquad = \bar{D}(\theta,x,\lambda_i) \left[\begin{array}{c} z_i^{c1} \\ z_i^{c0} \end{array}\right], \\ w_i^{c0}(\theta,x) = D(\theta,x,\lambda_i) z_i^{c0}, w_i^{c1}(\theta,x) = \lambda_i v_{1i}(\theta,x), \\ \qquad i = 1,2,\cdots,n+p, \end{cases} \tag{46}$$

and

$$\begin{cases} T(\theta,x) = \left[\begin{array}{cccc} t_1(\theta,x) & t_2(\theta,x) & \cdots & t_{n+p}(\theta,x) \end{array}\right], \\ t_i(\theta,x) = \left[\begin{array}{c} t_{0i}(\theta,x) \\ t_{1i}(\theta,x) \end{array}\right] = \bar{H}(\theta,x,\lambda_i) z_i^b \\ \qquad = \bar{H}(\theta,x,\lambda_i) \left[\begin{array}{c} z_i^{b1} \\ z_i^{b0} \end{array}\right], \\ t_{0i}(\theta,x) = H(\theta,x,\lambda_i) z_i^{b0}, t_{1i}(\theta,x) = z_i^{b1}, \\ \qquad i = 1,2,\cdots,n+p, \end{cases} \tag{47}$$

$$\begin{cases} W_b(\theta,x) = \left[\begin{array}{cccc} w_1^b(\theta,x) & w_2^b(\theta,x) & \cdots & w_{n+p}^b(\theta,x) \end{array}\right], \\ w_i^b(\theta,x) = \left[\begin{array}{c} w_i^{b0}(\theta,x) \\ w_i^{b1}(\theta,x) \end{array}\right] = \bar{L}(\theta,x,\lambda_i) z_i^b \\ \qquad = \bar{L}(\theta,x,\lambda_i) z_i^b = \bar{H}(\lambda_i) \left[\begin{array}{c} z_i^{b1} \\ z_i^{b0} \end{array}\right], \\ w_i^{b0}(\theta,x) = L(\theta,x,\lambda_i) z_i^{b0}, w_i^{b1}(\theta,x) = \lambda_i t_{1i}(\theta,x), \\ \qquad i = 1,2,\cdots,n+p, \end{cases} \tag{48}$$

with

$$\begin{cases} Z_c = \left[\begin{array}{cccc} z_1^c & z_2^c & \cdots & z_{n+p}^c \end{array}\right], \\ Z_b = \left[\begin{array}{cccc} z_1^b & z_2^b & \cdots & z_{n+p}^b \end{array}\right], \end{cases} \tag{49}$$

where $z_i^{b0} \in \mathbb{C}^m$, $z_i^{b1} \in \mathbb{C}^p$ and $z_i^{c0} \in \mathbb{C}^r$, $z_i^{c1} \in \mathbb{C}^p$, $i = 1,2,\cdots,n+p$ indicate the DOFs in parametric solutions. Then, we give the following Theorem to solve the Problem 2.2.

Theorem 3.2. Let $N(\theta,x,s)$, $D(\theta,x,s)$ and $H(\theta,x,s)$, $L(\theta,x,s)$ satisfy RCFs (13) and (16), meanwhile, $\bar{N}(\theta,x,s)$, $\bar{D}(\theta,x,s)$ and $\bar{H}(\theta,x,s)$, $\bar{L}(\theta,x,s)$ satisfy RCF (20), then,

1. Problem 2.2 has a solution if and only if there exist two groups of parameter vectors $z_i^b \in \mathbb{C}^{m+p}$ and $z_i^c \in \mathbb{C}^{r+p}$ satisfying

$$(z_i^b)^{\mathrm{T}} \bar{H}^{\mathrm{T}}(\lambda_i) \bar{N}(\lambda_j) z_j^c = \delta_{ij},\ i,j = 1,2,\cdots,n+p. \tag{50}$$

2. When satisfying the above condition, the generally parametric expressions of the left and right eigenvector matrices are obtained in Equations (45) and (47).

Constraint 3.3. $\det V(\lambda_i) \neq 0$.

Constraint 3.4. $(z_i^{b0})^{\mathrm{T}} H^{\mathrm{T}}(\lambda_i) N(\lambda_j) z_j^{c0} + (z_i^{b1})^{\mathrm{T}} z_j^{c1} = \delta_{ij},\ i,j = 1,2,\cdots,n+p$.

3. Based on the above deduction, the static output feedback gain matrix $K(\theta, x)$ is solved by Equation (28) or (29), where $W_c(\theta, x)$ and $W_b(\theta, x)$ are substituted by Equations (46) and (48).

Proof. On the basis of Theorem 3.1, when Λ is chosen to be a diagonal one as Equation (44), $T(\theta, x)$, $W_b(\theta, x)$ and $V(\theta, x)$, $W_c(\theta, x)$ can be the form of columns given in Equations (45)–(48), we prove the results in Theorem 3.2 easily. □

3.3. General procedure

Based on the results in Theorems 3.1 and 3.2, we propose a general procedure to design the generally parametric form of dynamic compensator (3) for quasi-linear systems (1).

Step 1 Determine an expected closed-loop eigenstructure.

Based on pole assignment theories [3, 15], Λ is required to be a Hurwitz matrix [3, 15], that is all closed-loop eigenvalues are located in the left-half s-plane,

$$\lambda_i(\Lambda) \in \mathbb{C}^-, \quad i = 1,2,\cdots,n+p. \tag{51}$$

Step 2 Obtain two groups of RCFs $\bar{N}(\theta, x, s)$, $\bar{D}(\theta, x, s)$ and $\bar{H}(\theta, x, s)$, $\bar{L}(\theta, x, s)$.

On the one hand, based on RCFs (13) and (16), acquire $N(\theta, x, s)$, $D(\theta, x, s)$ and $H(\theta, x, s)$, $L(\theta, x, s)$, further, according to Equation (19), obtain $\bar{N}(\theta, x, s)$, $\bar{D}(\theta, x, s)$ and $\bar{H}(\theta, x, s)$, $\bar{L}(\theta, x, s)$. On the other hand, we can also obtain two pairs of solutions according to (20) as

$$\begin{cases} \bar{N}(\theta, x, s) = \operatorname{adj}\left(sI - \bar{A}(\theta, x)\right) B(\theta, x), \\ \bar{D}(\theta, x, s) = \det\left(sI - \bar{A}(\theta, x)\right) I_{r+p}, \end{cases}$$

and

$$\begin{cases} \bar{H}(\theta, x, s) = \operatorname{adj}\left(sI - \bar{A}^{\mathrm{T}}(\theta, x)\right) C^{\mathrm{T}}(\theta, x), \\ \bar{L}(\theta, x, s) = \det\left(sI - \bar{A}^{\mathrm{T}}(\theta, x)\right) I_{m+p}. \end{cases}$$

Step 3 Establish a multi-objective optimization problem.

According to practical requirements of control systems, we establish multiple objectives as

$$J_i = J_i(Z_b, Z_c, \Lambda), i = 1,2,\cdots,l,$$

then, an objective function representing the comprehensive performance can be formulated as

$$J = \sum_{i=1}^{l} \varepsilon_i J_i,$$

where $\varepsilon_i \in [0,1], i = 1,2,\cdots,l$ are weight coefficients satisfying

$$\sum_{i=1}^{l} \varepsilon_i = 1. \tag{52}$$

Then, a multi-objective optimization problem is formulated as follows

$$\begin{cases} \min\ J, \\ \text{s.t. } (21), (51), (52), \end{cases} \tag{53}$$

Note that the optimization problem is related to the design parameters Z_b, Z_c and Λ, which will be further discussed in Section 4.

Step 4 Find parameters.

In this study, we see Z_b and Z_c are two arbitrary parameter matrices which give the DOFs in design process. In other words, seeking Z_b and Z_c is the key to solve optimization problem (53).

Step 5 Compute the static output feedback gain matrix $K(\theta, x)$ and get the coefficient matrices of dynamic compensator $F(\theta, x)$, $M(\theta, x)$ and $P(\theta, x)$, $Q(\theta, x)$.

By utilizing $T(\theta, x)$, $W_b(\theta, x)$ and $V(\theta, x)$, $W_c(\theta, x)$ given in Equations (24)–(27), (30)–(33) or (46)–(49), the static feedback gain matrix $K(\theta, x)$ is solved by Equation (28) or (29), therefore the coefficient matrices $F(\theta, x)$, $M(\theta, x)$ and $P(\theta, x)$, $Q(\theta, x)$ are obtained.

4. MULTI-OBJECTIVE DESIGN AND OPTIMIZATION

In this study, we successfully solve the Problem 2.2 through Theorems 3.1 and 3.2, further, there are the DOFs in arbitrary parameters Z_b and Z_c provided by the parametric approach, which can be utilized to improve the comprehensive performances of closed-loop system.

4.1. Regional pole assignment

We aim to locate these eigenvalues λ_i, $i = 1, 2, \cdots, n+p$ in an admissible set to satisfy the practical requirements of control systems. Actually, if placing these closed-loop eigenvalues within a small interval around the expected location, it can reduce the difficulties and improve flexibility when implementing controller. Generally, we choose the form $\underline{\lambda}_i < \lambda_i < \overline{\lambda}_i$, where $\underline{\lambda}_i, \overline{\lambda}_i \in \mathbb{C}$ are the lower and upper bound. Then, the closed-loop eigenvalues can be defined as [18]

$$\lambda_i = \underline{\lambda}_i + \left(\overline{\lambda}_i - \underline{\lambda}_i\right) \sin^2\left(\|z_i^c\|_2\right), i = 1, 2, \cdots, n+p, \tag{54}$$

which shows that all closed-loop eigenvalues can utilize arbitrary parameters to move in the region of s-plane defined by the lower and upper bounds during the design process, that is, the arbitrary parameters play an important role in defining the location of closed-loop eigenvalues. Thus, parameter vectors z_i^c, $i = 1, 2, \cdots, n+p$ become decisive factors in the multi-objective optimization problem.

4.2. Robustness criteria

4.2.1. Low sensitivity

In order that maintaining the stability robustness and performance robustness when parameters perturbations, a general method is to minimize the sensitivity function of closed-loop eigenvalues, under general circumstances, we choose the overall eigenvalues sensitivity as [22]

$$J_1 = \|V\|_2 \|T\|_2, \tag{55}$$

which gives an overall measurement of the condition for eigen-problem.

4.2.2. Disturbance attenuation

Considering the influence of non-modeled system dynamics and external disturbance on closed-loop system, a bounded disturbance $\bar{G}w(t)$ is led into closed-loop system (10) as

$$\begin{cases} \dot{X} = \bar{A}_c(\theta, x)X + \bar{G}w(t), \\ Y = \bar{C}(\theta, x)X, \end{cases} \tag{56}$$

for the above system, we obtain relation of Y and $w(t)$ as [6]

$$Y_w(s) = \bar{C}(sI - \bar{A}_c)^{-1}\bar{G} = \bar{C}T^{\mathrm{T}}(sI - \Lambda)^{-1}V\bar{G},$$

based on H_2 and H_∞ control theories, the value of $\|Y_w(s)\|_2$ and $\|Y_w(s)\|_\infty$ can be exploited to measure the effort of disturbance on output. In order to simplify the computation, an effective and easy way is [5]

$$J_2 = \|V\bar{G}\|_2, \; J_3 = \|V\bar{G}\|_\infty. \tag{57}$$

Based on the above discussion, we establish an objective function to express the robustness criteria as

$$J_R = \sum_{i=1}^{3} \varepsilon_i J_i. \tag{58}$$

4.3. Low compensation gain criteria

As we know, low compensation gain is an important index when designing compensator. Under the low compensation gain, the series amplifier can be reduced and it is difficult to produce self-oscillation, which is benefit for physical realization.

According to Equations (3) and (4), we can know that the consumption of energy is depended on value of compensation vectors. It is noteworthy that the smaller the compensation vector, the less energy consumed. To further reduce the energy loss during transient process, we choose the following index

$$J_4 = \frac{1}{2}\|F(\theta, x)\|_2 + \frac{1}{2}\|M(\theta, x)\|_2 \tag{59}$$

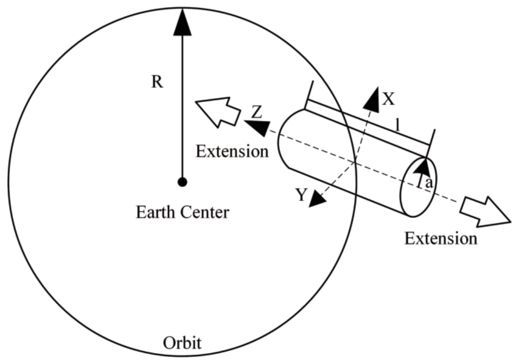

Fig. 1. Coordinate systems.

Based on the above discussion, we establish an objective function to express the low compensation gain criteria

$$J_L = \varepsilon_4 J_4. \tag{60}$$

According to Equations (58) and (60), a multi-objective optimization problem can be formulated as

$$\begin{cases} \min\ J, \\ J = J_R + J_L, \\ \text{s.t. } (21), (51), (52), (54), \end{cases} \tag{61}$$

which is exploited to express the comprehensive performance of system.

5. EXAMPLE — ATTITUDE CONTROL OF COMBINED SPACECRAFTS

Consider the attitude motion of the extending space structures in [16], its physical model is provided in Figure 1, we possess the following mathematical model as

$$\begin{bmatrix} J_x\ddot{\alpha} \\ J_y\ddot{\beta} \\ J_z\ddot{\gamma} \end{bmatrix} + \begin{bmatrix} \dot{J}_x\dot{\alpha} - \Omega(J_x - J_y + J_z)\dot{\gamma} + 4\Omega^2(J_y - J_z)\alpha - \Omega\dot{J}_x\gamma \\ \dot{J}_y\dot{\beta} + 3\Omega^2(J_x - J_z)\beta \\ \Omega(J_x - J_y + J_z)\dot{\alpha} + \dot{J}_z\dot{\gamma} + \Omega\dot{J}_z\alpha + \Omega^2(J_y - J_x)\gamma \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \tag{62}$$

where α, β and γ indicate the roll angle, pitch angle and yaw angle of the combined spacecrafts, J_x, J_y and J_z are the inertia variables, T_x, T_y and T_z are the external input torques, $\Omega = \sqrt{\mu/R^3}$ is orbital angular speed of combined spacecrafts, and μ is the gravitational parameter and R is the distance from the center of the earth to combined spacecrafts. It follows from Equation (62) that the in-plane motion (i. e., the $x - z$ subsystem) and the out-of-plane motion (i. e. the y subsystem) are independent. Therefore, they can be considered separately. Then, we choose the in-plane motion as

$$\begin{bmatrix} J_x\ddot{\alpha} \\ J_z\ddot{\gamma} \end{bmatrix} + \begin{bmatrix} \dot{J}_x\dot{\alpha} - \Omega(J_x - J_y + J_z)\dot{\gamma} + 4\Omega^2(J_y - J_z)\alpha - \Omega\dot{J}_x\gamma \\ \Omega(J_x - J_y + J_z)\dot{\alpha} + \dot{J}_z\dot{\gamma} + \Omega\dot{J}_z\alpha + \Omega^2(J_y - J_x)\gamma \end{bmatrix} = \begin{bmatrix} u_x \\ u_z \end{bmatrix}, \tag{63}$$

let

$$q = \begin{bmatrix} \alpha & \gamma & \dot{\alpha} & \dot{\gamma} \end{bmatrix}^{\mathrm{T}},$$

hence, the system (63) can be transformed into the following system

$$\begin{cases} \dot{q} = A(\theta, q)q + B(\theta, q)u, \\ y = C(\theta, q)q, \end{cases} \tag{64}$$

with

$$A(\theta, q) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \Theta_3 & \Theta_4 & \Theta_1 & \Theta_2 \\ \Theta_7 & \Theta_8 & \Theta_5 & \Theta_6 \end{bmatrix},$$

$$B(\theta, q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \ C(\theta, q) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

where

$$\begin{aligned} &\Theta_1 = -\dot{J}_x/J_x, \ \Theta_2 = \Omega(J_x - J_y + J_z)/J_x, \\ &\Theta_3 = 4\Omega^2(J_z - J_y)/J_x, \ \Theta_4 = \Omega\Theta_1, \\ &\Theta_5 = -\Omega(J_x - J_y + J_z)/J_z, \ \Theta_6 = -\dot{J}_z/J_z, \\ &\Theta_7 = \Omega\Theta_6, \ \Theta_8 = \Omega(J_x - J_y)/J_z. \end{aligned}$$

Considering Equation (56), a bounded disturbance $\bar{G}w(t)$ is led into the closed-loop system (64), where

$$\bar{G} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \ w(t) = \begin{cases} 1, & t \in [200, 250] \\ 0, & \text{else.} \end{cases}$$

Design a dynamic compensator in the form of Equation (3), let $p = 2$, then, we can obtain a closed-loop system like Equation (4).

The closed-loop eigenvalues are required to lie in ranges given by

$$\begin{aligned} &\lambda_1 \in [-0.1, 0], \\ &\lambda_2 \in [-0.2, -0.1], \\ &\lambda_3 \in [-0.3, -0.2], \\ &\lambda_4 \in [-0.4, -0.3], \\ &\lambda_5 \in [-0.5, -0.4], \\ &\lambda_6 \in [-0.6, -0.5]. \end{aligned}$$

According to RCFs (13) and (16), two groups of RCFs can be obtained as

$$\begin{cases} N(\theta, x, s) = \begin{bmatrix} 1 & 0 & s & 0 \\ 0 & 1 & 0 & s \end{bmatrix}^{\mathrm{T}}, \\ D(\theta, x, s) = \begin{bmatrix} s^2 - \Theta_1 s - \Theta_3 & -\Theta_2 s - \Theta_4 \\ -\Theta_5 s - \Theta_7 & s^2 - \Theta_6 s - \Theta_8 \end{bmatrix}, \end{cases}$$

and

$$\begin{cases} H(\theta, x, s) = I_4, \\ L(\theta, x, s) = \begin{bmatrix} s & 0 & -\Theta_3 & -\Theta_7 \\ 0 & s & -\Theta_4 & -\Theta_8 \\ 0 & -1 & -\Theta_2 & s-\Theta_6 \\ -1 & 0 & s-\Theta_1 & -\Theta_5 \end{bmatrix}. \end{cases}$$

5.1. Non-optimized solutions

Let

$$\Lambda = \operatorname{diag}\{-0.1, -0.2, -0.3, -0.4, -0.5, -0.6\},$$

and

$$Z_{c0} = \begin{bmatrix} -0.1 & 0 & -0.3 & 0 & -0.5 & 0 \\ 0 & -0.2 & 0 & -0.4 & 0 & -0.6 \end{bmatrix}, \tag{65}$$

based on Equations (24) and (25), $V_0(\theta, q)$ can be obtained as

$$V_0(\theta, q) = \begin{bmatrix} -0.1 & 0 & -0.3 & 0 & -0.5 & 0 \\ 0 & -0.2 & 0 & -0.4 & 0 & -0.6 \\ 0.01 & 0 & 0.09 & 0 & 0.25 & 0 \\ 0 & 0.04 & 0 & 0.16 & 0 & 0.36 \end{bmatrix},$$

then, choose

$$Z_{c1} = V_1(\theta, q) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \tag{66}$$

according to Equations (30) and (31), $W_{c0}(\theta, q)$ and $W_{c1}(\theta, q)$ can be obtained as

$$W_{c0}(\theta, q) = \begin{bmatrix} \frac{10\Theta_3-\Theta_1}{100} - \frac{1}{1000} & \frac{5\Theta_4-\Theta_2}{25} & \frac{30\Theta_3-9\Theta_1}{100} - \frac{27}{1000} & \frac{10\Theta_4-4\Theta_2}{25} & \frac{2\Theta_3-\Theta_1}{4} - \frac{1}{8} & \frac{15\Theta_4-9\Theta_2}{25} \\ \frac{10\Theta_7-\Theta_5}{100} & \frac{5\Theta_8-\Theta_6}{25} - \frac{1}{25} & \frac{30\Theta_7-9\Theta_5}{100} & \frac{10\Theta_8-4\Theta_6}{25} - \frac{8}{125} & \frac{4\Theta_7-\Theta_5}{4} & \frac{15\Theta_8-9\Theta_6}{25} - \frac{27}{125} \end{bmatrix},$$

and

$$W_{c1}(\theta, q) = \begin{bmatrix} -0.1 & 0 & -0.3 & 0 & -0.5 & -0.6 \\ 0 & -0.2 & -0.3 & -0.4 & -0.5 & 0 \end{bmatrix}.$$

Further, based on Equation (21) or (50), we can easily obtain Z_{b0} and Z_{b1} as

$$Z_{b0} = \frac{1}{14} \begin{bmatrix} -160 & 480 & -10 & -480 & 10 & 160 \\ -225 & 220 & 150 & -325 & -45 & 120 \\ -200 & 600 & -100 & -600 & 100 & 200 \\ -375 & 250 & 250 & -425 & -75 & 200 \end{bmatrix},$$

and

$$Z_{b1} = \frac{1}{7} \begin{bmatrix} 0 & 21 & 0 & -21 & 0 & 7 \\ -15 & 24 & 10 & -24 & -3 & 8 \end{bmatrix},$$

then, according to Equations (26) and (27), we have

$$T_0(\theta,q)=\frac{1}{14}\begin{bmatrix} -160 & 480 & -10 & -480 & 10 & 160\\ -225 & 220 & 150 & -325 & -45 & 120\\ -200 & 600 & -100 & -600 & 100 & 200\\ -375 & 250 & 250 & -425 & -75 & 200\end{bmatrix},$$

and

$$T_1(\theta,q)=\frac{1}{7}\begin{bmatrix} 0 & 21 & 0 & -21 & 0 & 7\\ -15 & 24 & 10 & -24 & -3 & 8\end{bmatrix},$$

meanwhile, based on Equations (32) and (33), we possess $W_{b0}(\theta,q)$ and $W_{b1}(\theta,q)$ as

$$W_{b0}(\theta,q)=\left[\begin{matrix} \frac{200\Theta_3+375\Theta_7}{14}+\frac{8}{7} & -\frac{300\Theta_3+125\Theta_7}{7}-\frac{48}{7} & \frac{50\Theta_3-125\Theta_7}{7}+\frac{3}{14}\\ \frac{200\Theta_4+375\Theta_8}{14}+\frac{45}{28} & -\frac{300\Theta_4+125\Theta_8}{7}-\frac{22}{7} & \frac{50\Theta_4-125\Theta_8}{7}-\frac{45}{14}\\ \frac{200\Theta_2+375\Theta_6}{14}+\frac{75}{4} & -\frac{300\Theta_2+125\Theta_6}{7}-\frac{135}{7} & \frac{50\Theta_2-125\Theta_6}{7}-\frac{225}{14}\\ \frac{200\Theta_1+375\Theta_5}{14}+\frac{90}{7} & -\frac{300\Theta_3+125\Theta_7}{7}-\frac{300}{7} & \frac{50\Theta_1-125\Theta_5}{7}+\frac{20}{7}\end{matrix}\right.$$
$$\left.\begin{matrix} \frac{600\Theta_3+425\Theta_7}{14}+\frac{96}{7} & \frac{25\Theta_7-100\Theta_3}{14}-\frac{5}{14} & -\frac{100(\Theta_3+\Theta_7)}{7}-\frac{48}{7}\\ \frac{600\Theta_4+425\Theta_8}{14}+\frac{65}{7} & \frac{25\Theta_8-100\Theta_4}{14}+\frac{45}{28} & -\frac{100(\Theta_3+\Theta_7)}{7}-\frac{36}{7}\\ \frac{600\Theta_2+425\Theta_6}{14}+\frac{495}{14} & \frac{25\Theta_6-100\Theta_2}{14}+\frac{165}{28} & -\frac{100(\Theta_2+\Theta_6)}{7}-\frac{120}{7}\\ \frac{600\Theta_1+425\Theta_5}{14}+\frac{360}{7} & \frac{25\Theta_5-100\Theta_1}{14}-\frac{30}{7} & -\frac{100(\Theta_1+\Theta_5)}{7}-20\end{matrix}\right],$$

and

$$W_{b1}(\theta,q)=\begin{bmatrix} 0 & -3/5 & 0 & 6/5 & 0 & -3/5\\ 3/14 & -24/35 & -3/7 & 48/35 & 3/14 & -24/35\end{bmatrix}.$$

Based on Equation (28) or (29), $K(\theta,q)$ can be obtained as

$$K(\theta,q)=\begin{bmatrix} -\Theta_3-\frac{41}{700} & \frac{9}{70}-\Theta_4 & \frac{3}{14}-\Theta_2 & -\Theta_1-\frac{24}{35} & 0 & \frac{3}{175}\\ -\Theta_7-\frac{96}{175} & -\Theta_8-\frac{86}{175} & -\Theta_6-\frac{9}{7} & -\Theta_5-\frac{24}{35} & -\frac{6}{125} & -\frac{48}{875}\\ -\frac{41}{7} & -\frac{36}{7} & -\frac{60}{7} & -\frac{60}{7} & -\frac{3}{5} & -\frac{24}{35}\\ \frac{47}{7} & \frac{127}{28} & \frac{165}{28} & \frac{50}{7} & \frac{3}{5} & \frac{33}{70}\end{bmatrix}, \tag{67}$$

that is,

$$\begin{aligned} &Q^{\mathrm{T}}(\theta,q)=\begin{bmatrix} -\Theta_3-\frac{41}{700} & -\Theta_7-\frac{96}{175}\\ \frac{9}{70}-\Theta_4 & -\Theta_8-\frac{86}{175}\\ \frac{3}{14}-\Theta_2 & -\Theta_6-\frac{9}{7}\\ -\Theta_1-\frac{24}{35} & -\Theta_5-\frac{24}{35}\end{bmatrix},\ P(\theta,q)=\begin{bmatrix} 0 & \frac{3}{175}\\ -\frac{6}{125} & -\frac{48}{875}\end{bmatrix},\\ &M(\theta,q)=\begin{bmatrix} -\frac{41}{7} & -\frac{36}{7} & -\frac{60}{7} & -\frac{60}{7}\\ \frac{47}{7} & \frac{127}{28} & \frac{165}{28} & \frac{50}{7}\end{bmatrix},\ F(\theta,q)=\begin{bmatrix} -\frac{3}{5} & -\frac{24}{35}\\ \frac{3}{5} & \frac{33}{70}\end{bmatrix},\end{aligned} \tag{68}$$

under the controller (67) or (68), the closed-loop system can be obtained as

$$\begin{bmatrix}\dot{q}\\ \dot{\xi}\end{bmatrix}=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ -41/700 & 9/70 & -24/35 & 3/14 & 0 & 3/175\\ -96/175 & -86/175 & -24/35 & -9/7 & -6/125 & -48/875\\ -41/7 & -36/7 & -60/7 & -60/7 & -3/5 & -24/35\\ 47/7 & 127/28 & 50/7 & 165/28 & 3/5 & 33/70\end{bmatrix}\begin{bmatrix} q\\ \xi\end{bmatrix}.$$

5.2. Optimized solutions

Consider the optimization problem in Equation (61), based on the MATLAB® function *fminsearch* in the MATLAB® Optimization Toolbox, choose initial value as Equation (65) and $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.1$, $\varepsilon_3 = 0.2$, $\varepsilon_4 = 0.1$, we can obtain Z_{c0} and Λ as

$$Z_{c0} = \begin{bmatrix} 0.6339 & 0 & -2.0210 & 0 & -3.1731 & 0 \\ 0 & 1.1287 & 0 & -1.1807 & 0 & -3.3006 \end{bmatrix}$$

and

$$\Lambda = \operatorname{diag}\{-0.0649, -0.1183, -0.2189, -0.3145, -0.4999, -0.5975\},$$

based on Equations (24) and (25), $V_0(\theta, q)$ can be obtained as

$$V_0(\theta, q) = \begin{bmatrix} 0.6339 & 0 & -2.0210 & 0 & -3.1731 & 0 \\ 0 & 1.1287 & 0 & -1.1807 & 0 & -3.3006 \\ -0.0411 & 0 & 0.4424 & 0 & 1.5862 & 0 \\ 0 & -0.1335 & 0 & 0.3713 & 0 & 1.9721 \end{bmatrix},$$

then, choose $Z_{c1} = V_1(\theta, q)$ as Equation (66), according to Equations (30) and (31), $W_{c0}(\theta, q)$ and $W_{c1}(\theta, q)$ can be obtained as

$$W_{c0}(\theta, q) = \left[\begin{matrix} 0.0411\Theta_1 - 0.6339\Theta_3 + 0.0027 & 0.1335\Theta_2 - 1.1287\Theta_4 \\ 0.0411\Theta_5 - 0.6339\Theta_7 & 0.1335\Theta_6 - 1.1287\Theta_8 + 0.0158 \end{matrix} \right.$$
$$\begin{matrix} 2.0210\Theta_3 - 0.4424\Theta_1 - 0.0968 & 1.1807\Theta_4 - 0.3713\Theta_2 \\ 2.0210\Theta_7 - 0.4424\Theta_5 & 1.1807\Theta_8 - 0.3713\Theta_6 - 0.1168 \end{matrix}$$
$$\left. \begin{matrix} 3.1731\Theta_3 - 1.5862\Theta_1 - 0.7930 & 3.3006\Theta_4 - 1.9721\Theta_2 \\ 3.1731\Theta_7 - 1.5862\Theta_5 & 3.3006\Theta_8 - 1.9721\Theta_6 - 1.1783 \end{matrix} \right],$$

and

$$W_{c1}(\theta, q) = \begin{bmatrix} -0.0649 & 0 & -0.2189 & 0 & -0.4999 & -0.5975 \\ 0 & -0.1183 & -0.2189 & -0.3145 & -0.4999 & 0 \end{bmatrix}.$$

Further, based on Equation (21) or (50), we can easily get Z_{b0} and Z_{b1} as

$$Z_{b0} = \begin{bmatrix} 0.4909 & 0.1719 & -0.6419 & 0.2782 & 0.1918 & -0.0408 \\ -0.1240 & 0.7007 & -0.0602 & -0.6540 & 0.0136 & 0.1706 \\ 0.4944 & 0.1731 & -1.5208 & 0.2803 & 1.0674 & -0.0410 \\ -0.5671 & 1.2243 & -0.2753 & -1.0110 & 0.0621 & 0.7803 \end{bmatrix},$$

and

$$Z_{b1} = \begin{bmatrix} 0.7092 & -0.1018 & 0.3443 & -0.1649 & -0.0776 & 0.0241 \\ 0.0642 & 0.3726 & 0.0312 & 0.6032 & -0.0070 & -0.0883 \end{bmatrix},$$

then, according to Equations (26) and (27), we have

$$T_0(\theta, q) = \begin{bmatrix} 0.4909 & 0.1719 & -0.6419 & 0.2782 & 0.1918 & -0.0408 \\ -0.1240 & 0.7007 & -0.0602 & -0.6540 & 0.0136 & 0.1706 \\ 0.4944 & 0.1731 & -1.5208 & 0.2803 & 1.0674 & -0.0410 \\ -0.5671 & 1.2243 & -0.2753 & -1.0110 & 0.0621 & 0.7803 \end{bmatrix},$$

and

$$T_1(\theta, q) = \begin{bmatrix} 0.7092 & -0.1018 & 0.3443 & -0.1649 & -0.0776 & 0.0241 \\ 0.0642 & 0.3726 & 0.0312 & 0.6032 & -0.0070 & -0.0883 \end{bmatrix},$$

meanwhile, based on Equations (32) and (33), we possess $W_{b0}(\theta, q)$ and $W_{b1}(\theta, q)$ as

$$W_{b0}(\theta, q) = \left[\begin{matrix} 0.5671\Theta_7 - 0.4944\Theta_3 - 0.0319 & -0.1731\Theta_3 - 1.2243\Theta_7 - 0.0203 \\ 0.5671\Theta_8 - 0.4944\Theta_4 + 0.0080 & -0.1731\Theta_4 - 1.2243\Theta_8 - 0.0829 \\ 0.5671\Theta_6 - 0.4944\Theta_2 + 0.1608 & -0.1731\Theta_2 - 1.2243\Theta_6 - 0.8455 \\ 0.5671\Theta_5 - 0.4944\Theta_1 - 0.5230 & -0.1731\Theta_1 - 1.2243\Theta_5 - 0.1924 \end{matrix}\right.$$

$$\begin{matrix} 1.5208\Theta_3 + 0.2753\Theta_7 + 0.1405 & 1.0110\Theta_7 - 0.2803\Theta_3 - 0.0875 \\ 1.5208\Theta_4 + 0.2753\Theta_8 + 0.0132 & 1.0110\Theta_8 - 0.2803\Theta_4 + 0.2057 \\ 1.5208\Theta_2 + 0.2753\Theta_6 + 0.1205 & 1.0110\Theta_6 - 0.2803\Theta_2 + 0.9720 \\ 1.5208\Theta_1 + 0.2753\Theta_5 + 0.9748 & 1.0110\Theta_5 - 0.2803\Theta_1 - 0.3664 \end{matrix}$$

$$\left.\begin{matrix} -1.0674\Theta_3 - 0.0621\Theta_7 - 0.0959 & 0.0410\Theta_3 - 0.7803\Theta_7 + 0.0244 \\ -1.0674\Theta_4 - 0.0621\Theta_8 - 0.0068 & 0.0410\Theta_4 - 0.7803\Theta_8 - 0.1019 \\ -1.0674\Theta_2 - 0.0621\Theta_6 - 0.0446 & 0.0410\Theta_2 - 0.7803\Theta_6 - 0.6368 \\ -1.0674\Theta_1 - 0.0621\Theta_5 - 0.7253 & 0.0410\Theta_1 - 0.7803\Theta_5 + 0.0653 \end{matrix}\right],$$

$$W_{b1}(\theta, q) = \begin{bmatrix} -0.0460 & 0.0120 & -0.0754 & 0.0518 & 0.0388 & -0.0144 \\ -0.0042 & -0.0441 & -0.0068 & -0.1897 & 0.0035 & 0.0528 \end{bmatrix}.$$

Based on Equation (28) or (29), $K(\theta, q)$ can be obtained as

$$K(\theta, q) = \begin{bmatrix} -\Theta_3 - 0.0886 & -\Theta_4 - 0.0053 & -\Theta_2 - 0.0241 & -\Theta_1 - 0.6978 & 0.0301 & 0.0027 \\ -\Theta_7 + 0.0182 & -\Theta_8 - 0.1136 & -\Theta_6 - 0.7821 & -\Theta_5 + 0.0184 & -0.0108 & 0.0395 \\ 0.0371 & -0.0875 & -0.4002 & -0.2083 & -0.0970 & 0.0453 \\ -0.0632 & 0.1292 & 0.2024 & -0.3093 & 0.0273 & -0.2371 \end{bmatrix}, \quad (69)$$

that is,

$$\begin{aligned} Q^{\mathrm{T}}(\theta, q) &= \begin{bmatrix} -\Theta_3 - 0.0886 & -\Theta_7 + 0.0182 \\ \Theta_4 - 0.0053 & -\Theta_8 - 0.1136 \\ -\Theta_2 - 0.0241 & -\Theta_6 - 0.7821 \\ -\Theta_1 - 0.6978 & -\Theta_5 + 0.0184 \end{bmatrix}, \\ P(\theta, q) &= \begin{bmatrix} 0.0301 & 0.0027 \\ -0.0108 & 0.0395 \end{bmatrix}, \\ M(\theta, q) &= \begin{bmatrix} 0.0351 & -0.0875 & -0.4002 & -0.2083 \\ -0.0632 & 0.1292 & 0.2024 & -0.3093 \end{bmatrix}, \\ F(\theta, q) &= \begin{bmatrix} -0.0970 & 0.0453 \\ 0.0273 & -0.2371 \end{bmatrix}, \end{aligned} \quad (70)$$

under the controller (69) or (70), the closed-loop system can be obtained as

$$\begin{bmatrix} \dot{q} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -0.0886 & -0.0053 & -0.6978 & -0.0241 & 0.0301 & 0.0027 \\ 0.0182 & -0.1136 & 0.0184 & -0.7821 & -0.0108 & 0.0395 \\ 0.0371 & -0.0875 & -0.2083 & -0.4002 & -0.0970 & 0.0453 \\ -0.0632 & 0.1292 & -0.3093 & 0.2024 & 0.0273 & -0.2371 \end{bmatrix} \begin{bmatrix} q \\ \xi \end{bmatrix}.$$

5.3. Simulation and comparison

Choose the initial values as

$$\begin{cases} q(0) = \begin{bmatrix} 3 & 3 & 3 & 3 \end{bmatrix}^{\mathrm{T}}, \\ \xi(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}}, \end{cases}$$

we can obtain the Figures 2 – 9.

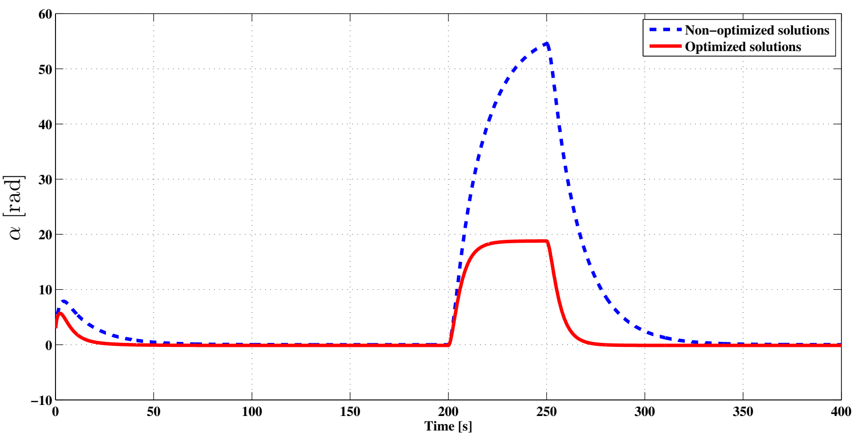

Fig. 2. Variation diagram of roll angle α.

In Figures 2 – 4, we can clearly see that the optimized dynamic compensator leads to a better transient performance than that of non-optimized one, meanwhile, in Figures 5 – 9, both compensation vectors and control signals of optimized dynamic compensator are smaller than that of non-optimized one, which means that the optimized dynamic compensator can achieve better control effects at the cost of less energy and magnitude of the control signals.

Define J_n as the non-optimized index and J_o as the optimized index, we possess

$$J_n = 151.8218,\ \ J_o = 7.2216.$$

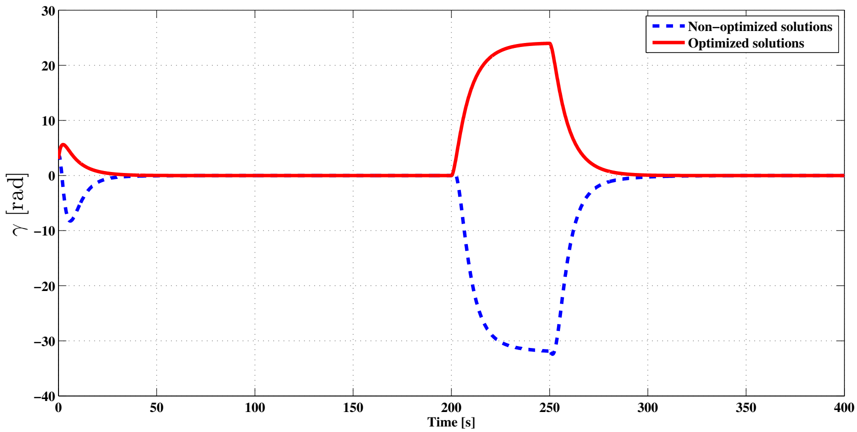

Fig. 3. Variation diagram of yaw angle γ.

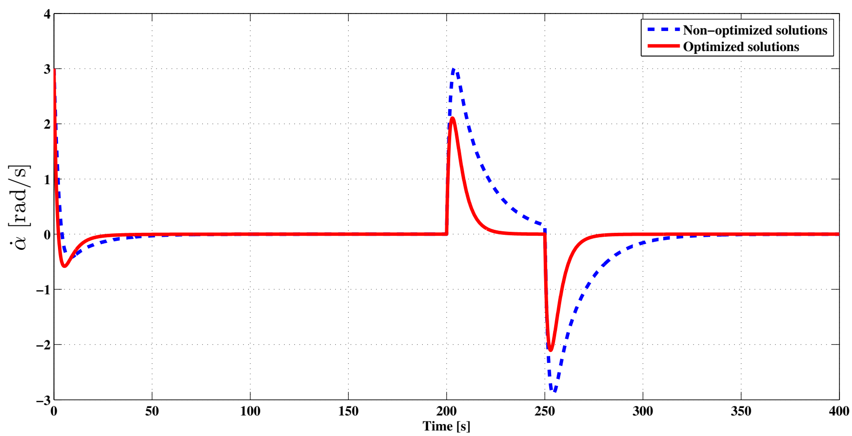

Fig. 4. Variation diagram of roll angular velocity $\dot{\alpha}$.

$J_o < J_n$, which means that the comprehensive performances of system has been significantly improved effectively under the multi-objective optimization.

6. CONCLUSIONS

In this study, a parametric approach is proposed to design dynamic compensator for quasi-linear systems. This parametric approach presents the completely parametric expressions of dynamic compensator and the left and right closed-loop eigenvector ma-

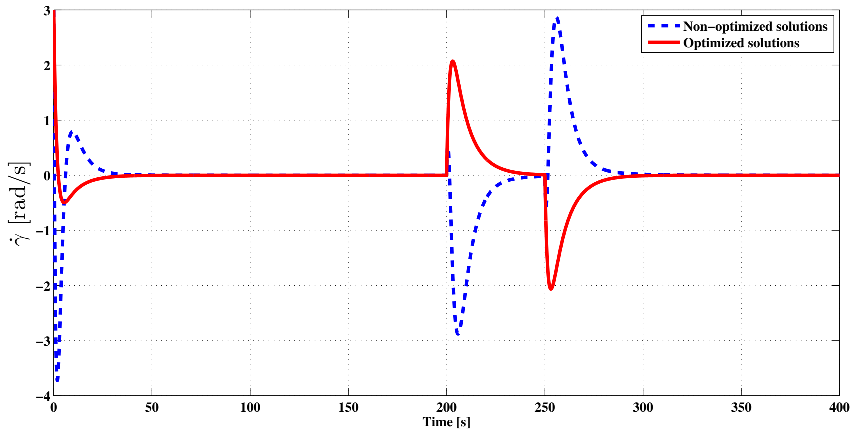

Fig. 5. Variation diagram of yaw angular velocity $\dot{\gamma}$.

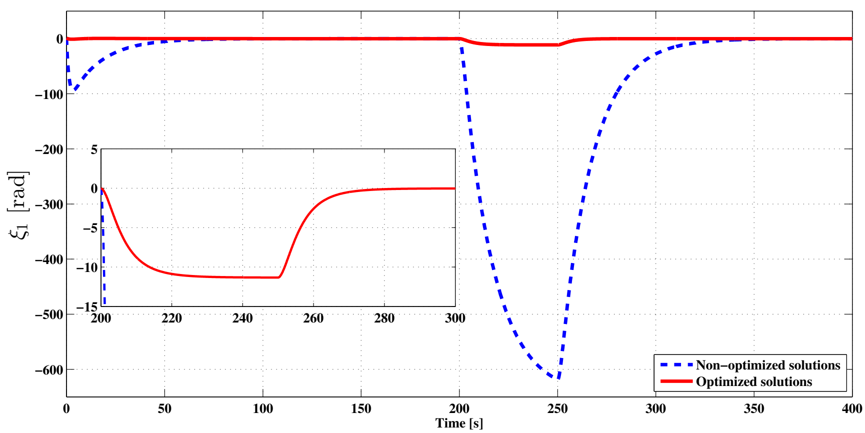

Fig. 6. Variation diagram of compensation vector ξ_1.

trices, which are both related to Λ, Z_b and Z_c.By using the parametric approach, the closed-loop system can be transformed into a linear time-invariant system with the desired eigenstructure. Moreover, arbitrary parameters Z_b and Z_c can be used to establish performance indexes such that a synthetic objective function is formulated to express the comprehensive performances. By using the DOFs of arbitrary parameters, a dynamic compensator can be found to satisfy robustness and low compensation gain criteria by solving a multi-objective optimization problem.

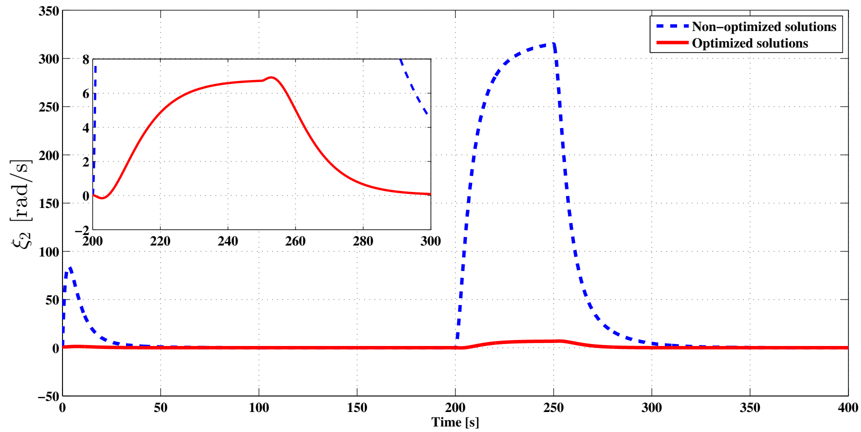

Fig. 7. Variation diagram of compensation vector ξ_2.

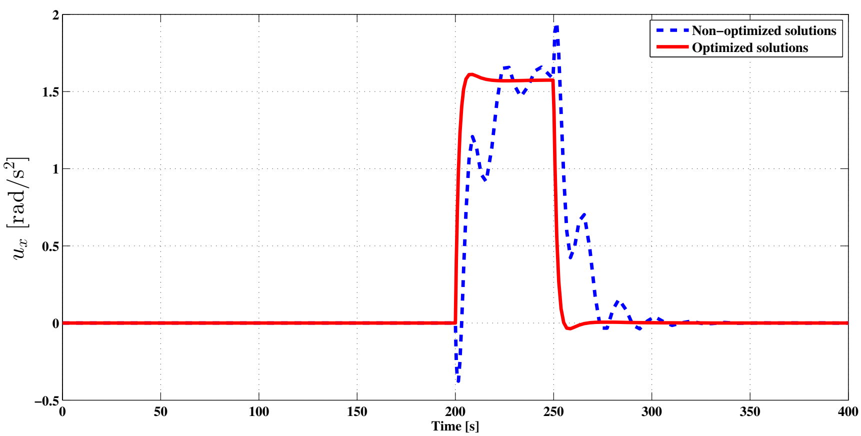

Fig. 8. Variation diagram of control signal u_x.

In the future, the main work focuses on two aspects. On the one hand, the presented results will be developed to second-order quasi-linear systems, which represent the dynamic process of many phenomena in nature and have a string of applications. On the other hand, a more generic multi-objective optimization method will be investigated.

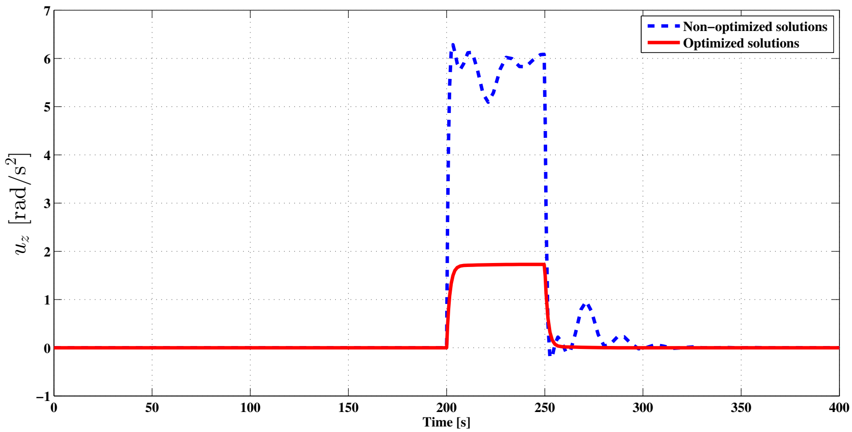

Fig. 9. Variation diagram of control signal u_z.

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