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PID AND FILTERED PID CONTROL DESIGN WITH APPLICATION TO A POSITIONAL SERVO DRIVE

Igor Bélai, Mikuláš Huba, Kevin Burn and Chris Cox

This paper discusses a novel approach to tuning 2DOF PID controllers for a positional control system, with a special focus on filters. It is based on the multiple real dominant pole method, applicable to both standard and series PID control. In the latter case it may be generalized by using binomial nth order filters. These offer filtering properties scalable in a much broader range than those allowed by a standard controller. It is shown that in terms of a modified total variance, controllers with higher order binomial filters allow a significant reduction of excessive control effort due to the measurement noise. When not limited by the sampling period choice, a significant performance increase may be achieved by using third order filters, which can be further boosted using higher order filters. Furthermore, all of the derived tuning procedures keep the controller design sufficiently simple so as to be attractive for industrial applications. The proposed approach is applied to the position control of electrical drives, where quantization noise can occur as a result of angular velocity reconstruction using the differentiated outputs of incremental position sensors.

Keywords: PID control, dominant pole placement, filtering, optimization

Classification: 93C-02

1. INTRODUCTION

Suppression of noise induced oscillations in control can radically reduce overall energy consumption, thermal losses, wear of mechanical parts, and acoustic noise. In some situations it may also significantly increase the control precision, due to the elimination of noise induced tracking error [5, 20]. Therefore, filtering of measurement and quantization noise has an important role in contemporary controller design. In the context of the most frequently used proportional-integral-derivative (PID) control, Segovia et al. [25] proposed the second order Butterworth filter for a first order plus dead time (FOPDT) system. Its time constant, expressed as a fraction of the integral time constant, was determined, together with other loop parameters, by an integral absolute error (IAE) or integral error (IE) optimization, subject to robustness constraints (M_s, M_t). The controller evaluation focused on the disturbance response.

The multiple real dominant pole (MRDP) method represents one of the first analytical methods used for controller tuning [23]. In numerous later applications to the design

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of simple controllers, we could also mention P and PD controller design for time delayed integral systems in [18], or the design of PI and PID controllers in [27, 28]. Whilst the above method directly gives both the dominant closed loop poles and the corresponding "optimal" controller parameters, in pole assignment control the choice of closed loop poles represents a crucial part of the controller tuning process. According to [1], the first publication devoted to this approach dates back to 1960 [24]. However, the pole assignment approach does not solve the problem of optimal controller tuning. It only brings a new formulation of the problem, where the basic question arises as to how the "optimal" pole vector can be chosen [4]. For example, in [30] the choice of a dominant triple loop pole under PID control has been based on an IAE optimization. In [31] the approach of minimizing the IAE criterion with a constraint of a damping factor and root ratios has been extended to a much broader class of time delayed systems by using a dimensionless model with similarity numbers of "laggardness" and "swingability". In [13, 14, 16] an approach to the optimal tuning of a PD controller for time delayed systems based on the triple real dominant pole method has been extended by the notions of equivalent poles and a delay equivalence. This made it possible to integrate PD controller tuning with an appropriate filter design. The filter calculation has also been included in the pole assignment controller design based on a choice of four closed loop poles by an iterative IAE optimization [7]. It has been carried out with a damping factor of 0.707 (characteristic for the Butterworth filter) and it included the filter time constant determination. However, the paper concluded that filters with a higher damping coefficient should also be considered in future research.

In [15] the MRDP method was applied to the integrated tuning of PI and PID controllers for FOPDT systems extended by additional noise filters (4). Such binomial filters, characterized by the order n and a single time constant T_f, have been proposed to keep a constant position of the dominant poles for any chosen n. The filter order ideally should be chosen to be as high as possible. However, since the T_f values corresponding to fixed loop dynamics decrease with increasing n, and for a chosen sampling period T_s there is a decreasing filtering ability for decreasing T_f/T_s, it is always possible to find an optimal filter order n. By simulation, it has been shown that for PI control, a filter order increase has resulted in significant control effort decrease up to $n = 2$. For PID control, the performance significantly improves with increasing n up to $n = 3$, which is in sharp contrast to the first order PID filters usually employed.

In this paper, the study carried out in [15], with noise generated in Matlab/Simulink by the Uniform Random Number block, is for the first time extended to a double integrator plus dead time (DIPDT) system. The optimal tuning rules for PID and filtered PID control (FPID_n), with different filter orders n, are derived by the multiple real dominant pole method and evaluated using simulation.

Next, ISA PID and generalized series FPID_n control are applied to a test bed with a servo positional system. This is a relatively fast motor torque generator, resulting in the necessity to consider so-called two pulse (2P) control, typically with acceleration and braking phases [13]. The control of such processes requires a determination of velocity, frequently implemented by differentiating the position signal from an incremental sensor. It represents a quantization noise source dominating both at low and high speeds of rotation. Whereas at low speed the velocity kicks produced by one position increment related to the time interval among the subsequent changes are lower than for a high speed of rotation, the relative quantization error values related to the absolute velocity values are lower for the high speed of rotation. Due to the highly non-linear nature of the speed dependent noise characteristics, the filtering approaches based on a priori known noise and system models (as, for example, the Kalman filter and other state observers [2, 3, 29]) usually present a complex problem. The situation is made more demanding by adverse effects of Coulomb friction, which in combination with integral controller action frequently leads to permanent oscillations. Without wishing to explore the nature of the processes that have arisen in detail, we will show that the proposed controller design can be successfully used in the given context. At the same time, however, we will also point to its substantial differences from the situation examined by the initial analysis.

The rest of the paper is structured as follows. Section 2 defines a model of the dominant plant dynamics and the binomial filters used for noise attenuation. In Section 3, optimal ideal PID controllers are derived for the double integrator system combined with a dead-time, or with a binomial filter $Q_n(s)$. Based on the requirement of a fixed position of the dominant pole, an equivalence of two types of delays in the considered closed loop is proposed, which enables an implementation of derivative action more reliable than that of the method usually applied in standard PID controllers. Section 4 defines the performance measures used for the subsequent control evaluation. Simulated and real time experimental results are derived and discussed in Section 5 and 6, and summarized in Section 7, together with an outline of future research directions.

2. CONSIDERED PLANT AND FILTER DYNAMICS

The modeled loop dynamics reflect a dominant subsystem approximated by the 2nd order transfer function

$$F(s) = \frac{K_s}{s^2 + a_1 s + a_0}. \tag{1}$$

The transfer function of a typical positional servo with $a_0 = 0, a_1 > 0$ may be expressed in the form

$$F(s) = \frac{K_s}{s^2 + a_1 s} = \frac{K}{s(T_p s + 1)} \tag{2}$$

with the time constant $T_p = 1/a_1$. To consider a variable load, unmodeled dynamics $F_a(s)$, and model uncertainties, an input disturbance d_i is added to the manipulated variable u_r

$$Y(s) = [U_r(s)F_a(s) + D_i(s)]\, F(s). \tag{3}$$

Thereby, the impact of torque generator (actuator) dynamics $F_a(s)$ will be considered by adjusting the controller parameters, not by modifications to its structure. The velocity signal of the derivative action is produced from the measured position $y(t)$ by differentiation. Both the velocity and the output signals may be filtered by an nth order binomial low pass filter $Q_n(s)$

$$Q_n(s) = \frac{1}{(T_f s + 1)^n}; \qquad 0 < T_f << T_p;\ n = 1, 2, \ldots \tag{4}$$

The approaches to identify the servo model with maximum precision usually work with $a_1 > 0$, which, for a constant input signal, corresponds to a limited steady-state angular velocity value. However, the approaches prioritizing simple, purely integral models neglect this parameter (which usually varies with time) by considering $a_1 = 0$. In this way, the loop dynamics including the dominant second order plant (2), actuator, and the necessary filters, will be simplified to a double integrator plus dead time (DIPDT) model

$$F_m(s) = \frac{K_m}{s^2} e^{-T_{\mathrm{dt}} s} \tag{5}$$

with a total dead time T_{dt} consisting of estimates of an actuator dead time T_a, a communication delay T_c and an equivalent filter delay estimate T_e

$$T_{\mathrm{dt}} = T_a + T_c + T_e. \tag{6}$$

Such "ultra-local" linear models[1] would exhibit linearly increasing angular velocity transients in their step responses. Thus, they may seem to be inadequate in the case of typical servo drives. However, when studying the control design based on the "flatness" theory [6, 8, 9], we may find that in so-called "model free", or "intelligent" PID control, such purely integral models are frequently used for an approximation of plants with constrained velocity signals in their step responses.

Purely integral models are also behind the "active disturbance rejection control" (ADRC) method using an extended state observer (ESO) [10, 11, 12, 19], which can approximate complex feedback dynamics by an equivalent input disturbance. They may also be found in several other robust control approaches, such as the ones mentioned by [17, 21].

3. PID CONTROL FOR A TIME DELAYED DOUBLE INTEGRATOR SYSTEM

By applying the MRDP method for tuning the ideal PID controller, approximations by DIPDT models (5) may be used for a broad spectrum of linear and non-linear, stable and unstable systems. To simplify the controller design, the complex and possibly non-linear plant internal feedbacks may be approximated by input or output disturbances.

3.1. 2DOF PID controller for the DIPDT plant by the QRDP method

According to the International Society of Automation (ISA), the traditional one-degree-of-freedom (1DOF) ideal PID controller[2] may be expressed as

$$C(s) = \frac{U(s)}{E(s)} = K_c \left(1 + \frac{1}{sT_i} + sT_D\right) = K_c + \frac{K_I}{s} + K_D s \tag{7}$$

K_c is the gain, T_i the integral and T_D the derivative time constant. Alternatively, it may also be expressed as a parallel PID with the gains K_c, K_I and K_D.

[1] which are intuitively expected to yield a required precision on a smaller neighborhood of an operating point than the "local" linear models considering nonzero parameters a_0 and a_1

[2] also called standard, or non-interacting, controller

Optimal controller parameters may be derived analytically by generalization of the approach applied in [27, 28]. For the DIPDT model, it starts with a derivation of the closed loop transfer functions of 1DOF PID control, where

$$\begin{aligned} F_{r0}(s) &= \frac{Y(s)}{R(s)} = \frac{K_c K_m(1+T_i s+T_i T_D s^2)}{T_i s^3 e^{T_{\mathrm{dt}} s}+K_c K_m(1+T_i s+T_i T_D s^2)} \\ F_i(s) &= \frac{Y(s)}{D_i(s)} = \frac{K_m T_i s}{T_i s^3 e^{T_{\mathrm{dt}} s}+K_c K_m(1+T_i s+T_i T_D s^2)}. \end{aligned} \quad (8)$$

The quadruple real dominant poles (QRDP) s_o of the characteristic quasi-polynomial

$$P(s) = T_i s^3 e^{T_{\mathrm{dt}} s} + K_c K_m(1+T_i s+T_i T_D s^2) \quad (9)$$

follow from the conditions

$$P(s_o)=0,\ \dot{P}(s_o)=0,\ \ddot{P}(s_o)=0,\ \dddot{P}(s_o)=0. \quad (10)$$

The optimal solution corresponds to

$$\begin{aligned} s_o &= -0.416/T_{\mathrm{dt}} \\ K_o &= K_{co}K_m T_{\mathrm{dt}}^2 = 0.125 \\ \tau_{io} &= T_{io}/T_{\mathrm{dt}} = 10.324 \\ \tau_{Do} &= T_{Do}/T_{\mathrm{dt}} = 4.043. \end{aligned} \quad (11)$$

The optimal controller parameters K_{co}, T_{io} and T_{Do} are given by the corresponding dimensionless (normed) parameters K_o, τ_{io} and τ_{Do}.

Overshoot, occurring typically in PID control of integral systems, may be eliminated by a 2DOF PID controller cancelling the numerator of the transfer function $F_{r0}(s)$ (8). By cancelling one or two of the dominant closed loop poles s_o (11) using the numerator of a prefilter

$$F_p(s) = \frac{cT_i T_D s^2 + bT_i s + 1}{T_i T_D s^2 + T_i s + 1} \quad (12)$$

the transients become faster. The “optimal” weighting cancelling a single closed loop pole s_o may be specified by

$$b_1 = \frac{1/|s_o|}{T_{io}} = -\frac{1}{\tau_{io} T_{\mathrm{dt}} s_o} = 0.233\ ;\quad c_1 = 0. \quad (13)$$

The fastest possible setpoint step responses correspond to the numerator of (12) factored to the form enabling cancellation of the double pole s_o, where

$$(s-s_o)^2 = s^2 - 2ss_o + s_o^2 = s^2 + bs/(cT_D) + 1/(cT_i T_D). \quad (14)$$

This yields the prefilter coefficients

$$b_2 = -2/(T_i s_o) = 0.466, c_2 = 1/(T_i T_D s_o^2) = 0.150 \quad (15)$$

The integral of absolute error (IAE) can be used to evaluate the loop performance for unit setpoint and input disturbance step inputs without the error sign changes given by the integrals of error (IE)

$$IAE_r = T_i(1-b);\ IAE_i = T_i/K_c. \quad (16)$$

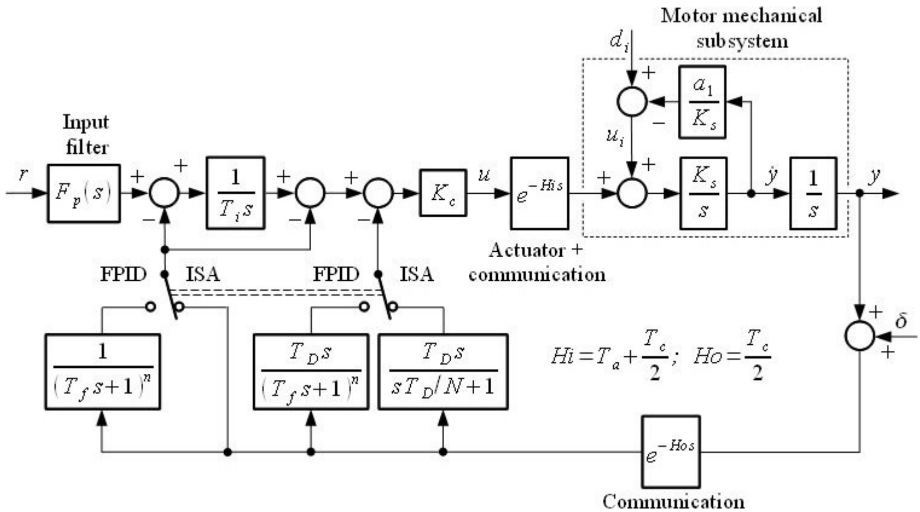

Fig. 1. Possible 2DOF ISA PID and FPID$_n$ implementation with filters in the feedback loop of the position servo control of the motor mechanical subsystem; δ – quantization noise, H_i, H_o - actuator and communication delays.

In optimal situations, with one or two poles cancelled

$$IAE_{r1} = 7.92T_{\mathrm{dt}};\; IAE_{r2} = 5.96T_{\mathrm{dt}};\; IAE_i = 82.73K_mT_{\mathrm{dt}}^3. \tag{17}$$

Thus, both the optimal controller tuning and the optimal loop performance have been simply expressed using a two-parameter model of the DIPDT plant (5).

3.2. 2DOF ISA PID controller

Further problems in the application to real systems are related to the implementation of the improper derivative term in (7). In the ISA 2DOF PID controller equation

$$U(s) = K_c\left\{bR(s) - Y(s) + \frac{R(s) - Y(s)}{T_is} + \frac{T_Ds[cR(s) - Y(s)]}{1 + sT_D/N}\right\}. \tag{18}$$

It is solved by introducing a differentiator filter with a time constant of T_D/N (see also Figure (1)). Thereby, $N \in [5, 20]$ is usually chosen. With higher values, $N \approx 20$, it is possible to come closer to the ideal responses (Figure 2). A decrease in this value to the range $N \in (5, 10)$ leads to performance degradation. However, amplification of higher frequencies introduced by the noise is proportional to N. Thus, in balancing the trade-off between noise attenuation and transient speed, the 2DOF ISA PID offers just limited possibilities.

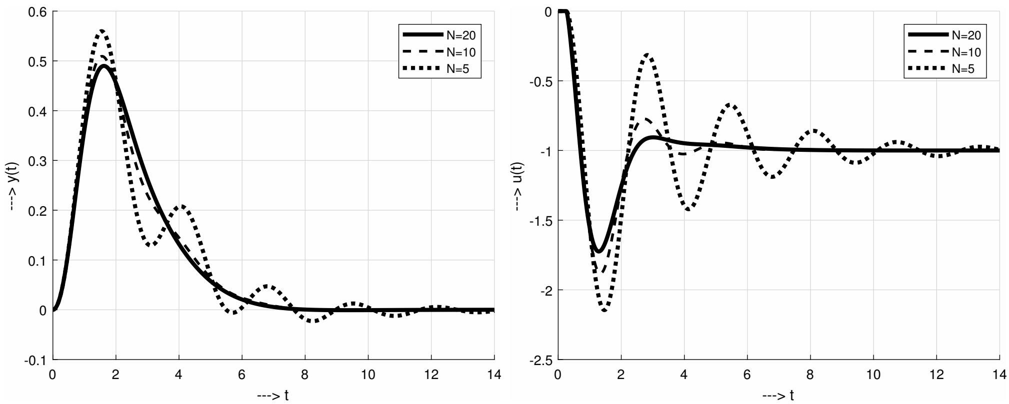

Fig. 2. Plant output and input after input disturbance step from $d_i = 0$ to $d_i = 0.08$ at $t = 0$: Impact of the parameter N of the ISA 2DOF PID controller on the loop performance in the DIPDT plant control, $K_m = 1; T_{\rm dt} = 0.25$; $n = 1$; $N = \{5, 10, 20\}$; $(T_D/N)/T_{\rm dt} = \{0.162, 0.404, 0.809\}$.

3.3. Generalized 2DOF series controller design by the MRDP method

In series 2DOF PID control, the filters affect all controller actions, when

$$U(s) = K_c\left\{\left[b + \frac{1}{sT_i}\right]\frac{1 + scT_D}{1 + sT_D/N}W(s) - \left[1 + \frac{1}{sT_i}\right]\frac{1 + sT_D}{1 + sT_D/N}Y(s)\right\}. \tag{19}$$

By replacing $T_D/N = T_f$, and considering the binomial filter (4) in combination with the ideal PID (7) and the prefilter (12) instead of $1/(1 + T_f s)$, a generalized solution may be derived (see Figure 1[3]). Since for loops containing filter dynamics combined with dead time, the multiple real dominant pole method leads to overly complex formulae, a simplified solution will be proposed based on an equivalence of the dead time and filter time constants. Similar to [15], this will be based upon firstly considering loops with just one type of delay, deriving for them optimal tuning guaranteeing the multiple real dominant poles, and then requiring for both situations the same position of the dominant closed loop poles.

The close loop transfer functions with the plant K_m/s^2, 1DOF PID control, and $Q_n(s)$ are

$$\begin{aligned} F_{r0}(s) &= \frac{Y(s)}{R(s)} = \frac{K_c K_m(1 + T_i s + T_i T_D s^2)}{T_i s^3(1 + T_f s)^n + K_c K_m(1 + T_i s + T_i T_D s^2)} \\ F_i(s) &= \frac{Y(s)}{D_i(s)} = \frac{K_m T_i s}{T_i s^3(1 + T_f s)^n + K_c K_m(1 + T_i s + T_i T_D s^2)}. \end{aligned} \tag{20}$$

[3]Location of the filters in the feedback loop has been chosen only for a more compact picture - the higher order filters may be assembled by putting together terms located at both the controller input and output.

n	$-s_nT_f$	$K_PK_mT_f^2$	T_D/T_f	T_i/T_f	b	c	$d_n = T_e/T_f$	$f_n = T_f/T_e$
1	0.2500	0.01798	12.161	35.226	0.641	0.297	1.663	0.6013
2	0.1551	0.02115	10.115	26.523	0.486	0.155	2.682	0.3729
3	0.1127	0.01057	14.188	36.928	0.481	0.150	3.689	0.2711
4	0.0886	0.00633	18.247	47.297	0.477	0.148	4.693	0.2131
5	0.0730	0.00421	22.301	57.649	0.475	0.146	5.696	0.1756

Tab. 1. Tuning parameters for particular n, $T_{\mathrm{dt}} = 0$.

According to the MRDP method, the roots s_n of the characteristic polynomial

$$P_n(s) = T_i s^3 (1 + T_f s)^n + K_c K_m (1 + T_i s + T_i T_D s^2) \tag{21}$$

have to fulfill conditions equivalent to (10). Then, from the requirement of a fixed position of the real dominant pole for both types of the considered delays

$$s_o = s_n \ , \quad n = 1, 2, 3, \ldots \tag{22}$$

it is possible to derive an equivalence between the loops containing only $Q_n(s)$ (specified by n and the time constants T_f) and only the equivalent dead time $T_{\mathrm{dt}} = T_e$ (6). Such an equivalence may be expressed in the form

$$T_f = f_n T_e \ , \quad n = 1, 2, 3, \ldots \tag{23}$$

or inversely as $T_e = d_n T_f$. The corresponding parameters f_n and $d_n = 1/f_n$ may be found in Table 1.

It is well known that for the sake of an optimal controller tuning, several dead times included in a control loop may be replaced by a single time delay corresponding to their sum. This might be used to formulate the following procedure for the controller tuning.

Proposition 3.1. (Integrated PID & Filter $Q_n(s)$ tuning procedure)
After specifying the dead time T_e, equivalent under (22) to a chosen $Q_n(s)$ as $T_e = d_n T_f$, the controller for a DIPDT plant model combined with the binomial filter, i. e. with a delay T_{dt} composed from several dead times (6), has to be tuned according to (11).

In many situations it may be convenient to choose some $T_e > 0$ and then to apply the tuning (11), and for a chosen n to calculate the filter time constant $T_f = f_n T_e$. Since it may be unrealistic to analytically design controllers in loops with significant measurement noise, it is often more practical to perform several iterations of the design process by trial and error.

Remark 3.2. ("Trial and error" character of the controller tuning)
Similarly the pole assignment method does not yield the optimal pole choice and the optimal filter parameters n and T_f are usually specified by trial and error methods.

Of course, the question also arises as to how precisely the derived equivalence holds for loops with both types of delays and for systems with a_1 or a_0 not equal to zero. Since an analytical proof to this proposition would be too complex (see, for example analysis of the optimal PD control in [13, 14, 16]), we are going to test this proposition experimentally.

4. PERFORMANCE MEASURES

In setpoint tracking, the task is to bring the system's output y from an initial value y_0 to a piecewise constant setpoint r monotonically and in the shortest possible time, with a minimal IAE (Integral of Absolute Error) value. For a step disturbance, $r(t) = 0$ and the excited output has to be returned monotonically with a minimal IAE, where

$$IAE_i = \int_0^\infty |e(t)| \, \mathrm{d}t; e(t) = r(t) - y(t). \tag{24}$$

To characterize the output deviations from monotonicity, the total variance (the total sum of absolute increments, [26]) exceeding the net output change $|y_\infty - y_0|$

$$yTV_0 = \int_0^\infty \left(\left| \frac{\mathrm{d}y}{\mathrm{d}t} \right| - sign(y_\infty - y_0) \frac{\mathrm{d}y}{\mathrm{d}t} \right) \mathrm{d}t \approx \sum_i \left(|y_{i+1} - y_i| \right) - |y_\infty - y_0| \tag{25}$$

which emphasises the contribution of excessive increments and yields the best measure of the "smoothness" of the output change. The value $yTV_0 = 0$ corresponds to an ideally smooth output change, else $yTV_0 > 0$.

Since an ideal output disturbance step response (the full curve in Figure 2, left) consists of two monotonic intervals separated by the maximum y_m, the deviations from such an ideal response denoted as "one pulse" (1P) will be evaluated according to

$$yTV_1 = \sum_i \left(|y_{i+1} - y_i| \right) - |2y_m - y_\infty - y_0| \, . \tag{26}$$

Also, during an input step response, excessive control effort exceeding the inevitable acceleration and braking should be kept as low as possible [13]. This focus on excessive control effort represents one of the most important differences from the traditional quadratic optimal control dealing with minimization of overall controller activity. Attention is paid to deviations from the input transients from an unavoidable two-pulse (2P) input shape given by the inversion of the plant model dynamics [13]. Geometrically, an ideal 2P input shape (the full curve in Figure 2 right) is specified by two extreme points u_{m1}, u_{m2} occurring for $t \in (0, \infty)$ and separating three monotonic control intervals. Excessive control effort is then evaluated according to

$$uTV_2 = \sum_i \left(|u_{i+1} - u_i| \right) - |2u_{m1} - 2u_{m2} - u_\infty - u_0| \, . \tag{27}$$

For ideal 2P control functions $u(t)$, $uTV_2 = 0$.

5. SIMULATED EVALUATION FOR A DIPDT PLANT

The impact of measurement noise on the loop performance and the validity of Proposition 3.1 will be tested in several steps.

5.1. Loop evaluation, no noise

Firstly, the controller tuning (11) derived by the QRDP method has been applied to the 2DOF ISA controller for a DIPDT system with a standard first order filter ($n = 1$) and three different values of N (Figure 2).

Then, the FPID_n controllers (with filters $Q_n(s)$) tuned by an equivalent dead time T_e, $T_f = f_n T_e, n \in [1,5]$ have been tested without any measurement noise for $T_e/T_d \in [0.01,2], T_d = 0.25, K_m = 1, T_s = 0.001$ (Figure 3). Obviously no observable deviations from 1P shapes at the plant output and 2P shapes at the plant input occur in the tested ranges of n and T_e.

By considering more detailed noise characteristics of the input disturbance step responses in Figure 2 and Figure 3 shown in Figure 4, it is apparent that they are always influenced by a low level internal noise resulting from imperfections in the numerical integration. The plant input noise characteristics in Figure 4, left illustrate the impact of

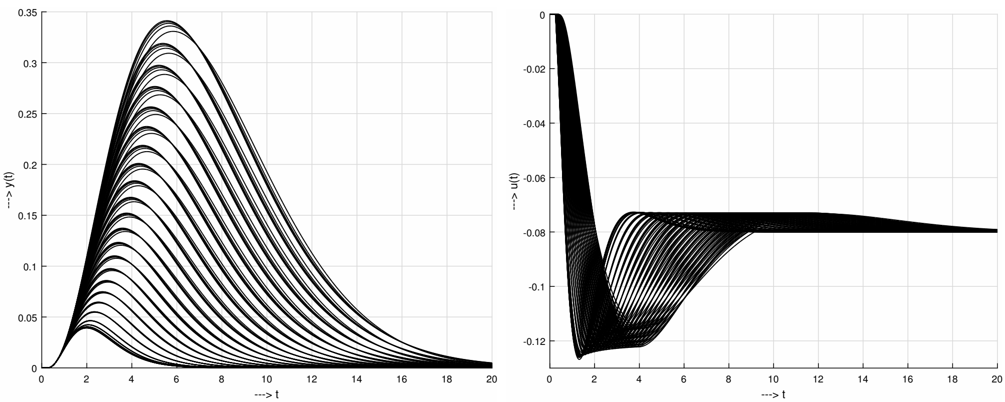

Fig. 3. Input and output of the loop with DIPDT plant with the FPID control for the input disturbance step change from 0 to 0.08, no external noise, $T_e/T_{\mathrm{dt}} \in [0.01,2]$, $n \in [1,5]$, $T_{\mathrm{dt}} = 0.25$, $K_m = 1$, $T_s = 0.001$.

the ISA controller parameter N and the impact of the generalized series FPID_n tuning parameter T_e on the loop performance expressed in the plane (uTV_2, IAE_i). For the series FPID_n, the impact of the internal noise resulting from the numerical integration is significantly lower than for the best ISA performance with $N = 20$. The dependence on n is negligible. Similar conclusions may be drawn from the plant output noise characteristics expressing dependence of IAE_i on yTV_1 (Figure 4, right).

Remark 5.1. (Choice of the ISA controller filter parameter N)
The choice of the differentiator filter parameter N strongly influences the loop performance (Figure 2), which may be demonstrated by the output shape deviations yTV_1. Although there seems to be no practical difference between $yTV1 = 10^{-20}$ or $yTV1 = 10^{-5}$, attention has to be paid to their ratio, indicating a strong difference in the noise attenuation. Thus, for $N = 5$ it is already possible to observe a significant performance

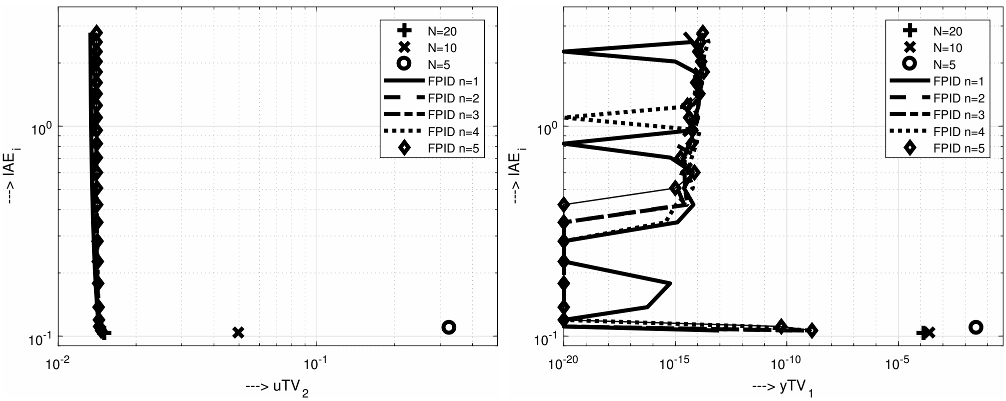

Fig. 4. Noise characteristics of the loop with DIPDT plant and input disturbance steps, no external noise, for the ISA PID with $n = 1$ and $N = \{5, 10, 20\}$; for FPID_n with $n \in [1, 5]$, $T_e/T_{dt} \in [0.01, 2]$; $T_{dt} = 0.25, K_m = 1, T_s = 0.001$.

degradation (Figure 2) expressed by the increase in shape related deviations, which are visible in the noise characteristics. As we will see when adjusting the noise attenuation in a loop with external noise, for the choice of the derivative filter with low shape related deviations, the ISA controller yields just a narrow working range.

Remark 5.2. (Impact of the simplified FPID_n tuning by T_e)
Under generalized series FPID_n control, the noise brought by the numerical simulation has much lower impact on the loop performance than for the ISA PID control and there appears to be no performance degradation due to the tuning parameter changes. An increase of the tuning parameter T_e contributes only to an IAE_i increase without causing any shape related deviations (Figures 3−4). The negligible shape related deviations at the plant input and output confirm that the simplified tuning procedure given by Proposition 3.1 offers an excellent tool for FIPD_n controller tuning. It may be used for a broad range of the tuning parameter T_e (broad range of the transient velocities) without a negative impact on the shape of the transient responses.

Remark 5.3. (Basic restrictions on FPID_n tuning)
In applying Proposition 3.1 to the DIPDT plant, the only restriction is that the filter time constants have to fulfil the requirement $T_f >> T_s$, whereby T_s denotes the sampling period used for the quasi-continuous-time controller implementation.

5.2. Evaluation under a measurement noise impact

Next, evaluation of the loop properties will continue with the loop simulation under a measurement noise generated by the Simulink block Uniform Random Number, with the integration step $T_s = 0.001$ and with amplitudes $|\delta| < 0.01$ and $|\delta| < 1$. This block

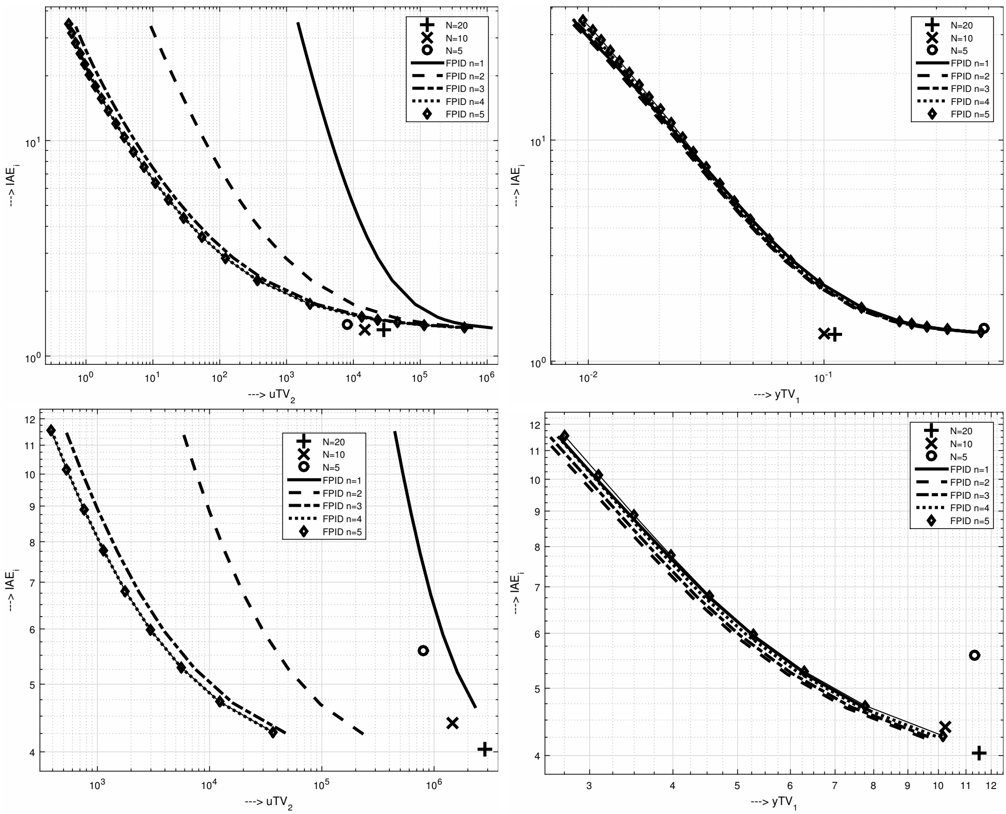

Fig. 5. Noise characteristics of the loop with DIPDT plant and input disturbance steps, external noise with amplitudes $|\delta| < 0.01$ (above) and $|\delta| < 1$ (below), $T_e/T_{dt} \in [0.01, 2]$, $T_{dt} = 0.25$, $K_m = 1$, $T_s = 0.001$.

Controller	uTV_2	%	$ISA/FPID$
ISA PID	$7.999.10^5$	100	-
$FPID_1$	$1.362.10^6$	170.27	0.587
$FPID_2$	$3.927.10^4$	4.91	20.37
$FPID_3$	$5.481.10^3$	0.69	145.94
$FPID_4$	$4.219.10^3$	0.53	189.60
$FPID_5$	$4.237.10^3$	0.53	188.79

Tab. 2. Comparing ISA PID, $N = 5$ and $FPID_n$, $n \in [1, 5]$ for $T_s = 1$ms, $IAE_i = 5.5807$, $K_m = 1$, $T_{dt} = 0.25$.

generates random numbers uniformly distributed over an interval $(-\delta, \delta)$. Its results are well illustrated by the noise characteristics in Figure 5 and by the following remarks.

Remark 5.4. (Application areas for the ISA PID and $FPID_n$ control)
Noise characteristics in Figure 5 show that under a relatively low noise level the fastest transients may be achieved by the ISA PID. $FPID_n$ control with $n \geq 3$ may significantly reduce the excessive control effort, however, at the cost of slower transients.

For higher noise amplitudes and $n > 1$, $FPID_n$ control may significantly reduce excessive control effort (to decrease the uTV_2 values) and simultaneously also yield sufficiently fast transients (with low IAE_i values). Still, this requires the possibility to work with $T_e << T_{dt}$ and, at the same time, with $T_f >> T_s$ (see Remark 5.3).

Remark 5.5. (Optimal choice of the filter order n)
For the given integration step (or the sampling period for noise generation) $T_s = 1$ms, significant performance improvements may be observed while increasing the $Q_n(s)$ order from $n = 1$ up to the value $n = 3$. The best performance corresponds to $n = 4$ (Table 2), when the excessive control effort uTV_2 produced by $FPID_n$ is nearly 190 times below the ISA PID noise level. For some noise and disturbance step amplitudes, this ratio may yet be significantly higher. Hence, the situation in industrial control, where typically first order filters are used, should be revised. This point may also give new impulse to several recent studies dealing mostly with 2nd order filters [25, 31].

Remark 5.6. (Impact of the noise distribution)
Performance improvements due to noise filtering have been achieved under the assumption that during the filtering of measurement noise with a zero mean value, its amplitude at the filter output for increasing n and T_f decreases to zero. Thus, the filter design does not need identification of the actual noise distribution, which in practice usually depends on the working point and varies with time. Furthermore, in the following illustration example, it will depend not only on the actual angular velocity, but also on the incremental sensor resolution and the applied sampling period.

Remark 5.7. (Noise elimination by averaging)
Performance improvements may be achieved by decreasing the sampling period, since by averaging higher numbers of output samples, the impact of the measurement noise

converges to zero. This, however, does not hold for the quantization noise considered in the subsequent example of servo drive control. In this case, by decreasing the sampling period, the velocity "kicks" produced by one increment divided by the sampling period actually increase.

6. APPLICATION TO SERVO DRIVE CONTROL

When wishing to demonstrate the contribution of the newly presented approach to controller design for dominant 2nd order plants, the most demanding solution would be to consider a plant with two unstable, or marginally stable, poles. Stable hydraulic or thermal processes considered by [25], or [22] do not represent such a challenge - they may be stabilized without the derivative action and thus it is unclear if they really test all features of the proposed controller. With respect to this, from the spectrum of processes available for an experimental verification of the above analysis and design, we have chosen a positional servo drive system. With $a_0 = 0$ and relatively short delays, similar to Figure 2, it routinely requires a 2P control signal [13] consisting of three almost monotonic intervals, which may not be achieved by a simpler control. However, as will be explained later, several nonlinear effects and properties of the included noise signal mean that this process is far from ideal for demonstrating the presented approach.

Position servo drives today typically use incremental sensors for output measurement. Velocity reconstruction is then mostly based on output differentiation using derivative action. Furthermore, when the decreased control precision leads to changes of the control error sign, in combination with Coulomb friction and the integral action, permanent oscillations occur. With respect to the precision of the velocity calculation, the sampling period T_s used for the differentiation should be the shortest possible. Usually it is set to the hardware limits, which determine the resulting loop performance. This, however, in combination with the output quantization, leads to significant velocity kicks, which are reciprocal to T_s. They result in current and generated torque kicks, possibly accompanied by acoustic noise. Since the effects of output quantization in the calculation of the control signal may not be eliminated by averaging, as for the random signals from previous analysis, there is a question as to whether it can influence the closed loop performance.

6.1. Experiment setup

An experimental test bed (Figure 6) contains two electric drives connected by their shafts. The controlled BLDC motor utilizes the second DC motor as the load drive enabling emulation of load torque changes. The shaft position is measured through an incremental rotary encoder (IRC) with 1024 imp./rev. The identified parameters are:
$J = 2\ 10^{-5}$ [kgm^2], moment of inertia;
$T_{L0} = 3.5\ 10^{-5}$ [Nm], Coulomb friction;
$K_s = 1/J = 5\ 10^4$ [kg^{-1}m^{-2}], plant model gain;
$a_1 = 0.481$ [N.s.rad^{-1}kg^{-1}m^{-1}], internal plant feedback coefficient;
$a_0 = 0$ (no position feedback, system has an integral character);
$T_a = 0.2$ [ms], time constant of the torque generator;
$\Delta\varphi = 0.001534$ [rad], position sensor resolution;

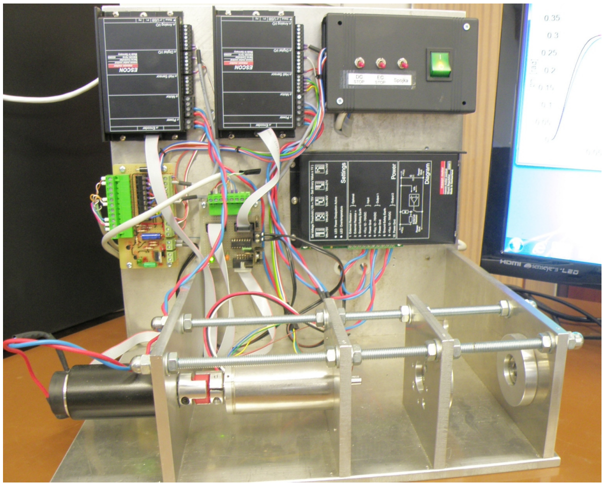

Fig. 6. Experimental setup.

The controller has been implemented in Matlab/Simulink by using a standard PC. For the controller design, the plant (1) plus the actuator model have been reduced to the simplest possible form - a double integrator model (5). Thus, the estimate of the internal feedback coefficient a_1 has not been used in the controller tuning.

6.2. Experiment 1, $T_s = 0.25$ [ms], $T_{dt} = T_a + T_s = 0.45$ [ms], $T_c = 0$

The sampling period has been set to $T_s = 0.25$ [ms]. The actuator delay, together with the sampling period T_s, were approximated by a transport delay $T_{dt} = T_a + T_s = 0.45$ [ms]. With respect to Remark 5.3 requiring $T_f >> T_s$, this configuration does not allow any repetition of the simulated experiment in its full, previously proposed extent with $n \in [1, 5]$ and $T_e/T_{dt} \in [0.01, 2]$. Instead, we worked with $T_e/T_{dt} \in [1, 20]$. It means that already for $n = 4$ and $T_e = T_{dt}$ the shortest filter time constant $T_f = f_n T_{dt} \approx 0.18(0.45 + 0.45) = 0.16$ms does not guarantee appropriate filtering (Remark 5.4) and at least for the shortest values T_e the use of higher order filters is not expected to bring any meaningful results.

A step change of the input disturbance d_i has been emulated by the load torque step change T_L from 0 to -0.08 [Nm].

The disturbance step responses with the ISA PID controller in Figure 7 demonstrate the dominant impact of the output quantization and of the Coulomb friction.

In contrast to the oscillatory response of the DIPDT plant for $N = 5$, the output performance is now (due to the presence of plant viscous damping) close to its behaviour for $N = 10$ (compare with Remark 5.1). Therefore, the noise characteristics of the FPID_n achieved for $T_e/T_{dt} \in [1, 20]$ (Figure 8) partially confirm the above results from

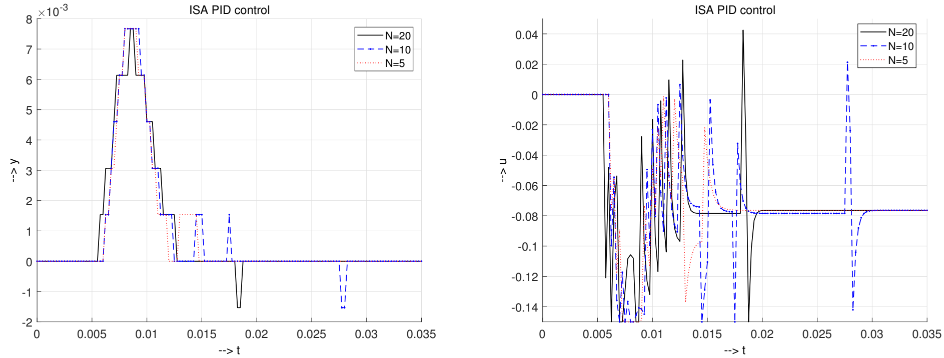

Fig. 7. Input disturbance step responses of the loop with ISA PID at the plant output (left) and input (right), $T_{dt} = 0.45$ms, $K_m = 50000$, $T_s = 0.25$ms.

the DIPDT analysis. Notwithstanding this, it has still been possible to reduce excessive control effort more than 50 times at the plant input (Figure 8 left). However, it is at the cost of increased IAE. To improve this performance, the experiment should be carried out with a significantly shorter sampling period.

Also, the nonlinear effects fully changed the yTV_1-IAE_i characteristic (Figure 8 right). By a weaker controller tuning (increased T_e) the shape-related deviations at the output increase.

6.3. Experiment 2, $T_s = 0.1$ [ms], $T_{dt} = T_a + T_s + T_c = 4.3$ [ms]

In order to be able to decrease the minimal value of T_e/T_{dt} applicable under the existing hardware limitations, the sampling period has been set to the hardware limit $T_s = 0.1$ [ms]. At the same time, a communication delay $T_c = 4$[ms] has been introduced in Simulink, which may correspond to situations typical for many network applications. One half of T_c has been added to the plant input, the other half to the plant output (Figure 1). Such a modification not only mimics real situations, but also makes the control problem more difficult and magnifies all design imperfections.

Evaluation of different situations using the ISA PID and FPID$_n$ controllers (see Figures $9-10$) yields conclusions which are now much closer to the comments from Remark 5.4.

The noise characteristics have been measured for $T_e/T_{dt} \in [0.1, 10]$. In comparison to the above experiment with $T_c = 0$, the prolonged loop delays increased the disturbance impact at the plant output (compare Figure 9 with Figure 7).

However, due to the decreased T_e/T_{dt} values, in some aspects the responses are now closer to the DIPDT simulation case and Remark 5.4 as above.

The noise characteristics in Figure 10, left, show that by using higher order filters the excessive control effort may be reduced significantly. However, a full correspondence with the simulation analysis would require T_e values to be 10 times shorter. In the sense

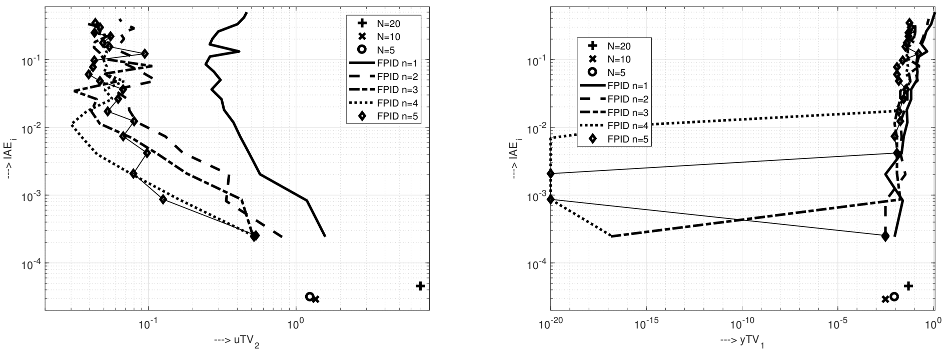

Fig. 8. Noise characteristics of the loop with real servosystem ,
$T_e/T_{dt} \in [1, 20]$, $n \in [1, 5]$, $T_d = 0.45$ms, $K_m = 50000$, $T_s = 0.25$ms.

of Remark 5.4 that would also mean sampling periods T_s of at least 10 times shorter.

On the other hand, the yTV_1-IAE_i characteristics (Figure 10 right) again illustrate that the application of a softer controller tuning leads to increased output oscillations. Thus, with respect to the existing hardware limitations and Remark 5.4, in applications with focus on minimal IAE values (i. e. on the fast transients) the ISA PID control might still seem to represent the optimal solution.

However, at this point it should be noted that the increased IAE_i value of the FPID_n control is due to the higher amplitude of the initial output pulse ($\max(y) > 0.6$), whereas for ISA PID $\max(y) < 0.6$. Surprisingly, this does not necessarily mean slower responses: when evaluating the speed of responses by the settling time t_s defined by a ± 0.01 broad error band around zero (the dashed line, see Figure 9), the FPID_n control with $t_s = 0.11$s is more than 50% faster than the ISA PID ($t_s = 0.17$s), and it offers smoother responses both at the plant input and output. In explaining these differences in behavior, it is to remember that in the ISA PID design the filter time constant has not been taken into account. In the FPID_n tuning, T_d has been increased by T_e, which according to (11) leads to increased T_i values.

7. CONCLUSIONS AND FUTURE WORK

It has been shown that the newly presented PID tuning procedure derived by the multiple real dominant pole (MRDP) method for the double integrator plus dead time (DIPDT) model may be applied to a broader class of systems with 2nd order dominant dynamics. Several problems related to the implementation of the ISA PID controller and an augmented series PID controller with a generalized nth order binomial filter (FPID_n) have been discussed. An integrated design of the FPID_n controller has been generalized from the FOPDT [15] to the DIPDT systems.

ISA PID control has been shown as appropriate for applications with a relatively low noise level which do not require an additional noise filter.

The advantages of the novel FPID_n control with respect to the commonly applied ISA

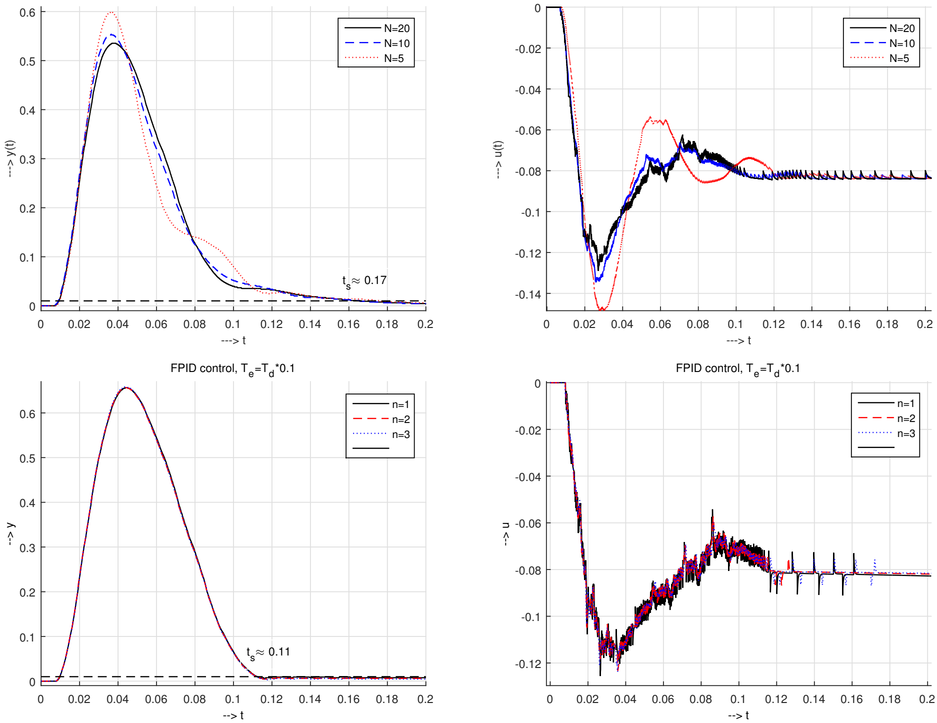

Fig. 9. Step responses with the ISA PID (above) and the FPID$_n$ control (below) $T_{dt} = 4.3$[ms], $K_m = 50000$, $T_s = 0.1$[ms], $T_c = 4$[ms], $T_e = 0.1T_{dt}$.

or series PID controllers with first order filters have been demonstrated by simulation and partially also by real time experiments. The analysis confirmed the conclusions formulated for FOPDT systems [15] which stated that significant loop performance improvements may be achieved by increasing the order of the binomial filters $Q_n(s)$ up to $n = 3$. These are, however, conditioned by use of sufficiently short sampling periods employed for the quasi-continuous controller implementation.

In order to fully demonstrate such improvements of FPID$_n$ control with higher n, we have been developing a new test bed for positional servo drives with a significantly faster FPGA based controller. The available servo system is going to be extended by a variable inertial moment, which will permit us to handle more effectively the robustness issues.

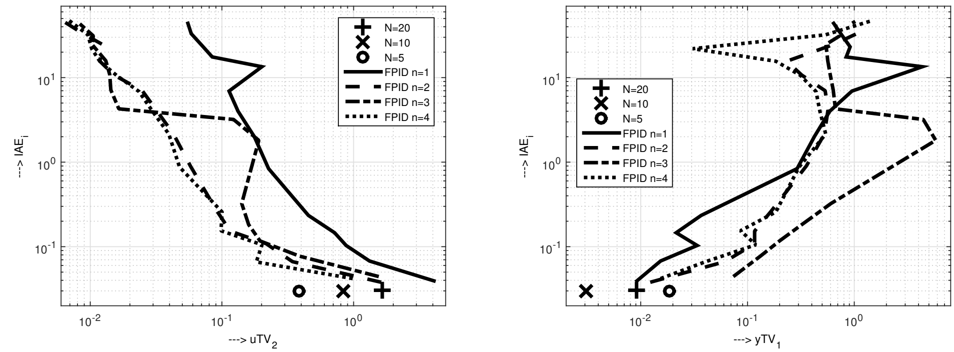

Fig. 10. Noise characteristics of the loop with real servosystem and a communication delay $T_c = 4$[ms], $T_e/T_{dt} \in [0.1, 10]$, $T_{dt} = 4.3$[ms], $K_m = 50000$, $T_s = 0.1$[ms], $n \in [1, 4]$.

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