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H_∞ SLIDING MODE CONTROL FOR MARKOV JUMP SYSTEMS WITH INTERVAL TIME-VARYING DELAYS AND GENERAL TRANSITION PROBABILITIES

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This paper is devoted to design H_∞ sliding mode controller for continuous-time Markov jump systems with interval time-varying delays and general transition probabilities. An integral sliding surface is constructed and its reachability is guaranteed via a sliding mode control law. Meanwhile, a linearisation strategy is applied to treat the nonlinearity induced by general transition probabilities. Using a separation method based on Finsler lemma to eliminate the coupling among Lyapunov variables and controller parameters, sufficient conditions for asymptotically stochastic stability of sliding mode dynamics are formulated in terms of linear matrix inequalities. Finally, a single-link robot arm system is simulated to demonstrate the effectiveness of the proposed method.

Keywords: Markov jump systems, time-varying delays, sliding mode control

Classification: 93E03, 93D09, 93D15

1. INTRODUCTION

Markov jump systems (MJSs) is a special class of hybrid systems modeled by differential equations and a finite-state Markov process. Due to its ability to model practical systems with random jumps, wide-spread applications of MJSs have been made in fault-tolerant control systems [1], networked control systems [2] and solar energy systems [3]. Regard this topic, many interesting theoretical results about the analysis and synthesis of the stability for MJSs can be found in [4, 5, 6, 7, 8]. However, the obtained results are based on the hypothesis that all transition probabilities (TPs) are completely known, which renders it hard to be employed to engineering problems because of the existence of partly unknown TPs in practice. Via a robust methodology, the uncertain TPs with norm bounded and partly known TPs are handled, please see in [9, 10, 11, 12, 13, 14, 15, 16] and the references therein. To mention a few, [11] derives the non-fragile finite-time H_∞ control of continuous-time MJSs with uncertain TPs. H_∞ filtering and finite-time H_∞ control of MJSs with partially unknown TPs are presented in [14, 15], respectively.

On the other hand, sliding mode control (SMC), as a particular type of variable structure control, has attracted significant research attention in past decades. Owing to

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its favorable features, such as fast response, easy realization, insensitivity to variation in plant parameters and complete rejection of external perturbations, SMC can be seen as an effective robust control method for nonlinear dynamic systems, please see [17, 18, 19] and the references therein. Specifically, [17] studies the robust H_∞ control problem of nonlinear stochastic systems via a sliding mode control method. In [18], considering MJSs with actuator degradation, a SMC based on an adaptive scheme to estimate the fault factor is developed. For uncertain nonlinear systems, [19] designs the robust finite-time sliding mode controller via a set of linear matrix inequalities (LMIs). It is noted that the time delay is not taken into account in most above works, which could degrade system performance and even cause system instability [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. To cope with this, [30] addresses the problem of robust H_∞ SMC for uncertain neutral-type MJSs with time-varying delays. [31, 32] investigates the SMC problem for Markov jump time-delay systems. [33] studies the issue of event-triggered SMC for discrete-time switched systems with time-varying network communication delay. However, TPs in these results are required to be completely known, which is hard to be obtained in practical systems. Once they are uncertain or unknown, the proposed approaches in [30, 31, 33, 32] will lose the effectiveness to handle the nonlinearities induced by unknown TPs. Apart from this, the method based on Jensen inequality to deal with the time delay in these works still can be improved to reduce design conservativeness.

Stimulated by the above mentioned points,the delay-dependent H_∞ SMC for MJSs is investigated with general TPs assumed as known, uncertain with known lower and upper bounds, and completely unknown. At first, an integral sliding surface is proposed. Then, the property of TP matrix is used to overcome the nonlinearity incurred by uncertain and unknown TPs. With the help of Finsler lemma, the coupling between Lyapunov variable and controller gain is eliminated. In the specified sliding surface, conditions of the stochastic stability of the closed-loop system is obtained in the form of LMIs. Furthermore, a synthesized SMC law is derived to guarantee the existence of the composite sliding motion. Finally, a single-link robot arm system is provided to demonstrate the validity of the established results.

The rest of the paper is divided into several parts. The problem statement and some preliminaries are arranged in section 2. Then, the stability analysis and the design of H_∞ sliding mode controller with complete known TPs and general TPs are discussed in Section 3, respectively. Section 4 shows the simulation result with a single-link robot arm system. Section 5 is a summary of the whole paper.

Notation: Throughout the paper $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes $n\times m$ real matrices. The notation $R > 0$ (< 0) stands for R is symmetric and positive (negative) definite. $(\cdot)^T$ indicates the transpose of a vector or matrix $(\cdot)$. The asterisk $*$ represent entries which are identifiable from symmetry. $\gamma(> 0)$ implies the level of disturbance attenuation. $E\{\cdot\}$ means the mathematical expectation operator. For any square matrices A and B, define $diag\{A, B\} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$. Moreover, for any square matrix $M \in \mathbb{R}^{n\times n}$, we define $He(M) = M^T + M$. The notation I stands for the identity matrix.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following continuous-time MJSs described as

$$\begin{aligned}\dot{x}(t) &= A(r_t)x(t) + A_d(r_t)x(t-d(t)) + B(r_t)u(t) + E(r_t)\omega(t)\\ z(t) &= C(r_t)x(t) + D(r_t)u(t) + F(r_t)\omega(t)\\ x(t) &= \psi(t), \quad t \in [-h_2, 0], \quad r(0) = r_0,\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system; $u(t) \in \mathbb{R}^m$ is control input; $z(t) \in \mathbb{R}^p$ is regulated output; $\omega(t) \in \mathbb{R}^q$ is the mismatched disturbance input which is unknown but bounded so that $\|\omega(t)\| \leq \bar{\omega}$; $d(t)$ is a time-varying delay satisfying $0 \leq h_1 \leq d(t) \leq h_2$ and $\dot{d}(t) \leq \mu < \infty$, meanwhile $h_{12} = h_2 - h_1$. In (1), $\psi(t)$ is vector-valued initial continuous function defined on the interval $[-h_2, 0]$ and $r_0 \in S$ is the initial conditions of the continuous state and the mode. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $E(r_t)$, $C(r_t)$, $D(r_t)$, and $F(r_t)$ are matrix functions of the random jumping process $\{r_t\}$. r_t is continuous-time Markov process taking values in a finite space $\mathcal{L} = \{1, 2, \ldots, N\}$ and satisfies

$$Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j\\ 1 + \pi_{ii}h + o(h), & i = j\end{cases}$$

where $h > 0$, $\pi_{ij} \geqslant 0$ for $i \neq j$ and $\pi_{ii} = -\sum\limits_{j=1, j\neq i}^{s} \pi_{ij}$ for each mode i, $\lim\limits_{h\to 0} o(h)/h = 0$.

Considering the fact that TPs may be known, uncertain with known lower and upper bounds and unknown [14], the incomplete TP matrix with five operation modes is presented below

$$\begin{bmatrix}\pi_{11} & ? & \pi_{13} & \pi_{14} & \alpha_{15}\\ \pi_{21} & ? & ? & \pi_{24} & \pi_{25}\\ ? & \pi_{32} & \pi_{33} & \pi_{34} & ?\\ \pi_{41} & ? & \alpha_{43} & \pi_{44} & \pi_{45}\\ ? & ? & \alpha_{53} & \pi_{54} & ?\end{bmatrix},$$

where α_{ij} ($\underline{\alpha} \leq \alpha \leq \overline{\alpha}$) represents the uncertain TPs with known lower and upper bounds, '?' denotes the completely unknown elements. Furthermore, L_k is used to denote the set of known and uncertain TPs in ith row, while L_{uk} represents the set of unknown ones as following:

$$\begin{cases} L_k \ = j | \pi_{ij} \text{ is known and uncertain,}\\ L_{uk} = j | \pi_{ij} \text{ is unknown.}\end{cases}$$

For $r_t = i \in \mathcal{L}$, the system matrices of the ith mode are simplified as A_i, A_{di}, B_i, E_i, C_i, D_i and F_i with appropriate dimensions.

Definition 2.1. System (1) satisfies the required H_∞ performance index γ if the inequality

$$\mathcal{E}\left\{\int_0^\infty z^T(t)z(t)\,\mathrm{d}t\right\} \leq \gamma^2 \mathcal{E}\left\{\int_0^\infty \omega^T(t)\omega(t)\,\mathrm{d}t\right\}$$

holds for zero initial condition.

Definition 2.2. System (1) is said to be stochastically stable if, for any finite $\psi(t) \in \mathbb{R}^n$ defined on $[-\tau, 0]$, and $r_0 \in S$ the following condition is satisfied

$$\lim_{t\to\infty} E\left\{\int_0^t x^T(t)x(t)\,\mathrm{d}t|\psi, r_0\right\} < \infty.$$

Lemma 2.3. (C. Zhang et al. [27]) For a diagonal matrix $\mathcal{G} = diag\{R_2, 3R_2\}$ with $R_2 > 0$ and any matrix S_1, $-\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s$ can be upper bounded as:

$$\begin{aligned} &-\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s \\ &\le -\frac{1}{h_{12}}\xi_t^T \begin{bmatrix} E1 \\ E2 \end{bmatrix}^T \left(\begin{bmatrix} \mathcal{G} & S_1 \\ * & \mathcal{G} \end{bmatrix} + \begin{bmatrix} \frac{h_2-d(t)}{h_{12}}T_1 & 0 \\ 0 & \frac{d(t)-h_1}{h_{12}}T_2 \end{bmatrix} \right) \begin{bmatrix} E1 \\ E2 \end{bmatrix} \xi_t \end{aligned} \tag{2}$$

where

$$\begin{aligned} \xi_t &= [x^T(t),\ \ x^T(t-d(t)),\ \ x^T(t-h_1),\ \ x^T(t-h_2),\ \ \frac{1}{d(t)-h_1}\int_{t-d(t)}^{t-h_1} x(s)^T\,\mathrm{d}s, \\ &\quad \frac{1}{h_2-d(t)}\int_{t-h_2}^{t-d(t)} x(s)^T\,\mathrm{d}s,\ \ \dot{x}^T(s),\ \ \omega^T(t),\ \ z^T(t)]^T, \\ e_i &= [0_{n(i-1)n}, I, 0_{n(10-i)n}]^T,\ \ i = 1,2,\dots,9, \\ E1 &= col\{-e_2+e_3, e_1+e_2+e_3-2e_5\}, \\ E2 &= col\{e_2-e_4, e_2+e_4-2e_6\}, \\ T_1 &= \mathcal{G} - S_1\mathcal{G}^{-1}S_1^T, \\ T_2 &= \mathcal{G} - S_1^T\mathcal{G}^{-1}S_1. \end{aligned}$$

P r o o f. The proposed inequalities can be obtained by following the similar idea in [27] with non-zero low bound. The details are omitted here. □

Remark 2.4. Different from the proposed inequalities to deal with time-varying delays in [27], the corresponding inequality (2) is obtained in Lemma 2.3 with non-zero low bound.

Remark 2.5. Compared with the Wirtinger-based inequality [28] and reciprocally convex lemma [29], the proposed inequality (2) could provide a closer estimated value of $\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s$ but without requiring any extra decision variable.

Lemma 2.6. (R. Skelton et al. [34]) Letting $v \in \mathbb{R}^n$, $\mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n\times n}$ and $\mathcal{H} \in \mathbb{R}^{m\times n}$, such that $rank(\mathcal{H}) = r < n$, the following statements are equivalent:
(*a*) $v^T\mathcal{P}v < 0$, for all $v \neq 0$, $\mathcal{H}v = 0$;
(*b*) $\exists\mathcal{X} \in \mathbb{R}^{n\times m}$ such that $\mathcal{P} + He(\mathcal{X}\mathcal{H}) < 0$.

Lemma 2.7. (M. C. D Oliveira [35]) Let $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{n\times n}$ and $\mathcal{B} \in \mathbb{R}^{m\times n}$ be given matrices, one can get the following equivalent statements:
(*a*) $\mathcal{M}^T\mathcal{Q}\mathcal{M} < 0$, and $\mathcal{B}\mathcal{M} = 0$;
(*b*) $\exists\mathcal{S} \in \mathbb{R}^{n\times m}$ such that $\mathcal{Q} + He(\mathcal{S}\mathcal{B}) < 0$.

3. MAIN RESULTS

3.1. Sliding surface design

First of all, construct a sliding surface functional as follows:

$$s_i(t) = G_i x(t) - \int_0^t G_i[(A_i + B_i K_i)x(t) + A_{\mathrm{d}i} x(t - \mathrm{d}(t))]\,\mathrm{d}t, \tag{3}$$

where $G_i \in \mathbb{R}^{m\times n}$ and $K_i \in \mathbb{R}^{m\times n}$ are constant matrices.

According to the so-called equivalent control principle of SMC theory, when the system trajectories reach onto the sliding surface, it follows that $s_i(t) = 0$ and $\dot{s}_i(t) = 0$. Therefore, the equivalent control $u_{eq}(t)$ is derived as

$$u_{eq}(t) = K_i x(t) - (G_i B_i)^{-1} G_i E_i \omega(t). \tag{4}$$

Substituting (4) into system (1), the sliding mode dynamic system is established as:

$$\begin{aligned} \dot{x}(t) &= (A_i + B_i K_i)x(t) + A_{di} x(t - d(t)) \\ z(t) &= (C_i + D_i K_i)x(t) + (F_i - D_i (G_i B_i)^{-1} G_i E_i)\omega(t). \end{aligned} \tag{5}$$

3.2. Robustly stochastic stability analysis

Now, we proceed to the first task which analyzes the robustly stochastic stability of the sliding motion described by (5) with completely known TPs, and derive a sufficient condition in terms of the LMIs.

Theorem 3.1. For given scalars $\tau_{1i}, \tau_{2i}, \tau_{3i}, \tau_{4i}$ and $\gamma > 0$, if there exist symmetric matrices P_i, Q_{1i}, Q_1, Q_2, Q_3, R_1, $R_2 > 0$ and matrices H_i, M_i, S_i, N_i, W_i, with appropriate dimensions, satisfying the following LMIs:

case 1: $d(t) = h_1$

$$\begin{bmatrix} -\gamma^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_{19} & 0 & 0 & 0 \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} & A_{di}^T M_i^T & A_{di}^T H_i^T & 0 & \Gamma_{210} & \Gamma_{211} & 0 \\ * & * & \Gamma_{33} & \Gamma_{34} & 6aR_2 & \Gamma_{36} & 0 & R_1 & 0 & \Gamma_{310} & \Gamma_{311} & 0 \\ * & * & * & \Gamma_{44} & \Gamma_{45} & 6bR_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -12aR_2 & -4S_{14} & 0 & 0 & 0 & -2S_{13} & -2S_{14} & 0 \\ * & * & * & * & * & -12bR_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Gamma_{77} & \Gamma_{78} & 0 & 0 & 0 & \Gamma_{712} \\ * & * & * & * & * & * & * & \Gamma_{88} & \Gamma_{89} & 0 & 0 & \Gamma_{812} \\ * & * & * & * & * & * & * & * & -I & 0 & 0 & \Gamma_{912} \\ * & * & * & * & * & * & * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -3R_2 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Gamma_{1212} \end{bmatrix} < 0, \tag{6}$$

case 2: $d(t) = h_2$

$$
\begin{bmatrix}
-\gamma^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_{19} & 0 & 0 & 0 \\
* & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} & A_{di}^T M_i^T & A_{di}^T H_i^T & 0 & \Gamma'_{210} & \Gamma'_{211} & 0 \\
* & * & \Gamma_{33} & \Gamma_{34} & 6aR_2 & \Gamma_{36} & 0 & R_1 & 0 & 0 & 0 & 0 \\
* & * & * & \Gamma_{44} & \Gamma_{45} & 6bR_2 & 0 & 0 & 0 & \Gamma'_{410} & \Gamma'_{411} & 0 \\
* & * & * & * & -12aR_2 & -4S_{14} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -12bR_2 & 0 & 0 & 0 & -2S_{12}^T & -2S_{14}^T & 0 \\
* & * & * & * & * & * & \Gamma_{77} & \Gamma_{78} & 0 & 0 & 0 & \Gamma_{712} \\
* & * & * & * & * & * & * & \Gamma_{88} & \Gamma_{89} & 0 & 0 & \Gamma_{812} \\
* & * & * & * & * & * & * & * & -I & 0 & 0 & \Gamma_{912} \\
* & * & * & * & * & * & * & * & * & -R_2 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -3R_2 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \Gamma_{1212}
\end{bmatrix} < 0, \tag{7}
$$

$$
\Sigma_{j=1}^{S} \pi_{ij} Q_{1j} - Q_3 \leq 0, \tag{8}
$$

where
$\Gamma_{19} = F_i^T - E_i^T((G_iB_i)^{-1}G_i)^T D_i^T$, $\Gamma_{22} = -(1-\mu)Q_{1i} - 4aR_2 + S_{11}^T + S_{12}^T - S_{13}^T - S_{14}^T + S_{11} - S_{13} - 4bR_2 + S_{12} - S_{14} + He(H_iA_{di})$, $\Gamma_{23} = -2aR_2 - S_{11}^T - S_{12}^T - S_{13}^T - S_{14}^T$, $\Gamma_{24} = -S_{11} + S_{13} - 2bR_2 + S_{12} - S_{14}$, $\Gamma_{25} = 6aR_2 + 2S_{13}^T + 2S_{14}^T$, $\Gamma_{26} = -2S_{12} + 2S_{14} + 6bR_2$, $\Gamma_{210} = -S_{11} + S_{13}$, $\Gamma'_{210} = S_{11}^T + S_{12}^T$, $\Gamma_{211} = -S_{12} + S_{14}$, $\Gamma'_{211} = S_{13}^T + S_{14}^T$, $\Gamma_{33} = -Q_1 - R_1 - 4aR_2$, $\Gamma_{34} = S_{11} + S_{13} - S_{12} - S_{14}$, $\Gamma_{36} = 2S_{12} + 2S_{14}$, $\Gamma_{310} = S_{11} + S_{13}$, $\Gamma_{311} = S_{12} + S_{14}$, $\Gamma_{44} = -Q_2 - 4bR_2$, $\Gamma_{45} = -2S_{13}^T + 2S_{14}^T$, $\Gamma'_{410} = -S_{11}^T + S_{12}^T$, $\Gamma'_{411} = -S_{13}^T + S_{14}^T$, $\Gamma_{77} = h_1^2R_1 + h_{12}^2R_2 + He(-M_i)$, $\Gamma_{78} = P_i^T - H_i^T + M_iA_i + \tau_{1i}B_iN_i$, $\Gamma_{712} = -\tau_{1i}B_iW_i + M_iB_i$, $\Gamma_{88} = \Sigma_{j=1}^{S}\pi_{ij}P_j + Q_{1i} + Q_1 + Q_2 - R_1 + h_2Q_3 + He(H_iA_i - \tau_{2i}B_iN_i)$, $\Gamma_{89} = C_i^T + \tau_{3i}N_i^TD_i^T$, $\Gamma_{812} = -\tau_{2i}B_iW_i - H_iB_i + \tau_{4i}N_i^T$, $\Gamma_{912} = -\tau_{3i}D_iW_i + D_i$, $\Gamma_{1212} = -\tau_{4i}W_i$.

Then system (1) with the controller (4) is stochastically stable with completely known TPs and satisfies the prescribed H_∞ performance index γ. Moreover, controller gain matrix K_i can be calculated as $K_i = W_i^{-1}N_i$ and the matrix $G_i = B_i^T$.

Proof. Consider the following Lyapunov–Krasovskii functional candidate:

$$
V(x(t), i) = \sum_{n=1}^{7} V_n(x(t), i), \tag{9}
$$

where

$$
V_1(x(t), i) = x^T(t)P_ix(t), \tag{10}
$$

$$
V_2(x(t), i) = \int_{t-d(t)}^{t} x^T(s)Q_{1i}x(s)\,\mathrm{d}s, \tag{11}
$$

$$
V_3(x(t), i) = \int_{t-h_1}^{t} x^T(s)Q_1x(s)\,\mathrm{d}s, \tag{12}
$$

$$V_4(x(t),i)=\int_{t-h_2}^{t} x^T(s)Q_2x(s)\,\mathrm{d}s, \tag{13}$$

$$V_5(x(t),i)=h_1\int_{-h_1}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)R_1\dot{x}(s)\,\mathrm{d}s\mathrm{d}\beta, \tag{14}$$

$$V_6(x(t),i)=h_{12}\int_{-h_2}^{-h_1}\int_{t+\beta}^{t} \dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s\mathrm{d}\beta, \tag{15}$$

$$V_7(x(t),i)=\int_{-d(t)}^{0}\int_{t+\beta}^{t} x^T(s)Q_3x(s)\,\mathrm{d}s\mathrm{d}\beta. \tag{16}$$

Calculating the derivative of $V(x(t),i)$ gives

$$\dot{V}_1(x(t),i)=2x^T(t)P_i\dot{x}(t)+x^T(t)\Sigma_{j=1}^{S}\pi_{ij}P_jx(t), \tag{17}$$

$$\begin{aligned}\dot{V}_2(x(t),i) \quad &\leq \quad x^T(t)Q_{1i}x(t)-(1-\mu)x^T(t-d(t))Q_{1i}x(t-d(t))\\ &+ \quad \int_{t-d(t)}^{t} x^T(s)\Sigma_{j=1}^{S}\pi_{ij}Q_{1j}x(s)\,\mathrm{d}s,\end{aligned} \tag{18}$$

$$\dot{V}_3(x(t),i)=x^T(t)Q_1x(t)-x^T(t-h_1)Q_1x(t-h_1), \tag{19}$$

$$\dot{V}_4(x(t),i)=x^T(t)Q_2x(t)-x^T(t-h_2)Q_2x(t-h_2), \tag{20}$$

$$\dot{V}_5(x(t),i)=h_1^2\dot{x}^T(t)R_1\dot{x}(t)-h_1\int_{t-h_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,\mathrm{d}s, \tag{21}$$

$$\dot{V}_6(x(t),i)=h_{12}^2\dot{x}^T(t)R_2\dot{x}(t)-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s, \tag{22}$$

$$\dot{V}_7(x(t),i)\leq h_2x^T(t)Q_3x(t)-\int_{t-d(t)}^{t}x^T(t)Q_3x(t)\,\mathrm{d}s. \tag{23}$$

Utilizing the Jensen inequality in (21) produces

$$\dot{V}_5(x(t),i)\leq h_1^2\dot{x}^T(t)R_1\dot{x}(t)-x^T(t)R_1x(t)-x^T(t-h_1)R_1x(t-h_1)+2x^T(t)R_1x(t-h_1). \tag{24}$$

To deal with $-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s$ in (22), applying Lemma 2.3 one can get

$$\begin{aligned}&-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)\,\mathrm{d}s\\ &\leq -\xi_t^T\begin{bmatrix}E1\\E2\end{bmatrix}^T\times\left(\begin{bmatrix}\mathcal{G} & S_1\\ * & \mathcal{G}\end{bmatrix}+\begin{bmatrix}\frac{h_2-d(t)}{h_{12}}T_1 & 0\\ 0 & \frac{d(t)-h_1}{h_{12}}T_2\end{bmatrix}\right)\times\begin{bmatrix}E1\\E2\end{bmatrix}\xi_t\\ &=-\xi_t^T\begin{bmatrix}E1\\E2\end{bmatrix}^T\times\begin{bmatrix}a\mathcal{G} & S_1\\ * & b\mathcal{G}\end{bmatrix}\times\begin{bmatrix}E1\\E2\end{bmatrix}\xi_t\\ &+\xi_t^T\left(\frac{h_2-d(t)}{h_{12}}E1^TS_1\mathcal{G}^{-1}S_1E1+\frac{d(t)-h_1}{h_{12}}E2^TS_1\mathcal{G}^{-1}S_1E2\right)\xi_t\end{aligned} \tag{25}$$

where $a = \frac{2h_2 - h_1 - d(t)}{h_{12}}$ and $b = \frac{h_2 - 2h_1 + d(t)}{h_{12}}$.

Firstly, to ensure the required H_∞ performance of system (5), one needs $\dot{V}(x(t), i) + z(t)^T z(t) - \gamma^2 \omega(t)^T \omega(t) < 0$, which can be guaranteed by

$$\xi_t^T(\Phi + \frac{h_2 - d(t)}{h_{12}} E1^T S_1 \mathcal{G}^{-1} S_1 E1 + \frac{d(t) - h_1}{h_{12}} E2^T S_1 \mathcal{G}^{-1} S_1 E2)\xi_t < 0, \tag{26}$$

$$\int_{t-d(t)}^{t} x(s)^T (\Sigma_{j=1}^S \pi_{ij} Q_{1j} - Q_3) x(s)\,\mathrm{d}s \le 0, \tag{27}$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & 0 & R_1 & 0 & 0 & 0 & \Phi_{17} & 0 & C_i^T \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & 0 & 0 & 0 \\ * & * & \Phi_{33} & \Phi_{34} & 6aR_2 & \Phi_{36} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 6bR_2 & 0 & 0 & 0 \\ * & * & * & * & -12aR_2 & -4S_{14} & 0 & 0 & 0 \\ * & * & * & * & * & -12bR_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Phi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 & F_i^T \\ * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$\Phi_{11} = \Sigma_{j=1}^S \pi_{ij} P_j + Q_{1i} + Q_1 + Q_2 - R_1 + h_2 Q_3$, $\Phi_{17} = P_i$, $\Phi_{22} = -(1-\mu)Q_{1i} - 4aR_2 + S_{11}^T + S_{12}^T - S_{13}^T - S_{14}^T + S_{11} - S_{13} - 4bR_2 + S_{12} - S_{14}$, $\Phi_{23} = -2aR_2 - S_{11}^T - S_{12}^T - S_{13}^T - S_{14}^T$, $\Phi_{24} = -S_{11} + S_{13} - 2bR_2 + S_{12} - S_{14}$, $\Phi_{25} = 6aR_2 + 2S_{13}^T + 2S_{14}^T$, $\Phi_{26} = -2S_{12} + 2S_{14} + 6bR_2$, $\Phi_{33} = -Q_1 - R_1 - 4aR_2$, $\Phi_{34} = S_{11} + S_{13} - S_{12} - S_{14}$, $\Phi_{36} = 2S_{12} + 2S_{14}$, $\Phi_{44} = -Q_2 - 4bR_2$, $\Phi_{45} = -2S_{13}^T + 2S_{14}^T$, $\Phi_{77} = h_1^2 R_1 + h_{12}^2 R_2$.

Then, (27) is equivalent to condition in (8). Since $0 \le h_1 \le d(t) \le h_2$, two cases are discussed as below to ensure the negative definiteness of (26).

Case I: If $d(t) = h_1, a = 2, b = 1$, (26) is equivalent to

$$\xi_t^T(\Phi + E1^T S_1 \mathcal{G}^{-1} S_1 E1)\xi_t < 0. \tag{28}$$

Adopting Lemma 2.6 to (28), one has

$$\underbrace{\Phi + E1^T S_1 \mathcal{G}^{-1} S_1 E1 + He(\mathcal{X}\mathcal{H})}_{\Omega} < 0, \tag{29}$$

where $\mathcal{X} = [H_i^T \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; M_i^T \;\; 0 \;\; 0]^T$, $\mathcal{H} = [A_i \;\; A_{di} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -I \;\; E_i \;\; 0]$,

$$\Omega = \begin{bmatrix} \Psi_{11} & H_i A_{di} & R_1 & 0 & 0 & 0 & \Psi_{17} & H_i E_i & \Psi_{19} & 0 & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & A_{di}^T M_i^T & 0 & 0 & \Psi_{210} & \Psi_{211} \\ * & * & \Psi_{33} & \Psi_{34} & 6aR_2 & \Psi_{36} & 0 & 0 & 0 & \Psi_{310} & \Psi_{311} \\ * & * & * & \Psi_{44} & \Psi_{45} & 6bR_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -12aR_2 & -4S_{14} & 0 & 0 & 0 & -2S_{13} & -2S_{14} \\ * & * & * & * & * & -12bR_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & M_i E_i & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 & F_i^T & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & * & * & * & * & * & -3R_2 \end{bmatrix},$$

$\Psi_{11} = \Sigma_{j=1}^{S}\pi_{ij}P_j + Q_{1i} + Q_1 + Q_2 - R_1 + h_2Q_3 + He(H_iA_i)$, $\Psi_{17} = P_i - H_i + A_i^T M_i^T$, $\Psi_{19} = C_i^T$, $\Psi_{22} = -(1-\mu)Q_{1i} - 4aR_2 + S_{11}^T + S_{12}^T - S_{13}^T - S_{14}^T + S_{11} - S_{13} - 4bR_2 + S_{12} - S_{14} + He(H_iA_{di})$, $\Psi_{23} = -2aR_2 - S_{11}^T - S_{12}^T - S_{13}^T - S_{14}^T$, $\Psi_{24} = -S_{11} + S_{13} - 2bR_2 + S_{12} - S_{14}$,$\Psi_{25} = 6aR_2 + 2S_{13}^T + 2S_{14}^T$,$\Psi_{26} = -2S_{12} + 2S_{14} + 6bR_2$,$\Psi_{210} = -S_{11} + S_{13}$, $\Psi_{211} = -S_{12} + S_{14}$, $\Psi_{33} = -Q_1 - R_1 - 4aR_2$, $\Psi_{34} = S_{11} + S_{13} - S_{12} - S_{14}$, $\Psi_{36} = 2S_{12}+2S_{14}$, $\Psi_{310} = S_{11}+S_{13}$, $\Psi_{311} = S_{12}+S_{14}$, $\Psi_{44} = -Q_2-4bR_2$, $\Psi_{45} = -2S_{13}^T+2S_{14}^T$, $\Psi_{77} = h_1^2R_1 + h_{12}^2R_2 + He(-M_i)$, $\Psi_{89} = F_i^T$.

Secondly, by pre and post multiplying with inverse matrix Λ, Ω is rewritten as

$$\begin{bmatrix} I_{11*11} \\ \Delta \end{bmatrix}^T \begin{bmatrix} \Xi & 0_{11\times1} \\ * & 0 \end{bmatrix} \begin{bmatrix} I_{11*11} \\ \Delta \end{bmatrix} < 0 \tag{30}$$

where $\Delta = [K_i \;\; 0_{1\times10}]$,

$$\Lambda = \begin{bmatrix}
0&0&0&0&0&0&0&I&0&0&0\\
0&I&0&0&0&0&0&0&0&0&0\\
0&0&I&0&0&0&0&0&0&0&0\\
0&0&0&I&0&0&0&0&0&0&0\\
0&0&0&0&I&0&0&0&0&0&0\\
0&0&0&0&0&I&0&0&0&0&0\\
0&0&0&0&0&0&I&0&0&0&0\\
I&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&I&0&0\\
0&0&0&0&0&0&0&0&0&I&0\\
0&0&0&0&0&0&0&0&0&0&I
\end{bmatrix}$$

$$\Xi = \begin{bmatrix}
\Psi_{11}^{'} & H_iA_{di} & R_1 & 0 & 0 & 0 & \Psi_{17}^{'} & 0 & \Psi_{19}^{'} & 0 & 0\\
* & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & A_{di}^T M_i^T & 0 & 0 & \Psi_{210} & \Psi_{211}\\
* & * & \Psi_{33} & \Psi_{34} & 6aR_2 & \Psi_{36} & 0 & 0 & 0 & \Psi_{310} & \Psi_{311}\\
* & * & * & \Psi_{44} & \Psi_{45} & 6bR_2 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & -12aR_2 & -4S_{14} & 0 & 0 & 0 & -2S_{13} & -2S_{14}\\
* & * & * & * & * & -12bR_2 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & \Psi_{77} & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & -\gamma^2 & \Psi_{89}^{'} & 0 & 0\\
* & * & * & * & * & * & * & * & -I & 0 & 0\\
* & * & * & * & * & * & * & * & * & -R_2 & 0\\
* & * & * & * & * & * & * & * & * & * & -3R_2
\end{bmatrix},$$

$\Psi_{11}^{'} = \Sigma_{j=1}^{S}\pi_{ij}P_j + Q_{1i} + Q_1 + Q_2 - R_1 + h_2Q_3 + He(H_iA_i + H_iB_iK_i)$, $\Psi_{17}^{'} = P_i - H_i + A_i^T M_i^T + (B_iK_i)^T M_i^T$, $\Psi_{19}^{'} = C_i^T + (D_iK_i)^T$, $\Psi_{89}^{'} = F_i^T - E_i^T((G_iB_i)^{-1}G_i)^T D_i^T$.

Let $N_i = K_iW_i$ and apply Lemma 2.7 to (30), P_i is separated from B_i and K_i as following

$$\begin{bmatrix} \Xi & 0_{11\times 1} \\ * & 0 \end{bmatrix} + He\left(\begin{bmatrix} 0_{6\times 1} \\ \tau_{1i}B_iW_i - M_iB_i \\ \tau_{2i}B_iW_i - H_iB_i \\ \tau_{3i}B_iW_i - D_i \\ 0_{2\times 1} \\ \tau_{4i}W_i \end{bmatrix} \begin{bmatrix} 0_{7\times 1} \\ K_i^T \\ 0_{3\times 1} \\ -I \end{bmatrix}^T \right) < 0, \tag{31}$$

which is equivalent to condition in (6).

Case II: If $d(t) = h_2, a = 1, b = 2$, (26) is equivalent to

$$\xi_t^T(\Phi + E2^T S_1 \mathcal{G}^{-1} S_1 E2)\xi_t < 0. \tag{32}$$

Taking a similar way, (7) can be obtained, which completes the proof. □

Remark 3.2. Unlike the free-weighting matrix approach and Wirtinger inequality, more accurate estimations for the cross terms are achieved in Theorem 3.1 by Lemma 2.3 without ignoring any useful terms. Therefore, the proposed Theorem 3.1 has potential to reduce the conservatism considerably.

Remark 3.3. To make the conditions in Theorem 3.1 in the framework of LMIs, a constructive strategy is developed to handle $P_iB_iK_i$ as $P_iB_iK_i = P_iB_iW_i^{-1} = (P_iB_i - B_iW_i)W_i^{-1}N_i + B_iN_i$. With this separated structure and Lemma 2.7, P_i is separated from B_i and K_i. It is worth noting that there is no need to pre- and post-multiply an inverse matrix and three sets of slack variables without specific structures are introduced.

In the following theorem, Theorem 3.1 is extended to derive a H_∞ sliding mode controller with general TPs assumed to be known, uncertain with known lower and upper bounds and unknown for (5).

Theorem 3.4. For given scalars $\tau_{1i}, \tau_{2i}, \tau_{3i}, \tau_{4i}$ and $\gamma > 0$, if there exist symmetric matrices P_i, Q_{1i}, Q_1, Q_2, Q_3, R_1, $R_2 > 0$ and matrices H_i, M_i, S_i, N_i, W_i with appropriate dimensions satisfying the following LMIs, then system (5) is stochastically stable with general TPs.

For $i \in L_k$

$$\begin{cases} (6)\ |_{\Gamma_{88}=\Gamma_{88}^k} < 0, \\ (7)\ |_{\Gamma_{88}=\Gamma_{88}^k} < 0, \\ \Sigma_{j\in L_k}\bar{\pi}_{ij}Q_{1j} + \left(-\underline{\pi}_{ii} - \Sigma_{j\in L_k}\underline{\pi}_{ij}\right)Q_{1l} - Q_3 < 0. \end{cases} \tag{33}$$

For $i \in L_{uk}$

$$\begin{cases} (6)\ |_{\Gamma_{88}=\Gamma_{88}^{uk}} < 0, \\ (7)\ |_{\Gamma_{88}=\Gamma_{88}^{uk}} < 0, \\ \Sigma_{j\in L_k}\bar{\pi}_{ij}Q_{1j} - \Sigma_{j\in L_k}\underline{\pi}_{ij}Q_{1i} - Q_3 < 0, \\ P_l \le P_i, \\ Q_{1l} \le Q_{1i}. \end{cases} \tag{34}$$

where
$\Gamma_{88}^{k} = Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L_k}\bar{\pi}_{ij}P_j+\left(-\underline{\pi}_{ii}-\Sigma_{j\in L_k}\underline{\pi}_{ij}\right)P_l$,
$\Gamma_{88}^{uk} = Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L_k}\bar{\pi}_{ij}P_j-\Sigma_{j\in L_k}\underline{\pi}_{ij}P_i$.

Proof. Considering the fact that $\underline{\pi}_{ij}\le\pi_{ij}\le\bar{\pi}_{ij}$, the following equalities can be obtained respectively

$$\begin{aligned}\Gamma_{88}^{k} &\geq Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)\\ &+ \Sigma_{j\in L_k}\pi_{ij}P_j+\left(-\pi_{ii}-\Sigma_{j\in L_k}\pi_{ij}\right)P_l\end{aligned}\tag{35}$$

$$\begin{aligned}\Gamma_{88}^{uk} &\geq Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)\\ &+ \Sigma_{j\in L_k}\pi_{ij}P_j-\Sigma_{j\in L_k}\pi_{ij}P_i\end{aligned}\tag{36}$$

Case I: $i\in L_k$. According to the property of $\Sigma_{j=1}^{N}\pi_{ij}=0$, $\Sigma_{l\in L_{uk}}\pi_{ij}=-\pi_{ii}-\Sigma_{j\in L_k}\pi_{ij}$ which also leads to $\frac{\Sigma_{l\in L_{uk}}\pi_{ij}}{-\pi_{ii}-\Sigma_{j\in L_k}\pi_{ij}}=1$. Taking these facts into (35) leads to

$$\begin{aligned}\Gamma_{88}^{k} &\geq Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)\\ &+ \Sigma_{j\in L_k}\pi_{ij}P_j+\left(-\pi_{ii}-\Sigma_{j\in L_k}\pi_{ij}\right)P_l\\ &= Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L_k}\pi_{ij}P_j\\ &+ \frac{\Sigma_{l\in L_{uk}}\pi_{ij}}{-\pi_{ii}-\Sigma_{j\in L_k}\pi_{ij}}\left(-\pi_{ii}-\Sigma_{j\in L_k}\pi_{ij}\right)P_l\\ &= Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L_k}\pi_{ij}P_j+\Sigma_{l\in L_{uk}}\pi_{ij}P_l\\ &= Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L}\pi_{ij}P_j.\end{aligned}\tag{37}$$

Case II: $i\in L_{uk}$. From $\Sigma_{l\in L_{uk},l\neq i}\pi_{il}>0$, multiplying the left and right side of $P_l\le P_i$ by $\Sigma_{l\in L_{uk},l\neq i}\pi_{il}$ gives

$$\Sigma_{l\in L_{uk},l\neq i}\pi_{il}P_l\le\Sigma_{l\in L_{uk},l\neq i}\pi_{il}P_i.\tag{38}$$

Substituting (38) to (36), it results in

$$\begin{aligned}\Gamma_{88}^{k} &\geq Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L_k}\pi_{ij}P_j\\ &- \Sigma_{j\in L_k}\pi_{ij}P_i+\Sigma_{l\in L_{uk},l\neq i}\pi_{il}P_l-\Sigma_{l\in L_{uk},l\neq i}\pi_iP_i\\ &= Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L_k}\pi_{ij}P_j\\ &+ \Sigma_{l\in L_{uk},l\neq i}\pi_{il}P_l-\left(\Sigma_{j\in L_k}\pi_{ij}+\Sigma_{l\in L_{uk},l\neq i}\pi_{il}\right)P_i\\ &= Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L,j\neq i}(\pi_{ij}P_j+\pi_{ii}P_i)\\ &= Q_{1i}+Q_1+Q_2-R_1+h_2Q_3+He(H_iA_i-\tau_{2i}B_iN_i)+\Sigma_{j\in L}\pi_{ij}P_j\end{aligned}\tag{39}$$

Therefore, whether $i\in L_k$ or $i\in L_{uk}$, (37) and (39) can guarantee (6) and (7) holds.

Finally, along the similar lines as (33) - (39) to dispose general TPs in (8), the system (5) with general TPs is stable if (33) and (34) hold. This completes the proof. □

Remark 3.5. In [30] and [32], TPs are treated as known which could lead to conservatism when all TPs are uncertain with known bounds or unknown. To fill up this deficiency, Theorem 3.4 derives a H_∞ sliding mode controller with incomplete TPs assumed to be known, uncertain with known lower and upper bounds and unknown.

3.3. SMC law design scheme

In this section, we will design a sliding mode control law, by which the trajectories of sliding mode dynamic system (5) can be driven onto the designed sliding surface $s(t,i)=0$ and satisfies the prescribed H_∞ performance index γ.

Theorem 3.6. With the constant matrixes K_i mentioned in Theorem 3.1 and the integral sliding surface given by (3), the trajectory of the closed-loop system (5) can be driven onto the sliding surface $s(t,i)=0$ with H_∞ performance index γ by the following controller:

$$u(t,i) = K_i x(t) - \parallel (G_iB_i)^{-1}G_iE_i \parallel \bar{\omega} sign(s_i(t)) - \frac{1}{2} \parallel \delta \parallel s_i(t) \tag{40}$$

where

$$\delta = \begin{cases} \Sigma_{j\in L_k}\bar{\pi}_{ij}(G_jB_j)^{-1} + \left(-\underline{\pi}_{ii} - \Sigma_{j\in L_k}\underline{\pi}_{ij}\right)(G_lB_l)^{-1}, & i\in L_k \\ \Sigma_{j\in L_k}\bar{\pi}_{ij}(G_jB_j)^{-1} - \Sigma_{j\in L_k}\underline{\pi}_{ij}(G_iB_i)^{-1}, & i \in L_{uk}. \end{cases} \tag{41}$$

P r o o f. Choose the following Lyapunov functional candidate as:

$$V_o(t) = \frac{1}{2}s_i(t)^T(G_iB_i)^{-1}s_i(t). \tag{42}$$

Computing the derivative of $V_o(t)$ with $\dot{s}_i(t)$ yields

$$\dot{V_o}(t) = s_i(t)^T(G_iB_i)^{-1}G_i[B_iu(t) - B_iK_ix(t) + E_i\omega(t)] + \frac{1}{2}s_i(t)^T\Sigma_{j=1}^S\pi_{ij}(G_jB_j)^{-1}s_i(t). \tag{43}$$

Taking the SMC controller $u(t,i) = K_ix(t) - \rho_i sign(s_i(t))$ into (43) gives

$$\dot{V}_o(t) = s_i(t)^T[\rho_i sign(s_i(t)) + (G_iB_i)^{-1}G_iE_i\omega(t)] + \frac{1}{2}s_i(t)^T\Sigma_{j=1}^S\pi_{ij}(G_jB_j)^{-1}s_i(t). \tag{44}$$

To meet $\dot{V}_o(t) < 0$, one needs to ensure following terms from (44)

$$s_i(t)^T[\rho_i sign(s_i(t)) + (G_iB_i)^{-1}G_iE_i\omega(t)] + \frac{1}{2}s_i(t)^T\Sigma_{j=1}^S\pi_{ij}(G_jB_j)^{-1}s_i(t) < 0. \tag{45}$$

Resorting to norm calculation, the left hand side of (45) is scaled as

$$\begin{aligned} &\rho_i|s_i(t)| + \|s_i(t)^T\|\|(G_iB_i)^{-1}G_iE_i\|\|\omega(t)\| \\ &\qquad + \frac{1}{2}\|s_i(t)^T\|\|\Sigma_{j=1}^S\pi_{ij}(G_jB_j)^{-1}\|\|s_i(t)\| < 0. \end{aligned} \tag{46}$$

Applying the fact that $|s_i(t)| \geq \parallel s_i(t) \parallel$, (46) holds if ρ_i satisfies

$$\rho_i \leq -\|(G_iB_i)^{-1}G_iE_i\|\|\omega(t)\| + \frac{1}{2}\|\Sigma_{j=1}^S\pi_{ij}(G_jB_j)^{-1}\|\|s_i(t)\|. \tag{47}$$

Since π_{ij} is incomplete, along the similar line as Theorem 3.4 to handle general TPs, two cases are discussed as below

Case I ($i \in L_k$):

$$\Sigma_{j\in L}\pi_{ij}(G_jB_j)^{-1} \le \Sigma_{j\in L_k}\bar{\pi}_{ij}(G_jB_j)^{-1} + \left(-\underline{\pi}_{ii} - \Sigma_{j\in L_k}\underline{\pi}_{ij}\right)(G_lB_l)^{-1}. \tag{48}$$

Case II ($i \in L_{uk}$):

$$\begin{cases} \Sigma_{j\in L}\pi_{ij}(G_jB_j)^{-1} \le \Sigma_{j\in L_k}\bar{\pi}_{ij}(G_jB_j)^{-1} - \Sigma_{j\in L_k}\underline{\pi}_{ij}(G_iB_i)^{-1} \\ \qquad\qquad\qquad\qquad\qquad\qquad (G_lB_l)^{-1} \le (G_iB_i)^{-1}. \end{cases} \tag{49}$$

Substituting (48) and (49) into (47), ρ_i should meet

$$\rho_i \le -\|(G_iB_i)^{-1}G_iE_i\|\|\omega(t)\| + \frac{1}{2}\|\delta\|\|s_i(t)\|. \tag{50}$$

Taking $\rho_i = -\|(G_iB_i)^{-1}G_iE_i\|\bar{\omega} + \frac{1}{2}\|\delta\|\|s_i(t)\|$ into $u(t,i) = K_ix(t) - \rho_i sign(s_i(t))$ leads to the condition in (41). This means that the system trajectories can reach onto the predefined switching surface. □

Remark 3.7. It is worth noting that the sliding mode controller could render the chattering problem. To avoid this phenomenon, $\rho_i sign(s_i(t))$ is substituted by $\rho_i \frac{s_i(t)}{\|s_i(t)\|+\iota}$, ($\iota > 0$) in the simulation part.

4. NUMERICAL EXAMPLE

Example 4.1. Considered a single-link robot arm system with a time delay borrowed from [12], the validity of the proposed method is illustrated. The time-delay model with uncertainties is given as follow.

$$\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} = -\frac{MgL}{J}sin(\theta(t)) - \frac{\mathcal{D}(t)}{J}\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} - \frac{\mathcal{D}(t)}{J_d}\frac{\mathrm{d}\theta(t-\mathrm{d}(t))}{\mathrm{d}t} + \frac{1}{J}u(t) + \frac{L}{J}\omega(t) \tag{51}$$

where $\theta(t), u(t), \omega(t)$ represent the arm's angle position, the control input and the external disturbance, respectively. M denotes the mass of the payload, while J is the inertia moment. The symbol g is the acceleration gravity, L is the arm length and $\mathcal{D}(t)$ denotes the uncertain coefficient of viscous friction. The parameters of g, L and $\mathcal{D}(t)$ are $9.81, 2$ and 0.5, respectively. The parameters M and J have three different modes. As follows, a linearized system model is given for (51).

$$\begin{cases} \dot{x}(t) = \left(\begin{bmatrix} 0 & 1 \\ -gL & -\frac{\mathcal{D}(t)}{J(r_t)} \end{bmatrix}\right)x(t) + \left(\begin{bmatrix} 0 & 1 \\ -gL & -\frac{\mathcal{D}(t)}{J_d(r_t)} \end{bmatrix}\right)x(t-d(t)) \\ \qquad\quad + \begin{bmatrix} 0 \\ \frac{1}{J(r_t)} \end{bmatrix}u(t) + \begin{bmatrix} 0 \\ \frac{L}{J(r_t)} \end{bmatrix}w(t) \\ z(t) = \begin{bmatrix} 1 & 1 \end{bmatrix}x(t) + u(t) + \omega(t) \end{cases} \tag{52}$$

where $x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix}$, $r_t = \{1, 2, 3\}$. $M(r_t), J(r_t)$ depends on the jump modes and $M(1) = J(1) = 0.2, M(2) = J(2) = 0.25, M(3) = J(3) = 0.4$.

The incomplete transition probability matrix is given as

$$\begin{bmatrix} -1.1 & 0.7 & 0.4 \\ 0.4 & ? & \alpha_{23} \\ \alpha_{31} & 1.9 & ? \end{bmatrix} \tag{53}$$

where α_{23} satisfies $0.35 \leq \alpha_{23} \leq 0.38$ and α_{31} satisfies $0.18 \leq \alpha_{31} \leq 0.21$, and ? represents the completely unknown TPs.

Our aim here is to verify the effectiveness of the proposed theoretical results in the previous sections. For initialization, the state vector is given $x_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Other parameters are selected as follows $\tau_{1i} = \tau_{2i} = \tau_{3i} = \tau_{4i} = 1$, $h_1 = 0.05, h_2 = 0.70$. $\omega(t) = 0.5sin(0.5t)$ for $5 \leq t \leq 7$(otherwise, $\omega(t) = 0$) such that $\bar{\omega} = 0.5$.

Solving the proposed synthesis conditions in Theorem 3.4 yields

$K_1 = \begin{bmatrix} 0.4811 & 0.6706 \end{bmatrix}, K_2 = \begin{bmatrix} 0.2407 & 0.4880 \end{bmatrix}, K_3 = \begin{bmatrix} 0.3627 & 0.4237 \end{bmatrix}$

With the aid of the switching surface function in (3), the sliding mode controller designed in (40) is obtained. Further, to reduce chattering in the control signals, the signum function $sign(s(t))$ is replaced by $\frac{s_i}{\|s_i\|+\iota}$. Applying the proposed sliding mode controller, the response curves of system states, sliding mode surface and control input are shown in Figures $1-3$, respectively.

It is seen that system state trajectories are stochastically stable in Figure 1 and Figure 2, the curve denotes the integral sliding mode surface function $s_i(t)$ with the possible mode evolution. The variation of the mode-dependent sliding mode control input is depicted in Figure 3. In summary, these figures have shown the validity of the proposed sliding mode control approach.

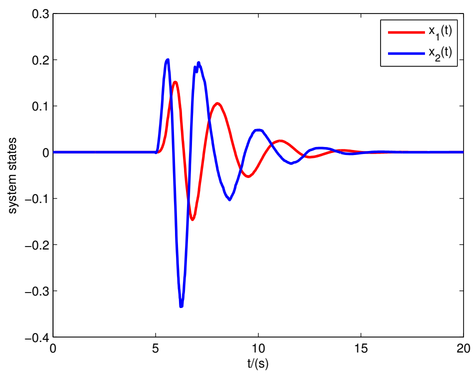

Fig. 1. Corresponding trajectories of system state.

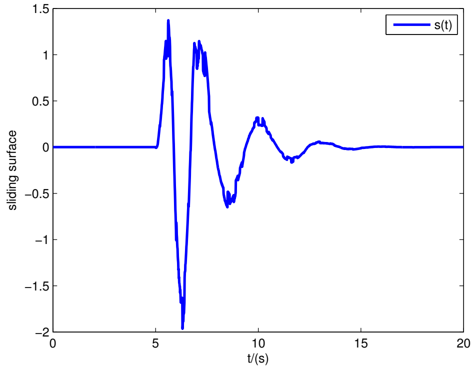

Fig. 2. Trajectory of the sliding surface.

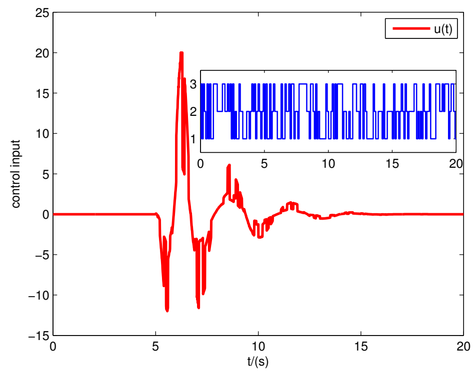

Fig. 3. Applied control input.

5. CONCLUSIONS

In this paper, H_∞ SMC for MJSs with interval time-varying delay and general TPs is discussed. Firstly, a sliding surface functional is constructed. Then, with the help of Finsler lemma and a relaxed inequality, a delay-dependent robust stability criterion has been established by LMI technique, which guarantees the sliding mode dynamic system to be robustly stochastically stable. Furthermore, a SMC law has been synthesized to ensure the stability of the closed-loop system. Finally, a single-link robot arm system has been provided to demonstrate the effectiveness of the obtained results.

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