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EXISTENCE, UNIQUENESS AND GLOBAL ASYMPTOTIC STABILITY FOR A CLASS OF COMPLEX-VALUED NEUTRAL-TYPE NEURAL NETWORKS WITH TIME DELAYS

Manchun Tan and Desheng Xu

This paper explores the problem of delay-independent and delay-dependent stability for a class of complex-valued neutral-type neural networks with time delays. Aiming at the neutral-type neural networks, an appropriate function is constructed to derive the existence of equilibrium point. On the basis of homeomorphism theory, Lyapunov functional method and linear matrix inequality techniques, several LMI-based sufficient conditions on the existence, uniqueness and global asymptotic stability of equilibrium point for complex-valued neutral-type neural networks are obtained. Finally, numerical examples are given to illustrate the feasibility and the effectiveness of the proposed theoretical results.

Keywords: complex-valued neutral-type neural networks, existence and uniqueness of equilibrium, global asymptotic stability, inequality techniques, Lyapunov functional

Classification: 37B25, 92B20

1. INTRODUCTION

In the past decades, various classes of neural networks (NNs) such as Hopfield neural networks, recurrent neural networks, Cohen–Grossberg neural networks, bidirectional associative memory neural networks have been investigated due to their extensive applications in associative memory, classification of pattern recognition, engineering optimization, image processing, signal processing and other areas (see [1, 2, 9, 19, 24, 25, 30, 31, 32, 34, 35, 40, 49] and references therein). In the designing of neural networks for such applications, it is significant to know the existence of equilibrium point and its stability of the neural networks. Therefore the stability analysis has been a hot topic in the studies of various NNs [2, 9, 19, 24, 30, 32, 34, 35, 40].

As an extension of real-valued neural networks, the complex-valued neural networks (CVNNs) have received increasing interests [3, 4, 6, 7, 10, 11, 12, 13, 14, 16, 17, 26, 27, 28, 29, 33, 37, 38, 43, 45, 46, 47, 48]. CVNNs, in which the states, connection weights, or activation functions are complex-valued, have more complicated properties than the real-valued NNs and have shown their advantages in real applications, e. g., solving

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the XOR problem and the detection of symmetry problem [7, 10, 17, 26, 28, 43]. In [14], a complex-valued recurrent neural network with time delays was investigated based on two classes of complex-valued activation functions, and some sufficient conditions for the existence of unique equilibrium point, global asymptotic stability and global exponential stability were derived. In [38], by separating complex-valued neural networks into real and imaginary parts, forming an equivalent real-valued system, some sufficient conditions to ascertain the existence, uniqueness and global stability of the equilibrium point of complex-valued BAM neural networks were provided in terms of linear matrix inequality. In [47], a class of complex-valued Cohen–Grossberg neural networks with time delays had been discussed, and some conditions had been derived to ascertain the existence, uniqueness, and global asymptotic stability of such networks. In [3], the activation dynamics of the complex-valued neural network with both leakage time delay and discrete time delay on time scales had been investigated.

It is well known that the time delay is an inherent feature of signal transmission between neurons, and the existence of time delays may lead to the instability or bad performance of systems. Two types of time delays are generally considered in nonlinear system in the literature, i. e., the retarded-type delays and neutral-type delays [15, 44]. Retarded-type NNs that have only time-delays in the states are not good enough to characterize precisely the complicated dynamic properties of the neural cells in real world, so neutral-type NNs that involve time delays in the time derivatives of states have attracted much attention [5, 8, 18, 20, 21, 22, 23, 36, 39, 41, 42].

Different kinds of approaches are employed for investigating the dynamics of neural networks, such as the Lyapunov function method, nonlinear measure approach, the matrix measure method and so on. For example, in [7], using switched Lyapunov functions on a complex field, the authors proposed some new stability criteria of complex-valued impulsive and switching systems, and designed the hybrid impulsive and switching feedback controllers for complex-valued chaotic Lu system. In [10], by using the nonlinear measure method, several sufficient criteria were obtained to ascertain the existence, uniqueness and global stability of the equilibrium point of the addressed complex-valued neural networks. In [18, 23, 31, 34], authors utilized the matrix measure method in the stability analysis of neural networks .

Although enormous works have been done on the complex-valued NNs and neutral-type NNs, respectively, the complex-valued neutral-type NNs have not been investigated in the literature. Motivated by the above discussion, the dynamic behavior of complex-valued neutral-type NNs has been addressed in this paper. Compared with the existing literature about stability of neural networks, the main contributions of this paper can be summarized as follows. Firstly, in the light of homeomorphism theory, Lyapunov functional method and linear matrix inequality techniques, several LMI-based sufficient conditions are proposed to guarantee the existence, uniqueness and global asymptotic stability of equilibrium point for complex-valued neutral-type neural networks. Secondly, the boundedness and differentiability of the activation function are no longer required. Lastly, for the sake of dealing with the effect of the neutral term, we define a significant mapping $H(w) = (I - \bar{E})^{-1}[-\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w) + \bar{u}]$ instead of $H(w) = -\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w) + \bar{u}$ which has frequently been considered in the existing literature.

The structure of this paper is summarized as follows. In Section 2, the complex-valued neutral-type model is presented, and some preliminaries are briefly outlined. Section 3 proposes the delay-independent and delay-dependent stability criteria for complex-valued neutral-type neural networks. In Section 4, two numerical examples are given to demonstrate the feasibility and the effectiveness of the results. Finally, some conclusions are drawn in Section 5.

Notation: The notation used throughout this paper is fairly standard. $\mathbb{R}$ and $\mathbb{C}$ show the set of real numbers and the set of complex numbers, respectively. $\mathbb{R}^n$ and $\mathbb{C}^n$ show, respectively, the n-dimensional Euclidean space and the n-dimensional unitary space. $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$ are, respectively, the set of all $n\times n$ real matrices and the set of all $n\times n$ complex matrices. If $A\in\mathbb{R}^{n\times n}$, A^T shows the transpose of A; $\lambda_M(A)$ and $\lambda_m(A)$ denote the maximum and minimum eigenvalue of a square matrix A, respectively. $A>0$ ($A\geq 0$) means that A is positive definite (positive semidefinite). Similarly, $A<0$ ($A\leq 0$) means that A is negative definite (negative semidefinite). z^* denotes the complex conjugate transpose of $z\in\mathbb{C}^n$. $\|z\|=\sqrt{z^*z}$. I is used to denote an identity matrix with proper dimension. The notation diagonal stands for a block-diagonal matrix. The symmetric term in a matrix is shown by $*$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM STATEMENT AND MATHEMATICAL PRELIMINARIES

In this paper, we consider the complex-valued neutral-type neural networks model described by the following set of differential equation:

$$\dot{z}(t)=-Dz(t)+Af(z(t))+Bg(z(t-\tau))+E\dot{z}(t-\tau)+u, \tag{1}$$

where $z(t)=(z_1(t),z_2(t),\dots,z_n(t))\in\mathbb{C}^n$ is the neural state vector; n corresponds to the number of neurons in layers; $f(z(t))=(f_1(z_1(t)),f_2(z_2(t)),\dots,f_n(z_n(t)))^T\in\mathbb{C}^n$, $g(z(t-\tau))=(g_1(z_1(t-\tau)),g_2(z_2(t-\tau)),\dots,g_n(z_n(t-\tau)))^T\in\mathbb{C}^n$ are the vector-valued neuron activation functions; $D=diag(d_1,d_2,\dots,d_n)\in\mathbb{R}^{n\times n}$ with $d_i>0$ denotes the interconnection weight matrix; $A\in\mathbb{C}^{n\times n}$ denotes the strengths of the neuron interconnections within the network; $B\in\mathbb{C}^{n\times n}$ denotes the strengths of the neuron interconnections with time delay parameters τ; $E\in\mathbb{C}^{n\times n}$ denotes coefficients of the time derivative of the delayed states; $u\in\mathbb{C}^n$ is the external input vector.

Let $f_j(\cdot)$, $g_j(\cdot)\in\mathbb{C}$ $(j=1,2,\dots,n)$ be complex-valued functions, which satisfy the following assumption.

Assumption 2.1. For any $j\in\{1,2,\dots,n\}$, there exist positive numbers $l_1,l_2,\dots,l_n$ and $m_1,m_2,\dots,m_n$ such that

$$\|f_j(z)-f_j(z')\|\leq l_j\|z-z'\|,\quad \|g_j(z)-g_j(z')\|\leq m_j\|z-z'\|, \tag{2}$$

for $\forall z,z'\in\mathbb{C}$.

In order to study (1), we separate it into its real and imaginary parts and transform it into a real-valued neural network model. Let $A=A^R+iA^I$, $B=B^R+iB^I$, $E=E^R+iE^I$, $u=u^R+iu^I$. Let $z(t)=x(t)+iy(t)$, where i denotes the imaginary unit, that is, $i = \sqrt{-1}$. $f_j(z(t))$ and $g_j(z(t))$ can be expressed respectively by separating into its real and imaginary part as

$$f_j(z(t)) = f_j^R(x(t), y(t)) + if_j^I(x(t), y(t)),$$
$$g_j(z(t-\tau)) = g_j^R(x(t-\tau), y(t-\tau)) + ig_j^I(x(t-\tau), y(t-\tau)),$$

where $f_j^R(\cdot,\cdot)$: $\mathbb{R}^2 \to \mathbb{R}$; $f_j^I(\cdot,\cdot)$: $\mathbb{R}^2 \to \mathbb{R}$; $g_j^R(\cdot,\cdot)$: $\mathbb{R}^2 \to \mathbb{R}$; $g_j^I(\cdot,\cdot)$: $\mathbb{R}^2 \to \mathbb{R}$. For simplicity, we show $x = x(t)$, $y = y(t)$, $x^\tau = x(t-\tau)$, $y^\tau = y(t-\tau)$. Thus, (1) can be separated into real and imaginary parts:

$$\begin{aligned}\dot{x} &= -Dx + A^R f^R(x,y) - A^I f^I(x,y) + B^R g^R(x^\tau, y^\tau) - B^I g^I(x^\tau, y^\tau) + E^R \dot{x}^\tau - E^I \dot{y}^\tau + u^R,\\ \dot{y} &= -Dy + A^R f^I(x,y) + A^I f^R(x,y) + B^R g^I(x^\tau, y^\tau) + B^I g^R(x^\tau, y^\tau) + E^R \dot{y}^\tau + E^I \dot{x}^\tau + u^I.\end{aligned} \tag{3}$$

Let

$$w = \begin{bmatrix} x \\ y \end{bmatrix}, \quad w^\tau = \begin{bmatrix} x^\tau \\ y^\tau \end{bmatrix}, \quad \bar{u} = \begin{bmatrix} u^R \\ u^I \end{bmatrix},$$
$$\bar{A} = \begin{bmatrix} A^R & -A^I \\ A^I & A^R \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B^R & -B^I \\ B^I & B^R \end{bmatrix},$$
$$\bar{D} = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} E^R & -E^I \\ E^I & E^R \end{bmatrix},$$
$$\bar{f}(w) = \begin{bmatrix} f^R(x,y) \\ f^I(x,y) \end{bmatrix}, \quad \bar{g}(x^\tau, y^\tau) = \begin{bmatrix} g^R(x^\tau, y^\tau) \\ g^I(x^\tau, y^\tau) \end{bmatrix},$$

then, (3) can be rewritten as

$$\dot{w} = -\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w^\tau) + \bar{E}\dot{w}^\tau + \bar{u}. \tag{4}$$

It is clear from (2) that

$$(f(z) - f(z'))^*(f(z) - f(z')) \le (z - z')^* L^T L (z - z'), \tag{5}$$
$$(g(z) - g(z'))^*(g(z) - g(z')) \le (z - z')^* M^T M (z - z'), \tag{6}$$

where $L = diag(l_1, l_2, \ldots, l_n)$ and $M = diag(m_1, m_2, \ldots, m_n)$. Equation (5) and (6) can be expressed by separating their real and imaginary parts as

$$(\bar{f}(w) - \bar{f}(w'))^T(\bar{f}(w) - \bar{f}(w')) \le (w - w')^T \bar{L}(w - w'), \tag{7}$$
$$(\bar{g}(w) - \bar{g}(w'))^T(\bar{g}(w) - \bar{g}(w')) \le (w - w')^T \bar{M}(w - w'), \tag{8}$$

where

$$\bar{L} = \begin{bmatrix} L^T L & 0 \\ 0 & L^T L \end{bmatrix}, \quad \bar{M} = \begin{bmatrix} M^T M & 0 \\ 0 & M^T M \end{bmatrix}, \quad w = \begin{bmatrix} x \\ y \end{bmatrix}, \quad w' = \begin{bmatrix} x' \\ y' \end{bmatrix}.$$

Notice that the equilibrium point of (1) is also the equilibrium of (4) and the stability of system (1) is equivalent to the stability of system (4). Therefore, in the sequel, we focus our study on the real-valued neural networks (4).

Remark 2.2. Some results require that the activation functions be differentiable or bounded in the references (e. g., [8, 14, 43]). Such restrictions are removed in this paper. Namely, both the real and the imaginary parts of the activation functions are no longer assumed to be differentiable and bounded in this paper.

The following lemmas will play an important role in the proof of the main results.

Lemma 2.3. (Forti and Tesi [9]) If $H(w) : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is a continuous map and satisfies the following conditions:

(1) $H(w) : \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is injective on $\mathbb{R}^{2n}$,

(2) $\|H(w)\| \to \infty$ as $\|w\| \to \infty$,

then $H(w)$ is a homeomorphism of $\mathbb{R}^{2n}$.

Lemma 2.4. Suppose $-I + \bar{E}^T\bar{E} < 0$, then $I - \bar{E}$ is a nonsingular matrix (or an invertible matrix).

Proof. By means of contradiction, assume that $I - \bar{E}$ is a singular matrix, then there exists vector $X \neq 0$ such that $(I - \bar{E})X = 0$. Thus, we get $\bar{E}X = X$, $X^T\bar{E}^T = X^T$, which lead to

$$X^T\bar{E}^T\bar{E}X = X^TX.$$

Then

$$X^T(\bar{E}^T\bar{E} - I)X = 0, \quad (X \neq 0)$$

which yields a contradiction to $-I + \bar{E}^T\bar{E} < 0$. The proof is completed. □

3. MAIN RESULTS

In this section, we present some LMI-based sufficient conditions for the existence, uniqueness, and globally asymptotical stability of the equilibrium point for system (1).

3.1. Delay-independent stability criteria

Theorem 3.1. Under Assumption 2.1, the neural network (1) has a unique equilibrium point and it is globally asymptotically stable if there exist scalars $\varepsilon_i > 0$ $(i = 1, 2)$ and positive symmetric matrices P and Q such that the following LMIs hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44} \end{bmatrix} < 0 \tag{9}$$

and

$$\Delta = -Q + \varepsilon_2\bar{M} \leq 0, \tag{10}$$

where $\Phi_{11} = -2P\bar{D} - \bar{D}^2 + \varepsilon_1\bar{L} + Q$, $\Phi_{12} = P\bar{A}$, $\Phi_{13} = P\bar{B}$, $\Phi_{14} = P\bar{E}$, $\Phi_{22} = -\varepsilon_1 I + \bar{A}^T\bar{A}$, $\Phi_{23} = \bar{A}^T\bar{B}$, $\Phi_{24} = \bar{A}^T\bar{E}$, $\Phi_{33} = -\varepsilon_2 I + \bar{B}^T\bar{B}$, $\Phi_{34} = \bar{B}^T\bar{E}$, $\Phi_{44} = -I + \bar{E}^T\bar{E}$.

P r o o f. Step 1: We prove the existence and uniqueness of equilibrium point of system (4) by using Lemma 2.3.

By Schur complement[1], from (9), one has $\Phi_{44} = -I + \bar{E}^T \bar{E} < 0$. Then, in the light of Lemma 2.4, we know that $I - \bar{E}$ is nonsingular.

Consider the following mapping associated with system (4):

$$H(w) = (I - \bar{E})^{-1}[-\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w) + \bar{u}]. \tag{11}$$

That is

$$H(w) = -\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w) + \bar{E}H(w) + \bar{u}. \tag{12}$$

It is obvious that $\hat{w} = (\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_{2n})^T$ is an equilibrium point of (4), if $\hat{w}$ satisfies the following equation:

$$H(\hat{w}) = -\bar{D}(\hat{w}) + \bar{A}\bar{f}(\hat{w}) + \bar{B}\bar{g}(\hat{w}) + \bar{E}H(\hat{w}) + \bar{u} = 0.$$

Thus, we can directly conclude from Lemma 2.3 that, for the system defined by (4), there exists a unique equilibrium point for every input vector $\bar{u}$ if $H(w)$ is homeomorphism of $\mathbb{R}^{2n}$.

First, we prove that the map $H(w)$ is injective. Suppose that there exist w and w' with $w \neq w'$. According to (12), we have

$$H(w) - H(w') = -\bar{D}(w - w') + \bar{A}(\bar{f}(w) - \bar{f}(w')) + \bar{B}(\bar{g}(w) - \bar{g}(w')) + \bar{E}(H(w) - H(w')). \tag{13}$$

Multiplying both sides of (13) by $[2(w - w')^T P + 2(w - w')^T \bar{D} + (H(w) - H(w'))^T]$, and we get

$$\begin{aligned}
&[2(w - w')^T P + 2(w - w')^T \bar{D} + (H(w) - H(w'))^T][H(w) - H(w')] \\
&\quad = 2(w - w')^T P \\
&\qquad \times [-\bar{D}(w - w') + \bar{A}(\bar{f}(w) - \bar{f}(w')) + \bar{B}(\bar{g}(w) - \bar{g}(w')) + \bar{E}(H(w) - H(w'))] \\
&\qquad + [\bar{D}(w - w') + \bar{A}(\bar{f}(w) - \bar{f}(w')) + \bar{B}(\bar{g}(w) - \bar{g}(w')) + \bar{E}(H(w) - H(w'))]^T \\
&\qquad \times [-\bar{D}(w - w') + \bar{A}(\bar{f}(w) - \bar{f}(w')) + \bar{B}(\bar{g}(w) - \bar{g}(w')) + \bar{E}(H(w) - H(w'))],
\end{aligned}$$

where P is positive symmetric matrix. The above equation is equivalent to:

$$\begin{aligned}
&2(w - w')^T (P + \bar{D})(H(w) - H(w')) \\
&\quad = -(H(w) - H(w'))^T (H(w) - H(w')) \\
&\qquad - 2(w - w')^T P\bar{D}(w - w') + 2(w - w')^T P\bar{A}(\bar{f}(w) - \bar{f}(w')) \\
&\qquad + 2(w - w')^T P\bar{B}(\bar{g}(w) - \bar{g}(w')) + 2(w - w')^T P\bar{E}(H(w) - H(w')) \\
&\qquad - (w - w')^T \bar{D}^2 (w - w') + (w - w')^T \bar{D}\bar{A}(\bar{f}(w) - \bar{f}(w')) \\
&\qquad + (w - w')^T \bar{D}\bar{B}(\bar{g}(w') - \bar{g}(w')) + (w - w')^T \bar{D}\bar{E}(H(w) - H(w')) \\
&\qquad - (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{D}(w - w') + (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{A}(\bar{f}(w) - \bar{f}(w'))
\end{aligned}$$

$$
\begin{aligned}
&+ (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{B}(\bar{g}(w) - \bar{g}(w')) + (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{E}(H(w) - H(w')) \\
&- (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{D}(w - w') + (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{A}(\bar{f}(w) - \bar{f}(w')) \\
&+ (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{B}(\bar{g}(w) - \bar{g}(w')) + (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{E}(H(w) - H(w')) \\
&- (H(w) - H(w'))^T \bar{E}^T \bar{D}(w - w') + (H(w) - H(w'))^T \bar{E}^T \bar{A}(\bar{f}(w) - \bar{f}(w')) \\
&+ (H(w) - H(w'))^T \bar{E}^T \bar{B}(\bar{g}(w) - \bar{g}(w')) + (H(w) - H(w'))^T \bar{E}^T \bar{E}(H(w) - H(w')).
\end{aligned} \tag{14}
$$

We note the following equalities:

$$
\begin{aligned}
&(w - w')^T \bar{D}\bar{A}(\bar{f}(w) - \bar{f}(w')) = (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{D}(w - w'), \\
&(w - w')^T \bar{D}\bar{B}(\bar{g}(w) - \bar{g}(w')) = (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{D}(w - w'), \\
&(w - w')^T \bar{D}\bar{E}(H(w) - H(w')) = (H(w) - H(w'))^T \bar{E}^T \bar{D}(w - w'), \\
&(\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{A}(\bar{f}(w) - \bar{f}(w')) = (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{B}(\bar{g}(w) - \bar{g}(w')), \\
&(H(w) - H(w'))^T \bar{E}^T \bar{A}(\bar{f}(w) - \bar{f}(w')) = (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{E}(H(w) - H(w')), \\
&(H(w) - H(w'))^T \bar{E}^T \bar{B}(\bar{g}(w) - \bar{g}(w')) = (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{E}(H(w) - H(w')).
\end{aligned}
$$

Using the above equalities in (14), we get

$$
\begin{aligned}
&2(w - w')^T (P + \bar{D})(H(w) - H(w')) \\
&\quad = -(H(w) - H(w'))^T (H(w) - H(w')) \\
&\qquad - (w - w')^T (2P\bar{D} + \bar{D}^2)(w - w') \\
&\qquad + 2(w - w')^T P\bar{A}(\bar{f}(w) - \bar{f}(w')) \\
&\qquad + 2(w - w')^T P\bar{B}(\bar{g}(w) - \bar{g}(w')) \\
&\qquad + 2(w - w')^T P\bar{E}(H(w) - H(w')) \\
&\qquad + (\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{A}(\bar{f}(w) - \bar{f}(w')) \\
&\qquad + (\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{B}(\bar{g}(w) - \bar{g}(w')) \\
&\qquad + (H(w) - H(w'))^T \bar{E}^T \bar{E}(H(w) - H(w')) \\
&\qquad + 2(\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{B}(\bar{g}(w) - \bar{g}(w')) \\
&\qquad + 2(\bar{f}(w) - \bar{f}(w'))^T \bar{A}^T \bar{E}(H(w') - H(w')) \\
&\qquad + 2(\bar{g}(w) - \bar{g}(w'))^T \bar{B}^T \bar{E}(H(w) - H(w')).
\end{aligned} \tag{15}
$$

Eqs. (7) – (8) guarantee that the following inequalities are true

$$
\begin{aligned}
&\varepsilon_1 \left[(w - w')^T \bar{L}(w - w') - (\bar{f}(w) - \bar{f}(w'))^T (\bar{f}(w) - \bar{f}(w')) \right] \ge 0, \\
&\varepsilon_2 \left[(w - w')^T \bar{M}(w - w') - (\bar{g}(w) - \bar{g}(w'))^T (\bar{g}(w) - \bar{g}(w')) \right] \ge 0.
\end{aligned} \tag{16}
$$

Combining (16) with (15), we obtain

$$\begin{aligned}
&2(w-w')^T(P+\bar{D})(H(w)-H(w'))\\
&\quad\le (w-w')^T(-2P\bar{D}-\bar{D}^2+\varepsilon_1\bar{L}+\varepsilon_2\bar{M})(w-w')\\
&\qquad+(\bar{f}(w)-\bar{f}(w'))^T(-\varepsilon_1 I+\bar{A}^T\bar{A})(\bar{f}(w)-\bar{f}(w'))\\
&\qquad+(\bar{g}(w)-\bar{g}(w'))^T(-\varepsilon_2 I+\bar{B}^T\bar{B})(\bar{g}(w)-\bar{g}(w'))\\
&\qquad+(H(w)-H(w'))^T(-I+\bar{E}^T\bar{E})(H(w)-H(w'))\\
&\qquad+2(w-w')^TP\bar{A}(\bar{f}(w)-\bar{f}(w'))\\
&\qquad+2(w-w')^TP\bar{B}(\bar{g}(w)-\bar{g}(w'))\\
&\qquad+2(w-w')^TP\bar{E}(H(w)-H(w'))\\
&\qquad+2(\bar{f}(w)-\bar{f}(w'))^T\bar{A}^T\bar{B}(\bar{g}(w)-\bar{g}(w'))\\
&\qquad+2(\bar{f}(w)-\bar{f}(w'))^T\bar{A}^T\bar{E}(H(w)-H(w'))\\
&\qquad+2(\bar{g}(w)-\bar{g}(w'))^T\bar{B}^T\bar{E}(H(w)-H(w'))\\
&\quad=\xi^T(t)\Omega\xi(t),
\end{aligned}\tag{17}$$

where $\xi^T(t)=\Big[(w-w')^T,(\bar{f}(w)-\bar{f}(w'))^T,(\bar{g}(w)-\bar{g}(w'))^T,(H(w)-H(w'))^T\Big]$ and

$$\Omega=\begin{bmatrix}\Omega_{11}&\Omega_{12}&\Omega_{13}&\Omega_{14}\\ *&\Omega_{22}&\Omega_{23}&\Omega_{24}\\ *&*&\Omega_{33}&\Omega_{34}\\ *&*&*&\Omega_{44}\end{bmatrix},$$

where $\Omega_{11}=-2P\bar{D}-\bar{D}^2+\varepsilon_1\bar{L}+\varepsilon_2\bar{M}$, $\Omega_{12}=P\bar{A}$, $\Omega_{13}=P\bar{B}$, $\Omega_{14}=P\bar{E}$, $\Omega_{22}=-\varepsilon_1 I+\bar{A}^T\bar{A}$, $\Omega_{23}=\bar{A}^T\bar{B}$, $\Omega_{24}=\bar{A}^T\bar{E}$, $\Omega_{33}=-\varepsilon_2 I+\bar{B}^T\bar{B}$, $\Omega_{34}=\bar{B}^T\bar{E}$, $\Omega_{44}=-I+\bar{E}^T\bar{E}$.

Since $\Phi<0$ holds, noting the relationship between Ω_{ij} and Φ_{ij} $(i,j=1,2,3,4)$, one can derive

$$\begin{aligned}
&X_1^T(\Omega_{11}+Q-\varepsilon_2\bar{M})X_1+(X_2^T\Omega_{21}+X_3^T\Omega_{31}+X_4^T\Omega_{41})X_1\\
&\quad+(X_1^T\Omega_{12}+X_2^T\Omega_{22}+X_3^T\Omega_{32}+X_4^T\Omega_{42})X_2\\
&\quad+(X_1^T\Omega_{13}+X_2^T\Omega_{23}+X_3^T\Omega_{33}+X_4^T\Omega_{43})X_3\\
&\quad+(X_1^T\Omega_{14}+X_2^T\Omega_{24}+X_3^T\Omega_{34}+X_4^T\Omega_{44})X_4<0,
\end{aligned}$$

for $\forall X=(X_1^T,X_2^T,X_3^T,X_4^T)^T$ and $X\neq 0$.

By virtue of (10), we get

$$\begin{aligned}
&(X_1^T\Omega_{11}+X_2^T\Omega_{21}+X_3^T\Omega_{31}+X_4^T\Omega_{41})X_1\\
&\quad+(X_1^T\Omega_{12}+X_2^T\Omega_{22}+X_3^T\Omega_{32}+X_4^T\Omega_{42})X_2\\
&\quad+(X_1^T\Omega_{13}+X_2^T\Omega_{23}+X_3^T\Omega_{33}+X_4^T\Omega_{43})X_3\\
&\quad+(X_1^T\Omega_{14}+X_2^T\Omega_{24}+X_3^T\Omega_{34}+X_4^T\Omega_{44})X_4<X_1^T(-Q+\varepsilon_2\bar{M})X_1\le 0.
\end{aligned}$$

That is, $X^T\Omega X < 0$ holds for $\forall X \neq 0$. Hence, we get

$$\Omega < 0. \tag{18}$$

Equation (17), (18) and $w \neq w'$ guarantee that the following inequality holds

$$2(w - w')^T(P + \bar{D})(H(w) - H(w')) < 0,$$

from which we conclude that $H(w) \neq H(w')$ for all $w \neq w'$. That is, the map $H(w)$ is injective.

Next we prove that $\|H(w)\| \to \infty$ as $\|w\| \to \infty$. Letting $w' = 0$ and $\gamma_0 = w^T$, then from (17) we deduce that

$$-2w^T(P + \bar{D})(H(w) - H(0)) \geq \lambda_m(-\Omega)\|\gamma_0\|^2.$$

Using Schwartz inequality, we have

$$2\|w\| \left\|P + \bar{D}\right\| (\|H(w)\| + \|H(0)\|) \geq \|\lambda_m(-\Omega)\| \|w\|^2,$$

i. e.,

$$2(\|H(w)\| + \|H(0)\|) \geq \frac{\|\lambda_m(-\Omega)\|}{\left\|P + \bar{D}\right\|} \|w\| .$$

That is to say $\|H(w)\| \to \infty$ as $\|w\| \to \infty$. By Lemma 2.3, the map $H(w)$ is a homeomorphism of $\mathbb{R}^{2n}$. Hence there exists a unique point $\hat{w}$ such that $H(\hat{w}) = 0$, that is, (4) has a unique equilibrium point $\hat{w}$.

Step 2: We prove that the unique equilibrium point of system (1) or (4) is globally asymptotically stable. Since there exists a unique equilibrium point $\hat{w}$ for (4), by the transformation $\tilde{w} = w - \hat{w}$, we can get

$$\dot{\tilde{w}} = -\bar{D}\tilde{w} + \bar{A}\tilde{f}(\tilde{w}) + \bar{B}\tilde{g}(\tilde{w}^\tau) + \bar{E}\dot{\tilde{w}}^\tau, \tag{19}$$

where $\tilde{f}(\tilde{w}) = \bar{f}(\tilde{w} + \hat{w}) - \bar{f}(\hat{w})$ and $\tilde{g}(\tilde{w}^\tau) = \bar{g}(\tilde{w}^\tau + \hat{w}) - \bar{g}(\hat{w})$. It is clear that the stability of the equilibrium point of (4) is equivalent to the stability of the origin of (19).

We construct the following Lyapunov functional:

$$V(\tilde{w}) = \tilde{w}^T P\tilde{w} + \tilde{w}^T\bar{D}\tilde{w} + \int_{t-\tau}^{t} \dot{\tilde{w}}^T(s)\dot{\tilde{w}}(s)\,\mathrm{d}s + \int_{t-\tau}^{t} \tilde{w}^T(s)Q\tilde{w}(s)\,\mathrm{d}s.$$

The time derivative of $V(\tilde{w})$ along the trajectory of (19) yields

$$\begin{aligned}
\dot{V}(\tilde{w}) &= 2\tilde{w}^T P\dot{\tilde{w}} + 2\dot{\tilde{w}}^T\bar{D}\tilde{w} + \dot{\tilde{w}}^T\dot{\tilde{w}} - \dot{\tilde{w}}^{\tau T}\dot{\tilde{w}}^\tau + \tilde{w}^T Q\tilde{w} - \tilde{w}^{\tau T}Q\tilde{w}^\tau \\
&= 2\tilde{w}^T P\dot{\tilde{w}} + \dot{\tilde{w}}^T(2\bar{D}\tilde{w} + \dot{\tilde{w}}) - \dot{\tilde{w}}^{\tau T}\dot{\tilde{w}} + \tilde{w}^T Q\tilde{w} - \tilde{w}^{\tau T}Q\tilde{w}^\tau \\
&= 2\tilde{w}^T P(-\bar{D}\tilde{w} + \bar{A}\tilde{f}(\tilde{w}) + \bar{B}\tilde{g}(\tilde{w}^\tau) + \bar{E}\dot{\tilde{w}}^\tau) \\
&\quad + (-\bar{D}\tilde{w} + \bar{A}\tilde{f}(\tilde{w}) + \bar{B}\tilde{g}(\tilde{w}^\tau) + \bar{E}\dot{\tilde{w}}^\tau)^T \times (\bar{D}\tilde{w} + \bar{A}\tilde{f}(\tilde{w}) + \bar{B}\tilde{g}(\tilde{w}^\tau) + \bar{E}\dot{\tilde{w}}^\tau) \\
&\quad - \dot{\tilde{w}}^{\tau T}\dot{\tilde{w}}^\tau + \tilde{w}^T Q\tilde{w} - \tilde{w}^{\tau T}Q\tilde{w}^\tau,
\end{aligned}$$

which gives

$$\begin{aligned}\dot{V}(\tilde{w}) = &-2\tilde{w}^T P\bar{D}\tilde{w} + 2\tilde{w}^T P\bar{A}\tilde{f}(\tilde{w}) + 2\tilde{w}^T P\bar{B}\tilde{g}(\tilde{w}^\tau) + 2\tilde{w}^T P\bar{E}\dot{\tilde{w}}^\tau\\
&- \tilde{w}^T\bar{D}^2\tilde{w} - \tilde{w}^T\bar{D}\bar{A}\tilde{f}(\tilde{w}) - \tilde{w}^T\bar{D}\bar{B}\tilde{g}(\tilde{w}^\tau) - \tilde{w}^T\bar{D}\bar{E}\dot{\tilde{w}}^\tau\\
&+ \tilde{f}^T(\tilde{w})\bar{A}^T\bar{D}\tilde{w} + \tilde{f}^T(\tilde{w})\bar{A}^T\bar{A}\tilde{f}(\tilde{w}) + \tilde{f}^T(\tilde{w})\bar{A}^T\bar{B}\tilde{g}(\tilde{w}^\tau) + \tilde{f}^T(\tilde{w})\bar{A}^T\bar{E}\dot{\tilde{w}}^\tau\\
&+ \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{D}\tilde{w} + \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{A}\tilde{f}(\tilde{w}) + \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{B}\tilde{g}(\tilde{w}^\tau) + \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{E}\dot{\tilde{w}}^\tau\\
&+ \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{D}\tilde{w} + \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{A}\tilde{f}(\tilde{w}) + \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{B}\tilde{g}(\tilde{w}^\tau) + \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{E}\dot{\tilde{w}}^\tau\\
&- \dot{\tilde{w}}^{\tau T}\dot{\tilde{w}}^\tau + \tilde{w}^T Q\tilde{w} - \tilde{w}^{\tau T} Q\tilde{w}^\tau. \end{aligned} \tag{20}$$

We note that the following equalities hold:

$$\begin{aligned}&\tilde{w}^T\bar{D}\bar{A}\tilde{f}(\tilde{w}) = \tilde{f}^T(\tilde{w})\bar{A}^T\bar{D}\tilde{w},\\
&\tilde{w}^T\bar{D}\bar{B}\tilde{g}(\tilde{w}^\tau) = \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{D}\tilde{w},\\
&\tilde{w}^T\bar{D}\bar{E}\dot{\tilde{w}}^\tau = \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{D}\tilde{w},\\
&\tilde{f}^T(\tilde{w})\bar{A}^T\bar{B}\tilde{g}(\tilde{w}^\tau) = \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{A}\tilde{f}(\tilde{w}),\\
&\tilde{f}^T(\tilde{w})\bar{A}^T\bar{E}\dot{\tilde{w}}^\tau = \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{A}\tilde{f}(\tilde{w}),\\
&\tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{E}\dot{\tilde{w}}^\tau = \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{B}\tilde{g}(\tilde{w}^\tau).\end{aligned} \tag{21}$$

Hence, (20) together with (21) gives

$$\begin{aligned}\dot{V}(\tilde{w}) = &-2\tilde{w}^T P\bar{D}\tilde{w} + 2\tilde{w}^T P\bar{A}\tilde{f}(\tilde{w}) + 2\tilde{w}^T P\bar{B}\tilde{g}(\tilde{w}^\tau) + 2\tilde{w}^T P\bar{E}\dot{\tilde{w}}^\tau\\
&- \tilde{w}^T\bar{D}^2\tilde{w} + \tilde{f}^T(\tilde{w})\bar{A}^T\bar{A}\tilde{f}(\tilde{w}) + \tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{B}\tilde{g}(\tilde{w}^\tau) + \dot{\tilde{w}}^{\tau T}\bar{E}^T\bar{E}\dot{\tilde{w}}^\tau\\
&+ 2\tilde{f}^T(\tilde{w})\bar{A}^T\bar{B}\tilde{g}(\tilde{w}^\tau) + 2\tilde{f}^T(\tilde{w})\bar{A}^T\bar{E}\dot{\tilde{w}}^\tau + 2\tilde{g}^T(\tilde{w}^\tau)\bar{B}^T\bar{E}\dot{\tilde{w}}^\tau\\
&- \dot{\tilde{w}}^{\tau T}\dot{\tilde{w}}^\tau + \tilde{w}^T Q\tilde{w} - \tilde{w}^{\tau T} Q\tilde{w}^\tau.\end{aligned} \tag{22}$$

Eqs. (7) – (8) guarantee that the following inequalities are true

$$\begin{aligned}&\varepsilon_1(\tilde{w}^T\bar{L}\tilde{w} - \tilde{f}^T(\tilde{w})\tilde{f}(\tilde{w})) \geq 0,\\
&\varepsilon_2(\tilde{w}^{\tau T}\bar{M}\tilde{w}^\tau - \tilde{g}^T(\tilde{w}^\tau)\tilde{g}(\tilde{w}^\tau)) \geq 0.\end{aligned} \tag{23}$$

Using (23) in (22), we obtain

$$\dot{V}(\tilde{w}) \leq \eta^T(t)\Phi\eta(t) + \tilde{w}^{\tau T}\Delta\tilde{w}^\tau, \tag{24}$$

where $\eta^T(t) = \left[\tilde{w}^T, \tilde{f}^T(\tilde{w}), \tilde{g}^T(\tilde{w}^\tau), \dot{\tilde{w}}^{\tau T}\right]$ and Φ, Δ are given in (9) and (10), respectively.

Thus, inequalities (9), (10) and (24) imply that $\dot{V}(\tilde{w})$ is negative definite, and the origin of system (19), or equivalently the equilibrium point of (4) is globally asymptotically stable. This completes the proof. □

If we select $Q = \varepsilon_2\bar{M}$, then from Theorem 3.1, we have the following corollary:

Corollary 3.2. Under Assumption 2.1, the neural network (1) has a unique equilibrium point and it is globally asymptotically stable if there exist scalars $\varepsilon_i > 0$ $(i = 1, 2)$ and positive symmetric matrix P such that the following LMIs hold:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix} < 0, \tag{25}$$

where $\Omega_{11} = -2P\bar{D} - \bar{D}^2 + \varepsilon_1\bar{L} + \varepsilon_2\bar{M}$, $\Omega_{12} = P\bar{A}$, $\Omega_{13} = P\bar{B}$, $\Omega_{14} = P\bar{E}$, $\Omega_{22} = -\varepsilon_1 I + \bar{A}^T\bar{A}$, $\Omega_{23} = \bar{A}^T\bar{B}$, $\Omega_{24} = \bar{A}^T\bar{E}$, $\Omega_{33} = -\varepsilon_2 I + \bar{B}^T\bar{B}$, $\Omega_{34} = \bar{B}^T\bar{E}$, $\Omega_{44} = -I + \bar{E}^T\bar{E}$.

Remark 3.3. The study of complex-valued neutral-type neural networks is more complicated than the usual recurrent neural networks because of its neutral term. Thus, how to deal with the effect of the neutral term on complex-valued neutral-type neural networks is a difficult problem. Aiming at the neutral term, by using Lemma 2.4 we define and employ a very important mapping (11), i.e., $H(w) = (I - \bar{E})^{-1}[-\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w) + \bar{u}]$ instead of $H(w) = -\bar{D}w + \bar{A}\bar{f}(w) + \bar{B}\bar{g}(w) + \bar{u}$ which has frequently been considered in the existing literature (e.g., [34, 38, 47]).

3.2. Delay-dependent stability criteria

Theorem 3.4. Under Assumption 2.1, the equilibrium point of system (1) is globally asymptotically stable if there exist four positive symmetric matrices P, Q, R and Λ, and three scalars $\gamma_1 > 0$, $\gamma_2 > 0$ and $\gamma_3 > 0$, and some constant matrices S_{ij} $(i = 1, 2, \cdots, 6$, $j = 1, 2, \cdots, 6)$, with appropriate dimensions

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ * & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ * & * & S_{33} & S_{34} & S_{35} & S_{36} \\ * & * & * & S_{44} & S_{45} & S_{46} \\ * & * & * & * & S_{55} & S_{56} \\ * & * & * & * & * & S_{66} \end{bmatrix} > 0,$$

and some free matrices N_i $(i = 1, 2, \ldots, 6)$ such that

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} \\ * & * & * & * & \Xi_{55} & \Xi_{56} \\ * & * & * & * & * & \Xi_{66} \end{bmatrix} < 0 \tag{26}$$

and

$$\Pi = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & N_1 \\ * & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & N_2 \\ * & * & S_{33} & S_{34} & S_{35} & S_{36} & N_3 \\ * & * & * & S_{44} & S_{45} & S_{46} & N_4 \\ * & * & * & * & S_{55} & S_{56} & N_5 \\ * & * & * & * & * & S_{66} & N_6 \\ * & * & * & * & * & * & R \end{bmatrix} > 0 \tag{27}$$

hold, where $\Xi_{11} = -2P\bar{D}-\bar{D}^2+Q+\gamma_1\bar{L}+\tau\bar{D}R\bar{D}+N_1+N_1^T+\tau S_{11}$, $\Xi_{12} = -N_1+N_2^T+\tau S_{12}$, $\Xi_{13} = P\bar{A}-\tau\bar{D}R\bar{A}+N_3^T+\tau S_{13}$, $\Xi_{14} = N_4^T+\tau S_{14}$, $\Xi_{15} = P\bar{B}-\tau\bar{D}R\bar{B}+N_5^T+\tau S_{15}$, $\Xi_{16} = P\bar{E}-\tau\bar{D}R\bar{E}+N_6^T+\tau S_{16}$, $\Xi_{22} = -Q+\gamma_2\bar{L}+\gamma_3\bar{M}-N_2^T-N_2+\tau S_{22}$, $\Xi_{23} = -N_3^T+\tau S_{23}$, $\Xi_{24} = -N_4^T+\tau S_{24}$, $\Xi_{25} = -N_5^T+\tau S_{25}$, $\Xi_{26} = -N_6^T+\tau S_{26}$, $\Xi_{33} = \bar{A}^T\bar{A}+\tau\bar{A}^TR\bar{A}+\tau S_{33}+\Lambda-\gamma_1 I$, $\Xi_{34} = \tau S_{34}$, $\Xi_{35} = \bar{A}^T\bar{B}+\tau\bar{A}^TR\bar{B}+\tau S_{35}$, $\Xi_{36} = \bar{A}^T\bar{E}+\tau\bar{A}^TR\bar{E}+\tau S_{36}$, $\Xi_{44} = \tau S_{44}-\Lambda-\gamma_2 I$, $\Xi_{45} = \tau S_{45}$, $\Xi_{46} = \tau S_{46}$, $\Xi_{55} = \bar{B}^T\bar{B}+\tau\bar{B}^TR\bar{B}+\tau S_{55}-\gamma_3 I$, $\Xi_{56} = \bar{B}^T\bar{E}+\tau\bar{B}^TR\bar{E}+\tau S_{56}$, $\Xi_{66} = -I+\bar{E}^T\bar{E}+\tau\bar{E}^TR\bar{E}+\tau S_{66}$.

Proof. We define the following positive definite Lyapunov functional:

$$V(\tilde{w}) = V_1(\tilde{w})+V_2(\tilde{w})+V_3(\tilde{w})+V_4(\tilde{w})+V_5(\tilde{w})+V_6(\tilde{w}),$$

where

$$\begin{aligned} V_1(\tilde{w}) &= \tilde{w}^TP\tilde{w}, \\ V_2(\tilde{w}) &= \tilde{w}^T\bar{D}\tilde{w}, \\ V_3(\tilde{w}) &= \int_{t-\tau}^{t} \dot{\tilde{w}}^T(s)\dot{\tilde{w}}(s)\,\mathrm{d}s, \\ V_4(\tilde{w}) &= \int_{t-\tau}^{t} \tilde{w}^T(s)Q\tilde{w}(s)\,\mathrm{d}s, \\ V_5(\tilde{w}) &= \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\tilde{w}}^T(s)R\dot{\tilde{w}}(s)\,\mathrm{d}s\mathrm{d}\theta, \\ V_6(\tilde{w}) &= \int_{t-\tau}^{t} \tilde{f}^T(\tilde{w}(s))\Lambda\tilde{f}(\tilde{w}(s))\,\mathrm{d}s. \end{aligned}$$

Deriving the derivative of $V(\tilde{w})$ along the trajectory of (19), we can obtain:

$$\begin{aligned} \dot{V}(\tilde{w}) &= 2\tilde{w}^TP\dot{\tilde{w}}+2\dot{\tilde{w}}^T\bar{D}\tilde{w}+\dot{\tilde{w}}^T\dot{\tilde{w}}-\dot{\tilde{w}}^{\tau T}\dot{\tilde{w}}^{\tau}+\tilde{w}^TQ\tilde{w}-\tilde{w}^{\tau T}Q\tilde{w}^{\tau} \\ &\quad +\tau\dot{\tilde{w}}^TR\dot{\tilde{w}}-\int_{t-\tau}^{t}\dot{\tilde{w}}^T(s)R\dot{\tilde{w}}(s)\,\mathrm{d}s+\tilde{f}^T(\tilde{w})\Lambda\tilde{f}(\tilde{w})-\tilde{f}^T(\tilde{w}^{\tau})\Lambda\tilde{f}(\tilde{w}^{\tau}). \end{aligned} \tag{28}$$

Using the following zero equation:

$$\tilde{w}(t)-\tilde{w}(t-\tau)-\int_{t-\tau}^{t}\dot{\tilde{w}}(s)\,\mathrm{d}s = 0.$$

Then for any constant matrices N_i $(i = 1, 2, \ldots, 5)$ with appropriate dimensions, we obtain:

$$2[\tilde{w}^T N_1 + \tilde{w}^{\tau T} N_2 + \tilde{f}^T(\tilde{w}) N_3 + \tilde{f}^T(\tilde{w}^\tau) N_4 + \tilde{g}^T(\tilde{w}^\tau) N_5 + \dot{\tilde{w}}^{\tau T} N_6] \times [\tilde{w} - \tilde{w}^\tau - \int_{t-\tau}^{t} \dot{\tilde{w}}(s)\,\mathrm{d}s] = 0. \quad (29)$$

On the other hand, for any constant matrices S_{ij} $(i = 1, 2, \ldots, 6;\ j = 1, 2, \ldots, 6)$ with appropriate dimensions, the following equation holds:

$$\varphi^T(t)\Psi\varphi(t) = 0, \quad (30)$$

where $\varphi^T(t) = [\tilde{w}^T, \tilde{w}^{\tau T}, \tilde{f}^T(\tilde{w}), \tilde{f}^T(\tilde{w}^\tau), \tilde{g}^T(\tilde{w}^\tau), \dot{\tilde{w}}^{\tau T}]$ and

$$\Psi = \begin{bmatrix} \tau(S_{11}-S_{11}) & \tau(S_{12}-S_{12}) & \tau(S_{13}-S_{13}) & \tau(S_{14}-S_{14}) & \tau(S_{15}-S_{15}) & \tau(S_{16}-S_{16}) \\ \tau(S_{21}-S_{21}) & \tau(S_{22}-S_{22}) & \tau(S_{23}-S_{23}) & \tau(S_{24}-S_{24}) & \tau(S_{25}-S_{25}) & \tau(S_{26}-S_{26}) \\ \tau(S_{31}-S_{31}) & \tau(S_{32}-S_{32}) & \tau(S_{33}-S_{33}) & \tau(S_{34}-S_{34}) & \tau(S_{35}-S_{35}) & \tau(S_{36}-S_{36}) \\ \tau(S_{41}-S_{41}) & \tau(S_{42}-S_{42}) & \tau(S_{43}-S_{43}) & \tau(S_{44}-S_{44}) & \tau(S_{45}-S_{45}) & \tau(S_{46}-S_{46}) \\ \tau(S_{51}-S_{51}) & \tau(S_{52}-S_{52}) & \tau(S_{53}-S_{53}) & \tau(S_{54}-S_{54}) & \tau(S_{55}-S_{55}) & \tau(S_{56}-S_{56}) \\ \tau(S_{61}-S_{61}) & \tau(S_{62}-S_{62}) & \tau(S_{63}-S_{63}) & \tau(S_{64}-S_{64}) & \tau(S_{65}-S_{65}) & \tau(S_{66}-S_{66}) \end{bmatrix}.$$

Eqs. (7) – (8) guarantee that the following inequalities are true

$$\begin{aligned} &\gamma_1(\tilde{w}^T \bar{L}\tilde{w} - \tilde{f}^T(\tilde{w})\tilde{f}(\tilde{w})) \geq 0, \\ &\gamma_2(\tilde{w}^{\tau T} \bar{L}\tilde{w}^\tau - \tilde{f}^T(\tilde{w}^\tau)\tilde{f}(\tilde{w}^\tau)) \geq 0, \\ &\gamma_3(\tilde{w}^{\tau T} \bar{M}\tilde{w}^\tau - \tilde{g}^T(\tilde{w}^\tau)\tilde{g}(\tilde{w}^\tau)) \geq 0. \end{aligned} \quad (31)$$

Submitting (19) and (21) to the right side of (28) and adding the left terms of (29) – (31), we can get the following inequality:

$$\dot{V}(\tilde{w}) \leq \phi^T(t)\Xi\phi(t) - \int_{t-\tau}^{t} \zeta^T(t,s)\Pi\zeta(t,s)\,\mathrm{d}s,$$

where $\phi^T(t) = [\tilde{w}^T, \tilde{w}^{\tau T}, \tilde{f}^T(\tilde{w}), \tilde{f}^T(\tilde{w}^\tau), \tilde{g}^T(\tilde{w}^\tau), \dot{\tilde{w}}^{\tau T}]$ and $\zeta^T(t,s) = [\phi^T(t), \dot{\tilde{w}}^T(s)]$.

Thus, we have $\dot{V}(\tilde{w}) < 0$ if the conditions (26) and (27) hold. It follows that the system (4) is globally stable, which means the equilibrium of system (1) is globally stable. This completes the proof. □

Remark 3.5. To the best of our knowledge, there are no stability results on complex-valued neutral-type neural networks with time delays in the existing literature. We just require that the activation functions satisfy the Lipschitz condition in this paper, so there is much room for further investigation.

4. NUMERICAL EXAMPLES

In this section, two numerical examples are provided to illustrate the effectiveness of the developed method for complex-valued neutral-type neural networks with time delays.

Example 4.1. Consider the system (1) that has two neurons, in which the parameters and nonlinear complex-valued functions are given as follows:

$$D = \begin{bmatrix} 80 & 0 \\ 0 & 60 \end{bmatrix}, \quad A = \begin{bmatrix} 05+0.5i & 1-i \\ 2-3i & 2+i \end{bmatrix},$$
$$B = \begin{bmatrix} 0.6-i & 1+0.8i \\ 2+2i & 2-2i \end{bmatrix}, \quad E = \begin{bmatrix} 0.1+0.3i & 0.2+0.1i \\ 0.3-0.4i & -0.5+0.2i \end{bmatrix},$$
$$\tau = 0.1, \quad u = \begin{bmatrix} 8-5i \\ -1+3i \end{bmatrix},$$

for $z_j = x_j + iy_j \in \mathbb{C}$, with $x_j, y_j \in \mathbb{R}$. The activation functions are given as follows:

$$f_j(z_j) = \frac{1-e^{-x_j}}{1+e^{-x_j}} + i\frac{1}{1+e^{-y_j}},$$
$$g_j(z_j) = \frac{1-e^{-x_j}}{1+e^{-x_j}} + i\frac{1}{1+e^{-y_j}},$$

and we can choose

$$L = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

Using Matlab LMI toolbox to solve (9) and (10), we can obtain the feasible solutions: $\varepsilon_1 = 525.5086$, $\varepsilon_2 = 536.8626$ and

$$P = \begin{bmatrix} 30.4912 & 9.4503 & -0.0000 & 2.3023 \\ 9.4503 & 41.6187 & -2.3023 & 0.0000 \\ -0.0000 & -2.3023 & 30.4912 & 9.4503 \\ 2.3023 & 0.0000 & 9.4503 & 41.6187 \end{bmatrix},$$
$$Q = 10^3 \times \begin{bmatrix} 5.0570 & 0.5587 & 0.0000 & 0.1342 \\ 0.5587 & 2.9856 & -0.1342 & -0.0000 \\ 0.0000 & -0.1342 & 5.0570 & 0.5587 \\ 0.1342 & -0.0000 & 0.5587 & 2.9856 \end{bmatrix}.$$

According to Theorem 3.1, the neutral-type neural network (1) has a unique equilibrium point, which is asymptotically stable. Figure 1 shows the real and imaginary parts of the states for neural network (1), where the initial conditions are taken as $z_1(0) = 8.8 + 9.5i$ and $z_2(0) = -6.3 - 4.5i$.

Example 4.2. Consider the system (1) that has two neurons, in which the parameters and nonlinear complex-valued functions are given as follows:

$$D = \begin{bmatrix} 25 & 0 \\ 0 & 40 \end{bmatrix}, \quad A = \begin{bmatrix} 1+i & -2-2i \\ 0.5+0.8i & 2+i \end{bmatrix},$$
$$B = \begin{bmatrix} 0.9+1.5i & 1+i \\ -2+2i & 2-2i \end{bmatrix}, \quad E = \begin{bmatrix} 0.2+0.2i & 0.1+0.3i \\ 0.4-0.5i & -0.3+0.1i \end{bmatrix},$$
$$\tau = 0.3, \quad u = \begin{bmatrix} 10-8i \\ -1+2i \end{bmatrix},$$

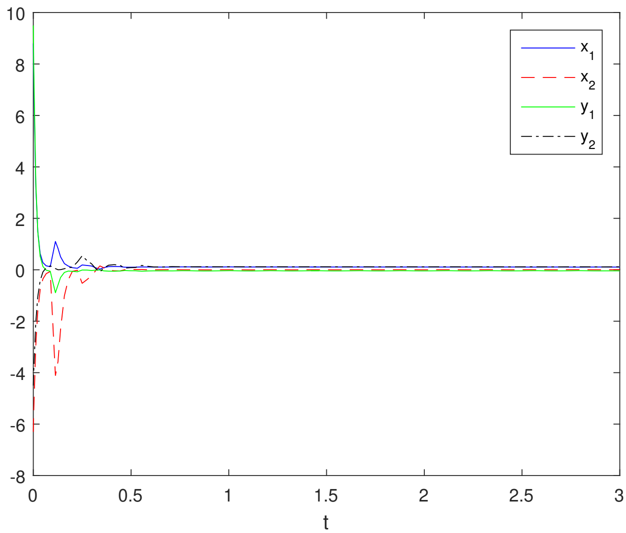

Fig. 1. Curves of the real and imaginary part of z_1 and z_2 in Example 4.1.

for $z_j = x_j + iy_j \in \mathbb{C}$, with $x_j, y_j \in \mathbb{R}$. The activation functions are given as follows:

$$f_j(z_j) = \frac{1 - e^{-x_j}}{1 + e^{-x_j}} + i\frac{1}{1+e^{-y_j}},$$

$$g_j(z_j) = \frac{1 - e^{-x_j}}{1 + e^{-x_j}} + i\frac{1}{1+e^{-y_j}},$$

and we can choose

$$L = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

Using Matlab LMI toolbox to solve (26) and (27), we can obtain the feasible solutions: $\gamma_1 = 268.4961$, $\gamma_2 = 8.9309$, $\gamma_3 = 98.8004$ and

$$P = \begin{bmatrix} 43.1511 & 1.0213 & 0.0000 & 1.7542 \\ 1.0213 & 5.0207 & -1.7542 & 0.0000 \\ 0.0000 & -1.7542 & 43.1511 & 1.0213 \\ 1.7542 & 0.0000 & 1.0213 & 5.0207 \end{bmatrix},$$

$$Q = \begin{bmatrix} 555.3960 & 7.5999 & 0.0000 & 12.4358 \\ 7.5999 & 569.4956 & -12.4358 & 0.0000 \\ 0.0000 & -12.4358 & 555.3960 & 7.5999 \\ 12.4358 & 0.0000 & 7.5999 & 569.4956 \end{bmatrix},$$

$$R = \begin{bmatrix} 1.5140 & 0.0531 & 0.0000 & 0.0925 \\ 0.0531 & 0.0891 & -0.0925 & 0.0000 \\ 0.0000 & -0.0925 & 1.5140 & 0.0531 \\ 0.0925 & 0.0000 & 0.0531 & 0.0891 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 120.3967 & 8.5685 & -0.0000 & 0.9818 \\ 8.5685 & 97.1789 & -0.9818 & -0.0000 \\ -0.0000 & -0.9818 & 120.3967 & 8.5685 \\ 0.9818 & -0.0000 & 8.5685 & 97.1789 \end{bmatrix}.$$

The conditions in Theorem 3.4 are satisfied. By Theorem 3.4 we can conclude that the equilibrium point of system (1) is globally asymptotically stable. Figure 2 shows the real and imaginary parts of the states for neural network (1), where the initial conditions are taken as $z_1(0) = 9.6 + 10.2i$ and $z_2(0) = 8.4 - 11.3i$.

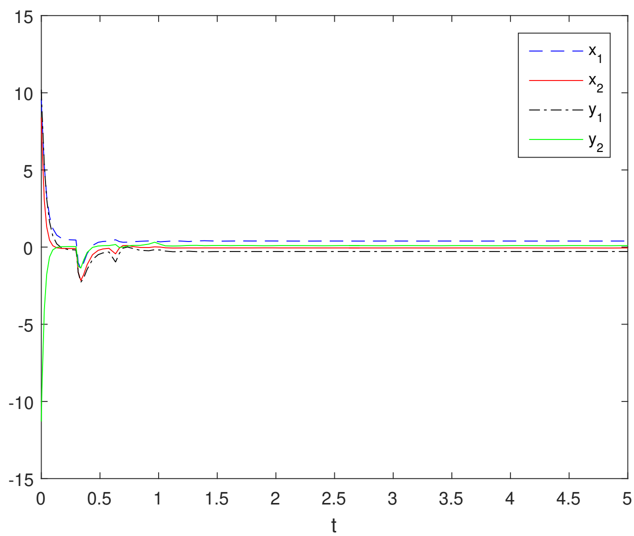

Fig. 2. Curves of the real and imaginary part of z_1 and z_2 in Example 4.2.

Remark 4.3. The boundedness and differentiability of the activation function are removed in this paper. Hence, the results in [8, 14, 43] cannot be utilized to assure the global asymptotic stability of neural network (1).

Remark 4.4. Due to the existence of neutral term, the LMI-based conditions in Theorem 3.1 and Theorem 3.4 are different from those in [6, 14, 45, 46]. Thus, for the parameters given in Examples 4.1 and 4.2, the global asymptotic stability of system (1) cannot be verified by the results in [6, 14, 45, 46].

5. CONCLUSIONS

To deal with the neutral term in the right side of neural network (1), a proper function (11) is constructed to prove the existence of equilibrium. By separating complex-valued neural networks into its real and imaginary parts in this paper, some criteria that guarantee the existence, uniqueness and global asymptotic stability of equilibrium point for the complex-valued neutral-type neural networks are obtained, without assuming that the activate functions are bounded or differentiable. The effectiveness of the obtained theoretical results is verified by two numerical examples.

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