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4-CYCLE PROPERTIES FOR CHARACTERIZING RECTAGRAPHS AND HYPERCUBES

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Abstract. A $(0,2)$-graph is a connected graph, where each pair of vertices has either 0 or 2 common neighbours. These graphs constitute a subclass of $(0,\lambda)$-graphs introduced by Mulder in 1979. A rectagraph, well known in diagram geometry, is a triangle-free $(0,2)$-graph. $(0,2)$-graphs include hypercubes, folded cube graphs and some particular graphs such as icosahedral graph, Shrikhande graph, Klein graph, Gewirtz graph, etc. In this paper, we give some local properties of 4-cycles in $(0,\lambda)$-graphs and more specifically in $(0,2)$-graphs, leading to new characterizations of rectagraphs and hypercubes.

Keywords: hypercube; $(0,2)$-graph; rectagraph; 4-cycle; characterization

MSC 2010: 05C75

1. Introduction

A $(0,2)$-*graph* is a connected graph, where each pair of vertices has either 0 or 2 common neighbours. This class of graphs is a special case of $(0,\lambda)$-*graphs*, introduced by Mulder, who showed that they are regular in [6]. A *rectagraph*, well-known in diagram geometry, see [9], is a $(0,2)$-graph without any triangles.

Since every pair of adjacent edges engenders a unique 4-cycle, we can easily compute the number of 4-cycles in a $(0,2)$-graph. In fact, there are $|V(G)|d(d-1)/8$ 4-cycles in a $(0,2)$-graph G of order $|V(G)|$ and degree d.

$(0,2)$-graphs were studied in various contexts; existence and construction of $(0,2)$-graphs were intensively studied by several researchers, we can cite for instance [2], [3]. Another important aspect is characterizing hypercubes as $(0,2)$-graphs; due to their remarkable properties and multiple applications, many authors have investigated this topic in order to give a new point of view which can be used for recognizing and constructing hypercubes, see [1], [5], [6], [7], [10].

In this paper, we give new characterizations of hypercubes and rectagraphs, in addition to many local properties of 4-cycles in $(0, \lambda)$-graphs and more specifically in $(0, 2)$-graphs, using an edge binary relation introduced by Sabidussi in his work [11] on Cartesian graph products.

2. Definitions and notation

Let G be a simple connected graph with $V(G)$ its vertex set and $E(G)$ its edge set, let x and y be two vertices in $V(G)$. We denote by $N(x)$ the set of neighbours of x and by $N(x, y)$ the set of common neighbours of x and y. An edge $\{x, y\}$ will be denoted xy, where vertices x and y are its *extremities*.

A *hypercube* or a d-dimensional cube is the graph, denoted Q_d, with $V(Q_d) = \{0,1\}^d$ where the d-tuples x and y are joined if and only if they differ in exactly one coordinate. Let G be a graph and let $u, v \in V(G)$. The *distance* $d(u, v)$ between two vertices u and v is the length of a shortest (u, v)-path. The *diameter* of the graph G, denoted $\mathrm{diam}(G)$, is the maximum distance between any pairs of vertices. The *interval* $I(u, v)$ can be defined by $I(u, v) = \{w \in V(G)\colon\ d(u, v) = d(u, w) + d(w, v)\}$.

A vertex $\bar{u}$ is an *antipode* of a vertex u if $I(u, \bar{u}) = V(G)$. A graph G is an *antipodal graph* if each vertex has a unique antipode. A subset $W \subseteq V(G)$ is *convex* if $I(u, v) \subseteq W$ for any $u, v \in W$. A graph G is *interval monotone* if each of its intervals is convex. A graph G is *interval regular* if $|I(u, v) \cap N(u)| = d(u, v)$ for any two vertices u and v in $V(G)$. Note that a hypercube is characterized as a $(0, 2)$-graph, see [5], [6], interval monotone [4], interval regular [8] and an antipodal graph [10].

Let G be a graph. A 4-cycle denoted C is defined by the sequence of its four vertices. The set of all 4-cycles in G is denoted $\Phi(G)$.

Let $e, é \in E(G)$, we define a binary relation θ on $E(G)$, where $e\theta é$ if there is a 4-cycle $C \in \Phi(G)$ such that $e, é$ are nonadjacent edges of C. We say that the edge e is *parallel* to $é$ and denote $\theta(e) = \{é \in E(G)\colon\ e\theta é\}$. Note that θ was introduced by Sabidussi in [11] as the relation $\sim^{(0)}$.

Let G be a $(0, 2)$-graph and let e be an edge in $E(G)$. We define the *edge level decomposition relative to* e by defining the subsets $N_i(e) \subseteq E(G)$, $0 \leqslant i \leqslant m$, which constitute the *levels* of the decomposition, where $N_0(e) = \{e\}$ and $e_i \in N_i(e)$, if there is at least one edge $e_{i-1} \in N_{i-1}(e)$ such that $e_i \in \theta(e_{i-1})$ for $1 \leqslant i \leqslant m$, where m denotes the index of the last level in the decomposition. It is then clear that in any edge level decomposition relative to e, every edge $f = xy$ in $E(G)$ is either in $N_k(e)$, $0 \leqslant k \leqslant m$, or belongs to $E(G) \setminus \bigcup\limits_{i=1}^{m} N_i(e)$. In this case, its two extremities

x and y are incident with two different edges from $\bigcup\limits_{i=1}^{m} N_i(e)$, which are either in the same level or different levels.

3. Some 4-cycle properties in $(0, 2)$-graphs

In this section, we give first some local properties of 4-cycles in $(0, \lambda)$-graphs. After that, other results, using the relation θ, are presented for the specific case $\lambda = 2$, including a characterization of rectagraphs.

Lemma 3.1. *If G is a $(0, \lambda)$-graph, then every vertex in $V(G)$ is in $d(d-1) \times (\lambda - 1)/2$ 4-cycles.*

Proof. Let u be a vertex. Since G is regular, $|N(u)| = d$ and for each pair of vertices $v, w \in N(u)$, there are $(\lambda - 1)$ 4-cycles containing u, v, w at one time. Thus the number of 4-cycles containing u equals $(\lambda - 1)\binom{d}{2}$. $\square$

Lemma 3.2. *If G is a $(0, \lambda)$-graph, then every edge e is contained in $(\lambda-1)(d-1)$ 4-cycles.*

Proof. Let $e = uv$ be an edge. For each edge e_u incident to u, there are $\lambda - 1$ edges $e_1, e_2, \ldots, e_{\lambda-1}$ incident to v such that e, e_u and e_i, $1 \leqslant i \leqslant \lambda - 1$ are contained in a 4-cycle. Since G is d-regular, e is then contained in $(\lambda - 1)(d - 1)$ 4-cycles. $\square$

Lemma 3.3. *Let G be a $(0, \lambda)$-graph. If there is an edge $e \in E(G)$ such that $\theta(e)$ is a nonempty matching, then $\lambda = 2$.*

Proof. Let G be a $(0, \lambda)$-graph. It is easy to see that if $\lambda = 1$ then $\Phi(G) = \emptyset$. Therefore, $\theta(e) = \emptyset$ for all edges in $E(G)$. Note that if $\lambda \geqslant 2$ and $|E(G)| \geqslant 2$, then every two adjacent edges are contained in $(\lambda - 1)$ 4-cycles, thus for any edge e, there are at least $\lambda - 1$ adjacent edges that are parallel to e. Therefore, if there is an edge e such that $\theta(e)$ is a matching, then we have necessarily $\lambda = 2$. $\square$

Lemma 3.4. *In a $(0, 2)$-graph G, we have*

$$d - 2 \leqslant |\theta(e)| \leqslant d - 1, \quad e \in E(G).$$

Furthermore, G is K_4 free if and only if $|\theta(e)| = d - 1$ for all $e \in E(G)$.

Proof. Let G be a $(0, 2)$-graph. First, we prove that $d - 2 \leqslant |\theta(e)| \leqslant d - 1$ for all $e \in E(G)$.

Let $e = uv$ be an edge in $E(G)$. According to Lemma 3.2, e is contained in $(d-1)$ 4-cycles, and by definition of the relation θ, we can easily deduce that $|\theta(e)| \leqslant d-1$. On the other hand, assume that $|\theta(e)| \leqslant d-3$, then there are $e_1 = xy$, $e_2 = zt \in \theta(e)$, $e_1 \neq e_2$, such that e and e_1 are contained in two 4-cycles C_1, C_2 and e and e_2 are contained in two other 4-cycles C_3, C_4. Thus on one hand we have $N(u,v) = \{x,y\}$, and on the other hand we have $N(u,v) = \{z,t\}$. By the $(0,2)$-property, this implies that $\{x,y\} = \{z,t\}$ which contradicts the fact that $e_1 \neq e_2$.

Note that if the edge $e = uv \in E(G)$ is such that $|\theta(e)| < d-1$, then there is an edge $\bar{e} \in \theta(e)$ such that e and $\bar{e}$ are contained in two different 4-cycles C_1, C_2, with $C_1 = uvwt$ and $C_2 = uvtw$, which means that vertices u, v, w, and t are pairwise adjacent and the subgraph induced by the set $\{u,v,w,t\}$ is isomorphic to K_4.

Conversely, if the subgraph induced by $Y = \{u,v,w,t\} \subseteq V(G)$ is isomorphic to K_4, then there are two 4-cycles $C_1, C_2 \in \Phi(G)$ containing e and $\bar{e}$ with $C_1 = uvwt$ and $C_2 = uvtw$, hence $|\theta(e)| = d-2 < d-1$. □

Lemma 3.5. *Let G be a $(0,2)$-graph and let $\acute{e} \in E(G)$. If $\bar{e} \in \theta(\acute{e})$, then there is at most one edge $\bar{\bar{e}} \in \theta(\acute{e})$ such that $\bar{e}, \bar{\bar{e}}$ are adjacent. In this case, G contains $K_4 - e$ as an induced subgraph.*

P r o o f. Let G be a $(0,2)$-graph. Let $\acute{e} = uv \in E(G)$ and $\bar{e} = wz \in \theta(\acute{e})$. Suppose that there are two edges e_1, e_2 adjacent to $\bar{e}$ such that $e_1, e_2 \in \theta(\acute{e})$. We have then, without loss of generality, two cases:

Case 1: $e_1 = wt$ and $e_2 = wy$. Since $\bar{e} \in \theta(\acute{e})$, there is a 4-cycle $C_1 = uvwz$ and $N(u,w) = \{v,z\}$. On the other hand, $e_1 = wt \in \theta(\acute{e})$ implies that $ut \notin E(G)$ (otherwise $N(u,w) = \{v,z,t\}$) thus there is a 4-cycle $C_2 = uvtw$ and $N(v,w) = \{u,t\}$. Since $e_2 = wy \in \theta(\acute{e})$, there is a 4-cycle C_3 such that $C_3 = uvyw$ or $C_3 = uvwy$. But if $C_3 = uvyw$, then $N(v,w) = \{u,t,y\}$ and if $C_3 = uvwy$, then $N(u,w) = \{v,z,y\}$, which is impossible in both the cases.

Case 2: $e_1 = wt$ and $e_2 = zy$. Since $\bar{e} \in \theta(\acute{e})$, there is a 4-cycle $C_1 = uvwz$ and $N(v,z) = \{u,w\}$. We have also $e_1 = wt \in \theta(\acute{e})$ and $ut \notin E(G)$ (otherwise $N(u,w) = \{v,z,t\}$) which implies that there is $C_2 = uvtw$ and thus $N(v,w) = \{u,t\}$. On the other hand, $e_2 = zy \in \theta(\acute{e})$ implies that there is $C_3 \in \Phi(G)$ such that $C_3 = uvyz$ or $C_3 = uvzy$. But if $C_3 = uvyz$, then $N(v,z) = \{u,w,y\}$, and if $C_3 = uvzy$, then $N(v,w) = \{u,z,t\}$, which is also impossible.

Note that if $\bar{e} = wz$ and $e_1 = wt$ are parallel to $\acute{e} = uv$, then there are two 4-cycles $C_1 = uvwz$ and $C_2 = uvtw$ in $\Phi(G)$ and the subgraph induced by $\{u,v,z,w\}$ is isomorphic to $K_4 - e$ with $e = uz$, which concludes this proof. □

Theorem 3.6. *Let G be a $(0,\lambda)$-graph of degree d. G is a rectagraph if and only if $\theta(\acute{e})$ is a matching of $(d-1)$ edges for any edge $\acute{e}$.*

Proof. Let G be a rectagraph and let $\acute{e} \in E(G)$. Since G is triangle free, G is also K_4 free and $(K_4 - e)$ free. According to Lemma 3.4, $|\theta(\acute{e})| = d - 1$ and by Lemma 3.5, there are no adjacent edges in $\theta(\acute{e})$. It follows that $\theta(\acute{e})$ is a matching of $d - 1$ edges.

Conversely, if $\theta(\acute{e})$ is a matching of $d - 1$ edges for every edge $\acute{e} \in E(G)$ then according to Lemma 3.3, G is a $(0,2)$-graph. Now assume that G is a $(0,2)$-graph that contains triangles, and let x, y, z be three vertices inducing a triangle. Let $e_1 = xy$, $e_2 = yz$ and $e_3 = xz$. Since $z \in N(x,y)$, there is a unique vertex t such that $N(x,y) = \{t,z\}$. Hence there is $C_1 \in \Phi(G)$ such that $C_1 = xzyt$ and it follows that $\hat{e} = xt \in \theta(e_2)$. On the other hand, since $y \in N(x,z)$, there is a unique vertex w such that $N(x,z) = \{w,y\}$. Note that $w \neq t$, otherwise the subgraph induced by $\{x,y,z,t\}$ is isomorphic to K_4 and according to Lemma 3.4, $|\theta(\hat{e})| = d - 2 < d - 1$. Consequently, there is a 4-cycle $C_2 \in \Phi(G)$ such that $C_2 = xyzw$ and $\tilde{e} = xw \in \theta(e_2)$. This means that $\theta(e_2)$ is not a matching, since $\hat{e}$ and $\tilde{e}$ are adjacent, which contradicts our assumption. □

4. New characterizations of hypercubes

Hypercubes constitute a remarkable class of graphs with very interesting properties, including the $(0,2)$-property. In this section, we show new characterizations of a hypercube as a $(0,2)$-graph. But first, let us recall important results due to Mulder and Laborde and Rao Hebbare that we shall use:

Proposition 4.1 (Mulder [6], Laborde and Rao Hebbare [5]). *Let G be a $(0,2)$-graph of degree d, then $|V(G)| \leqslant 2^d$. Furthermore, G is a hypercube of dimension d if and only if $|V(G)| = 2^d$.*

Mulder has also shown the following result:

Proposition 4.2 (Mulder [8]). *Let G be a connected graph. G is a hypercube if and only if G is bipartite and interval regular.*

Theorem 4.3. *Every interval monotone rectagraph is interval regular.*

Proof. Let G be an interval monotone rectagraph. The proof is by induction on $d(u,v)$ where $u,v \in V(G)$. If $d(u,v) = 2$, then $I(u,v) = \{x,y\}$ and $|N(u) \cap I(u,v)| = |N(u,v)| = 2$. Let $u,v \in V(G)$ such that $d(u,v) = k \geqslant 3$, and let $w \in N(u) \cap I(u,v)$.

By induction hypothesis, we have $|N(w) \cap I(w,v)| = d(w,v) = k-1$. We denote by $t_1, t_2, \ldots, t_{k-1}$, the neighbours of w in $I(w,v)$. Since $d(u,t_i) = 2$, $1 \leqslant i \leqslant k-1$, there are $w_1, w_2, \ldots, w_{k-1} \in V(G)$ with $w_i \neq w$, $1 \leqslant i \leqslant k-1$, such that $N(u,t_i) = \{w, w_i\}$. Hence $d(w_i, v) = d(t_i, v) + 1 = k-1$ and $w_i \in N(u) \cap I(u,v)$. Note that $w_i \neq w_j$ for $i \neq j$, otherwise $N(w, w_i) = N(w, w_j) = \{u, t_i, t_j\}$. Assume now that there is a vertex $x \in N(u) \cap I(u,v)$ such that $x \neq w, w_1, w_2, \ldots, w_{k-1}$. Therefore $u \in N(x,w)$ and there is a unique vertex $y \neq u$ such that $y \in N(x,w)$. Note that $y \neq t_i$, $1 \leqslant i \leqslant k-1$, otherwise $N(u,t_i) = \{x, w, w_i\}$. Since G is interval monotone and triangle free, $y \in I(x,w) \subset I(u,v)$, and consequently, $y \in N(w) \cap I(u,v)$, which contradicts the induction hypothesis. □

Corollary 4.4. *Let G be a graph. G is a hypercube if and only if G is a bipartite interval monotone rectagraph.*

Proof. A hypercube is a bipartite interval monotone rectagraph. Conversely, according to Theorem 4.3, an interval monotone rectagraph G is interval regular. Since G is bipartite, G is a hypercube by Proposition 4.2. □

Theorem 4.5. *Let G be a rectagraph of degree d. G is a hypercube of dimension d if and only if in an edge level decomposition relative to any edge e, we have $|N_0(e)| = |N_m(e)| = 1$, $|N_{i-1}(e) \cap \theta(e_i)| = i$ and $|N_{i+1}(e) \cap \theta(e_i)| = d-1-i$ for any edge $e_i \in N_i(e)$, $1 \leqslant i \leqslant m$.*

Proof. Let Q_d be a hypercube of dimension d, let $e = uw$ be an edge in $E(Q_d)$ and let $N_0(e), N_1(e), \ldots, N_m(e)$ be the subsets of edges in the level decomposition relative to the edge e.

First, we prove that $|N_m(e)| = 1$ and $m = d-1$. Let v be the unique antipode of u in Q_d and t the unique antipode of w, thus $I(u,v) = I(w,t) = V(Q_d)$ and $d(u,v) = d(w,t) = d$. Since $v \in I(w,t)$, we have $d(w,t) = d = d(w,v) + d(v,t)$. On the other hand, $w \in I(u,v)$ implies that $d(u,v) = d = d(u,w) + d(w,v)$. We can easily deduce that $d(u,w) = d(v,t)$, and v, t are adjacent. Since $d(u,t) = d(w,v) = d-1$, the edge $vt \in N_m(e)$ and $m = d-1$. This edge is unique in $N_{d-1}(e)$, otherwise there would be another pair of vertices x and y which are antipodes of u and v, respectively. Hence $|N_m(e)| = 1$.

Now let $e_i = xy \in N_i(e)$. Since $d(u,x) = i$ and Q_d is interval regular, $|N(x) \cap I(u,x)| = i$, thus there are i neighbours $x_1, x_2, \ldots, x_i$ of x in $I(u,x)$. Note that $d(u,x_j) = i-1$ and $x \in N(y,x_j)$, $1 \leqslant j \leqslant i$, which implies that there are i vertices, $y_1, y_2, \ldots, y_i$ such that $y_j \in N(y,x_j)$, $1 \leqslant j \leqslant i$. It follows that $x_j y_j \in \theta(e_i)$, $1 \leqslant j \leqslant i$ and we have necessarily the edges $x_j y_j \in N_{i-1}(e)$, $1 \leqslant j \leqslant i$. Thus $|N_{i-1}(e) \cap \theta(e_i)| = i$. In the same way, let $vt \in N_{d-1}(e)$ be such that $d(u,t) =$

$d(w,v) = d-1$. So $d(x,t) = d-1-i$ and there are $d-1-i$ neighbours of the vertex x (say $\acute{x}_1, \acute{x}_2, \ldots, \acute{x}_{d-1-i}$) in $I(x,t)$. Since $x \in N(y, \acute{x}_j), 1 \leqslant j \leqslant d-1-i$, there are vertices $\acute{y}_1, \acute{y}_2, \ldots, \acute{y}_{d-1-i}$, such that $\acute{y}_j \in N(y, \acute{x}_j)$, $1 \leqslant j \leqslant d-1-i$. Thus $\acute{x}_j\acute{y}_j \in N_{i+1}(e)$, $1 \leqslant j \leqslant d-1-i$ and $|N_{i+1}(e) \cap \theta(e_i)| = d-1-i$.

Conversely, let G be a rectagraph satisfying the hypothesis of Theorem 4.5 and let $e \in E(G)$. Let $N_0(e), N_1(e), \ldots, N_m(e)$ be the subsets of edges in the level decomposition relative to the edge e. Note that for any edge $e_i \in N_i(e)$, every edge of $\theta(e_i)$ is either in $N_{i-1}(e)$ or $N_{i+1}(e)$.

First we prove that $N_i(e)$ is a matching for $1 \leqslant i \leqslant m$. So let us assume the contrary, and choose the smallest k such that $N_k(e)$ is not a matching. So there are at least two adjacent edges in $N_k(e)$, say $e_k = xy$ and $\acute{e}_k = yz$. Since $|N_{k-1}(e) \cap \theta(e_k)| = k$, there is an edge $e_{k-1} = \acute{x}\acute{y}$ parallel to e_k in $N_{k-1}(e)$. Note that $\acute{e}_k \notin \theta(e_{k-1})$, since G is a rectagraph. Therefore, since $y \in N(\acute{y}, z)$, there is necessarily another vertex $\acute{z} \in N(\acute{y}, z)$, thus the edge $\acute{e}_{k-1} = \acute{y}\acute{z} \in \theta(\acute{e}_k)$ and consequently, $\acute{e}_{k-1} \in N_{k-1}(e)$. But edges e_{k-1} and $\acute{e}_{k-1}$ are adjacent, which contradicts the fact that k is the smallest integer such that $N_k(e)$ contains adjacent edges. It follows that $N_i(e)$ is a matching for $1 \leqslant i \leqslant m$.

Now let us prove, by induction on i, that $|N_i(e)| = \binom{d-1}{i}$, $0 \leqslant i \leqslant m$.

For $i = 0$, we have $|N_0(e)| = 1 = \binom{d-1}{0}$. Assume now that for all $k \leqslant i-1$, $|N_k(e)| = \binom{d-1}{k}$. Since each edge $e_{i-1} \in N_{i-1}(e)$ is parallel to $d-i$ edges in $N_i(e)$, the total number of 4-cycles lying between $N_{i-1}(e)$ and $N_i(e)$ equals $\binom{d-1}{i-1}(d-i)$.

On the other hand, each edge $e_i \in N_i(e)$ is parallel to i edges from $N_{i-1}(e)$. Thus, the number of 4-cycles lying between $N_i(e)$ and $N_{i-1}(e)$ equals $|N_i(e)|i$, and it follows that

$$|N_i(e)| = \frac{\binom{d-1}{i-1}(d-i)}{i} = \binom{d-1}{i}.$$

Note that $|N_m(e)| = 1 = \binom{d-1}{m}$ implies $m = d-1$.

By counting the total number of edges in the levels, we have

$$|N_0(e)| + |N_1(e)| + \ldots + |N_{d-1}(e)| = \sum_{i=0}^{d-1} \binom{d-1}{i} = 2^{d-1}.$$

Since each level is a matching, it follows that $|V(G)| = 2 \cdot 2^{d-1} = 2^d$. According to Proposition 4.1, G is a hypercube of dimension d. □

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