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# EQUILIBRIUM SEARCH MODEL WITH ENDOGENOUS GROWTH RATE OF HUMAN CAPITAL

Wansheng Tang, Chi Zhou, Chaoqun Xiao and Ruiqing Zhao

This article studies an equilibrium search problem when jobs provided by firms can be either unskilled or skilled and when workers differing in their education level can be either low-educated or high-educated. The structure proportion of jobs affects the equilibrium which indicates a threshold that can distinguish whether the equilibrium is separating or cross-skill. In addition, the cross-skill equilibrium solution implies the high-educated workers are more likely to obtain higher pay rates than the low-educated workers with same tenure. Moreover, the profits of the firms decrease as the number of the low-educated workers increases. The most interesting conclusion is that the growth rate of the human capital is an endogenous variable which is different from previous work.

*Keywords:* dynamic programming, contracts, job search, market equilibrium, human capital

*Classification:* 49L20, 90C39, 91B40, 97M40

## 1. INTRODUCTION

The unsymmetrical distribution between job opportunities and labor resources makes the real labor market different from other markets. In the labor market, workers hope to find an ideal job in the shortest time, while firms expect to maximize their profit through offering workers the wage as low as possible. In practice, the human capital of workers not only affects the efficiency of job search and matching, but also becomes the most important influencing factors of corporate value. Among 11,601 construction enterprises in Taiwan in 2008, 8017 (i. e., 69%) of them are small construction enterprises. And only a few of the construction workers employed had bachelor's degree, while many of them have only received junior high school education or even lower education, resulting in low safety awareness and inability to increase profits [9]. Therefore, both human capital and labor market structure, which have a close relation, have significant effects on job search and recruitment strategies.

Job search theory developed in the 1960s has been an important theory in the labor market which is the study of dynamic decision making problems involving decision makers' optimal search strategy under incomplete information. Since the pioneering research of Stigler [24, 25], many researchers have done tremendously significant works

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in the search theory. McCall [17, 18] developed a sequential search model to analyze the job search behaviors of the new entrants. Phelps et al. [20] who is the first one to propose the concept of job search theory assumed that firms offer diverse wages and each worker learns about the distribution of wages from her search outcomes. Eriksson et al. [11] studied gender differences in job search. Zhou et al. [26, 27] investigated a search model to study job search and recruitment problem. Their results have a certain practical significance. Recently, job search models relax the assumptions of the canonical model by allowing for heterogeneous workers, an endogenous wage distribution, on-the-job search and unsteady-state conditions, to have better explanations for the search behaviors of workers, the process of the firms offering wage, and the phenomenon of job creation and job destruction in the labor market.

Over the past few decades, there has been a number of significant developments in the study of labor market equilibrium. Maybe the most commonly used equilibrium search models are on the basis of the work of Burdett and Mortensen [6] (for simplicity B/M). B/M assumed that each firm posts a single wage, both employed and unemployed workers can search for better job opportunities. Having recognized that, Burdett and Coles [2] (for simplicity B/C) and Stevens [22] extended B/M by assuming that firms post wage-tenure contract rather than a single wage. Burdett and Coles [4] generalized previous work by assuming firms have different productivities. Carrillo-Tudela [7] made an important assumption that firms cannot decide their wage offers on unemployment and employment duration. Shi [23] analyzed the equilibrium in a labor market in which firms offer wage-tenure contracts to direct the search by risk-averse workers. This paper differs from the aforementioned papers in the aspects of contract form that the firms offer wage-human capital contracts in order to analyze the effects of human capital accumulation on the labor market equilibrium.

Recent years, a number of theses have combined human capital accumulation with the B/M as studied here. For instance, Rubinstein and Weiss [21] analyzed the accumulation of human capital and on-the-job search, however, not considering market equilibrium. Dolado et al. [10] examined the effects of transitory skill mismatches in a matching model with heterogeneous jobs and workers. Gonzalez and Shi [14] integrated learning from search into an equilibrium framework and applied lattice-theoretic techniques to analyze learning from experience. In an insightful study, Burdett and Coles [3] assumed that workers accumulate general human capital through learning-by-doing, and they found the equilibrium approach to identify the effects between experience and tenure on workers' wages. Bringing in training, Fu [13] yielded new insights on wage dispersion and wage dynamics which shows that endogenous training breaks the perfect correlation between work experience and human capital.

The model addressed in this thesis is studied based on the previous research. For instance, Burdett et al. [5] structured an equilibrium labor market by allowing workers to learn by doing. Their paper implied that learning-by-doing increases equilibrium wage dispersion and considered that all firms are equally productive, and workers are heterogeneous defined by initial productivity. In contrast to the previous discussion, we discuss an equilibrium search model in which firms offer unskilled jobs or skilled jobs with different distributions and workers with low-education or high-education search for better job opportunities. The introduction of heterogeneous firms refines the structure of the labor market which has important consequences for the optimal search profits. High-educated workers are assumed to accept unskilled jobs for which they are over-qualified. The structure proportion of jobs affects the equilibrium which shows there is a threshold that can distinguish whether there exists a separating equilibrium or a cross-skill equilibrium. In addition, the cross-skill equilibrium solution implies that the high-educated workers are more likely to own higher pay rates than that the low-educated workers with the same tenure are likely to. It also implies both the profits of offering the unskilled jobs and offering skilled jobs decrease with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than the profit of the firms offering the unskilled jobs until there is only very few high-educated workers. Another important difference is that we allow for the growth rate of an employed worker's human capital to be an endogenous variable. Unlike other papers such as [3, 5] and [13] in which the growth rate of an employed worker's productivity is given exogenously, this allows us to clarify the endogenous growth rate of human capital which can be determined by death shock, job destruction shock, the fraction of the unskilled jobs and the arrival rate of jobs.

This paper shows how the endogenous human capital affects the labor market equilibrium. Firstly, this paper discusses an equilibrium search problem in which jobs provided by firms can be either unskilled or skilled while workers differing in their education level can be either low-educated or high-educated. The high-educated workers are assumed to be able to accept either the unskilled jobs or the skilled jobs for which they are over-qualified, while the low-educated workers can only accept the unskilled jobs. There is little literature to deal with this problem. Secondly, we imply that the structure proportion of the offered jobs affects the equilibrium which shows there exists a threshold that can distinguish whether the equilibrium is separating or cross-skill. In addition, the cross-skill equilibrium solution implies that the high-educated workers are more likely to own higher pay rates than that the low-educated workers with the same tenure are likely to. Thirdly, it also yields a new insight into the effect of the structure proportion of workers on the profits which implies that both the profits of firms offering the unskilled jobs and offering the skilled jobs decrease with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than the profit of the firms offering the unskilled jobs until there is only very few high-educated workers. Fourthly, the growth rate of the human capital is an endogenous variable which is different from previous work.

The rest of this paper is organized as follows. Section 2 outlines a framework describing the workers' job search strategies with different education levels and the firms' optimal contracts. Section 3 gives a definition of market equilibrium and illustrates the distributions of pay rate in different types of jobs. Section 4 presents the effect of human capital accumulation on equilibrium pay rate. Using numerical example, Section 5 describes how the proportion of the low-educated workers affects profit and the support of the pay rate. The paper is concluded in Section 6 with further work pointed out.

## 2. PROBLEM DESCRIPTION

Consider a labor market in steady state in which time is discrete. There is a continuum of risk neutral workers (worker is denoted by 'she') and firms (firm is denoted by 'he'), each of measure one.

The workers differ in their education level $j$ who can be either low-educated ($j = 0$) or high-educated ($j = 1$). An exogenous fraction $\alpha \in [0, 1]$ of the workers is low-educated, while the remaining fraction $1 - \alpha$ is high-educated. After a low-educated worker has worked for $\tau$ periods up to period $t$, her human capital is denoted by $h_t = (1 + g)^\tau$, where $0 < g < 1$, and the variable $\tau$ is the worker's experience and closely related to the period $t$. While for a high-educated worker, her human capital is denoted by $h_t = e^{g\tau}$ [5, 13]. For each new entrant, $\tau = 0$ and $h_t = 1$. The lives of workers are of uncertain duration. Any worker dies with probability $\delta$ per period, which describes the inflow rate of new entrants per period. All the dead workers are replaced with newly joined workers, so the population is balanced. There are job destruction shocks in which each employed worker is displaced into unemployment with probability $\sigma$ per period. However, the human capital of the unemployed worker does not grow. Assume that there is no recall if a worker quits or rejects a job offer. The objective of the worker is to maximize her total expected lifetime utility.

Jobs are either unskilled or skilled. A fraction $\beta$ of all the jobs in the labor market which can be performed by both types of workers are called unskilled jobs; while $1 - \beta$ of all the jobs which only requires high-educated workers are called skilled jobs. Any firm generates revenue $p$ from each unit of human capital he employs. Each firm posts a job offer contract $\theta^i$ at zero cost in the period $t$. That is, $\theta^i$ is the wage per unit of human capital and $i$ specifies the type of jobs, which can be either unskilled jobs ($i = 0$) or skilled jobs ($i = 1$). Let $F_0(\theta^0)$ describe the probability of receiving an unskilled job offer which is no greater than $\theta^0$. Likewise, $F_1(\theta^1)$ describes the probability of receiving a skilled job offer which is no greater than $\theta^1$. Further, let $\underline{\theta}^i$ and $\overline{\theta}^i$ denote the infimum and supremum of the support of $F_i$. Let $\lambda$ denote the Poisson arrival rate of these offers. Low-educated workers meet these offers at the same rate as high-educated do, but they do not qualify for the skilled jobs. The effective arrival rate of unskilled job offers faced by any low-educated worker is $\beta\lambda$, independent of their employment status; while for high-educated workers, the effective arrival rate depends on the type of jobs. If a job is from unskilled jobs, the arrival rate is $\beta\lambda$; if it is from skilled jobs, the arrival rate is $(1 - \beta)\lambda$. For simplicity, we assume that firms only offer one type of job and the choice is irreversible. The objective of each firm is to maximize steady state flow profit.

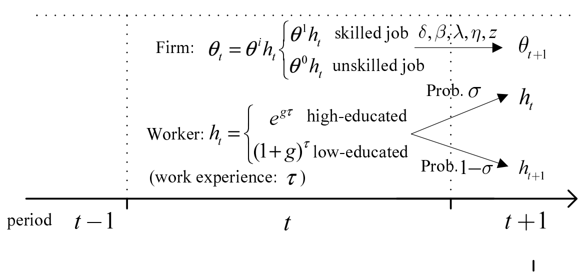



**Fig. 1.** The timeline of events.

Figure 1 explicitly presents the timeline of events. Assume workers and firms meet randomly. For a low-educated worker with human capital $h_t$, she will meet an outside firm which offers unskilled job contract $\theta^0$ with probability $\beta\lambda$. If the match succeeds, the worker is paid $\theta^0 h_t$ while the firm's profit is $\left(p-\theta^0\right)h_t$ at the production stage. Meanwhile for a high-educated worker with human capital $h_t$, it depends on the type of the outside job. With probability $\beta\lambda$ the worker will meet an unskilled job offer $\theta^0$. Then the worker is paid $\theta^0 h_t$ and the firm's profit is $\left(p-\theta^0\right)h_t$ at the production stage. With probability $(1-\beta)\lambda$ the worker will meet a skilled job offer $\theta^1$. If the match succeeds, the worker is paid $\theta^1 h_t$ and the firm's profit is $\left(p-\theta^1\right)h_t$. If the match does not succeed, the worker returns to her previous status. In this situation, the outside firm doesn't have any profit. For an unemployed worker, she has income $bh_t$, where $b$ can be interpreted as home production or leisure, and $0<b<p$.

### 2.1. The worker's problem

When a job offer arrives, a worker must choose between keeping the current status and accepting an outside offer. Let $x_t$ be the worker's decision variable which can be either 0 or 1. Furthermore, $x_t=0$ means that the worker will keep the current status, while $x_t=1$ means that the worker will accept the new offer.

#### 2.1.1. Low-educated workers' job search strategies

In this subsection, the search behavior of only low-educated workers is considered. The low-educated workers' objective function is

$$\max_{x_t} \mathrm{E}\sum_{t=0}^{\infty} u\left(\theta_t, h_t, x_t\right),$$

where $\theta_t$, $h_t$ are the state variables in period $t$.

First consider the unemployed workers with low-education. The pay rate $\theta_{t+1}$ of the workers satisfies

$$\theta_{t+1}=\xi\left\{(1-\zeta)b+\zeta\left[(1-x_t)b+x_t\theta^0\right]\right\},$$

i.e., $\theta_{t+1}$ also represents the payment per unit of human capital in period $t+1$. We assume that $\xi$ is a random variable denoting whether a low-educated worker survives at that period, which $\Pr\{\xi=0\}=\delta$ represents the worker dies and $\Pr\{\xi=1\}=1-\delta$ represents the worker is still in the labor market. And $\zeta$ is also a random variable, denoting whether a low-educated worker receives an unskilled job with $\Pr\{\zeta=1\}=\beta\lambda$ or $\Pr\{\zeta=0\}=1-\beta\lambda$. The expected lifetime utility of an unemployed worker with the human capital $h_t$ is denoted by $V_{u0}(h_t)$. When an unemployed worker with low-education meets an outside job offer $\theta^0$, she would compare the expected lifetime utility of accepting the offer, denoted by $V_0(\theta^0;h_t)$, with that of unemployment $V_{u0}(h_t)$. If the worker chooses $V_0(\theta^0;h_t)$, she would accept a new offer $\theta^0$. Otherwise, she still keeps the current status. Therefore, the Bellman equation is formulated as follows,

$$V_{u0}(h_t)=\max\{bh_t+\mathrm{E}_{(\theta^0)}V_0(\theta_{t+1};h_{t+1})\},$$

which can be rewritten as

$$\begin{aligned} V_{u0}(h_t) &= bh_t + (1-\delta-\beta\lambda)V_{u0}(h_t) + \beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_{u0}(h_t), V_0(\theta^0; h_t)\} + \delta \cdot 0 \\ &= bh_t + (1-\delta-\beta\lambda)V_{u0}(h_t) + \beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_{u0}(h_t), V_0(\theta^0; h_t)\}. \end{aligned} \tag{1}$$

Next consider the employed workers with low-education. Different from the unemployed workers, there is a job destruction shock for the employed workers with low-education. The pay rate $\theta_{t+1}$ of the employed worker with low-education satisfies

$$\theta_{t+1} = \xi \left\{(1-\eta)b + \eta \left[(1-\zeta)\theta_t + \zeta \left((1-x_t)\theta_t + x_t\theta^0\right)\right]\right\},$$

where $\eta$ is a random variable denoting whether there exists a job destruction shock. $\Pr\{\eta = 0\} = \sigma$ means the worker is displaced into unemployment and $\Pr\{\eta = 1\} = 1-\sigma$ means that there is no job destruction shock, and independent of $\xi$ and $\zeta$. Let $V_0(\theta_t; h_t)$ denote the expected lifetime utility of a worker when she is employed by obtaining pay rate $\theta_t$ in period $t$ and her human capital is $h_t$. Give an employed worker with human capital $h_t$ as an example, she needs to choose the larger one between $V_0(\theta_t; h_{t+1})$ and $V_0(\theta^0; h_{t+1})$. In addition, her human capital $h_t$ will become $(1+g)h_t$ at next period. Therefore, the Bellman equation is formulated as follows,

$$V_0(\theta_t; h_t) = \max\{\theta_t h_t + \mathrm{E}_{(\theta^0)} V_0(\theta_{t+1}; h_{t+1})\},$$

which can be written as

$$\begin{aligned} V_0(\theta_t; h_t) = \; & \theta_t h_t + \sigma V_{u0}((1+g)h_t) + (1-\delta-\sigma-\beta\lambda)V_0(\theta_t; (1+g)h_t) \\ & + \beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_0(\theta_t; (1+g)h_t), V_0(\theta^0; (1+g)h_t)\}. \end{aligned} \tag{2}$$

The employed worker has a job from which she gets payment $\theta_t$ per unit of human capital in period $t$, which corresponds to the first term on the right-hand side in Eq.(2). With probability $\sigma$, the worker is displaced into unemployment by a job destruction shock, which is the second term on the right-hand side in Eq.(2). The third term on the right-hand side in Eq.(2) implies that the worker keeps with the current firm if she does not receive any offer. At next period the worker gets a new offer with probability $\beta\lambda$, upon which she chooses whether to stay with the current job or to accept the new job, which is the last term on the right-hand side in Eq.(2).

Once she is displaced into unemployment, dies or receives an outside offer which is higher than her current pay rate, the employed worker with low-education will leave the firm. Hence, the separate probability of the employed worker with low-education by obtaining $\theta_t$ is described as follows.

$$\psi_0(\theta_t) = \delta + \sigma + \beta\lambda(1 - F_0(\theta_t)),$$

where $F_0(\theta_t)$ represents the probability of receiving an outside job offer which is no greater than the current pay rate $\theta_t$.

Employed workers would stop the search and accept any outsider offer which is higher than her current pay rate; while for unemployed worker, they would stop the search and accept any outsider offer which is no less than the reservation pay rate. By adopting the optimal stopping rule [19], we obtain the following proposition.

**Proposition 2.1.** For the unemployed workers with low-education, the reservation pay rate, denoted by $\theta_r^0$, can be characterized by the following equation

$$\frac{\delta(b(1+g)-\theta_r^0)}{g} = b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\psi_0(\theta)\right)} \mathrm{d}\theta. \tag{3}$$

Moreover, the optimal job search implies that any unemployed worker with low-education accepts job offer $\theta^0$ if and only if $\theta^0 \geq \theta_r^0$.

Proposition 2.1 implies that any unemployed worker with low-education accepts job offer when the offer is higher than the reservation pay rate. We assume that jobs provided by firms can be either unskilled or skilled and that workers differing in their education level can be either low-educated or high-educated. The human capital accumulation causes that the reservation pay rate is related to the human capital.

**Proposition 2.2.** The growth rate of human capital $g$ is an endogenous variable which is increasing with the death shock $\delta$ and the job destruction shock $\sigma$, while decreasing with the fraction of the unskilled jobs $\beta$ and the arrival rate of jobs $\lambda$.

Faced with the high death rate or the staff reduction in large, workers normally raise the growth rate of the human capital actively which increases production indirectly in order to continue to be employed. That is why the rise of the death shock or the job destruction shock results in the growth rate of the human capital increases. If there are more unskilled jobs in the labor market, resulting in demotivation, the growth rate of the human capital reduces. When the situation of employment is better, such as the arrival rate of the jobs is higher, the workers obviously have less competition which leads the growth rate of the human capital to become slower.

**Proposition 2.3.** The reservation pay rate of unemployed workers with low-education $\theta_r^0$ which satisfies Eq.(3) is unique.

Obviously, the term on the left-hand side of Eq.(3) is linear in $\theta_r^0$ with slope $-\frac{\delta}{g}$, and the straight line intercepts the $x$-axis at $\theta_0^r = b(1+g)$. The terms on the right-hand side of Eq.(3) describe a curve which is continuous and strictly decreasing with first derivative

$$-\frac{\beta\lambda[1-F_0(\theta_r^0)]}{[1-(1+g)\left(1-\psi_0(\theta_r^0)\right)]} = -\frac{\delta}{g} < 0,$$

that means the straight line is tangent to the curve at $\theta_r^0$. It is clear that the terms on the right-hand side are positive strictly. Therefore, the straight line and the curve must have a unique intersection at some $\theta_r^0 < b(1+g)$ described in Figure 2 which implies $\theta_r^0$ is unique.

2.1.2. High-educated workers' job search strategies

For the high-educated workers, they can receive both the unskilled jobs and the skilled jobs, and their objective function is

$$\max_{x_t} \mathrm{E} \sum_{t=0}^{\infty} u\left(\theta_t, h_t, x_t\right),$$

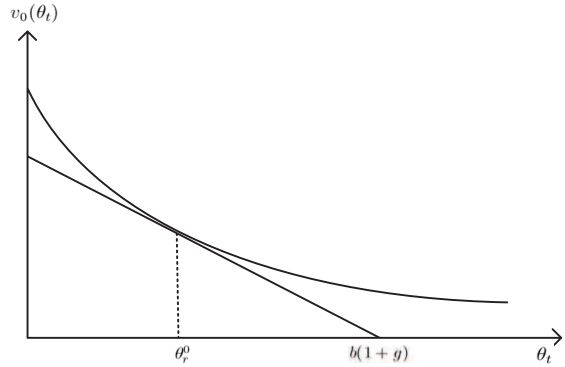



**Fig. 2.** Reservation pay rate of unemployed workers with low-education.

where $\theta_t$ and $h_t$ are the state variables in period $t$.

First consider the unemployed workers with high-education. The worker's pay rate $\theta_{t+1}$ satisfies

$$\theta_{t+1} = \xi \left\{(1-y)b + y\left[(1-x_t)b + x_t\left((1-z)(\theta^0 + c) + z\theta^1\right)\right]\right\},$$

where $\xi$, $\eta$, $\zeta$, $y$ and $z$ are all independent random variables. $\xi$, $\eta$ and $\zeta$ are described as the same as the above section, $y$ denotes whether the unemployed worker with high-education receives an outsider offer or not. $\Pr\{y=0\} = 1-\lambda$ means that the worker does not receive any outside job and $\Pr\{y=1\} = \lambda$ means that the worker receives an outside offer, and $z$ represents the kind of the outside offer received with $\Pr\{z=0\} = \beta\lambda$ and $\Pr\{z=1\} = (1-\beta)\lambda$, which means the offer is from unskilled jobs and skilled jobs. In addition, the human capital of the unemployed worker does not grow. The Bellman equation is written as follows.

$$V_{u1}(h_t) = \max\{bh_t + \mathrm{E}_{(\theta^0)}V_1(\theta_{t+1}; h_{t+1})\},$$

which can be rewritten as

$$\begin{aligned} V_{u1}(h_t) = \ & bh_t + (1-\delta-\lambda)V_{u1}(h_t) + \beta\lambda\mathrm{E}_{(\theta^0)}\max\{V_{u1}(h_t), V_1(\theta^0; h_t)\} \\ & + (1-\beta)\lambda\mathrm{E}_{(\theta^1)}\max\{V_{u1}(h_t), V_1(\theta^1; h_t)\}. \end{aligned} \tag{4}$$

Next consider the employed workers with high-education. The worker's pay rate $\theta_{t+1}$ satisfies

$$\theta_{t+1} = \xi\left\{(1-\eta)b + \eta\left[(1-y)\theta_t + y\left((1-x_t)\theta_t + x_t\left((1-z)\theta^0 + z\theta^1\right)\right)\right]\right\}.$$

Given an employed worker with the human capital $h_t$ and the pay rate $\theta_t$ at period $t$, the Bellman equation is also formulated as

$$V_1(\theta_t; h_t) = \max\{\theta_t h_t + \mathrm{E}_{(\theta^1)}V_1(\theta_{t+1}; h_{t+1})\},$$

which can be rewritten as

$$\begin{aligned} V_1(\theta_t; h_t) = \theta_t h_t + \sigma V_{u1}(e^g h_t) + (1-\delta-\sigma-\lambda) V_1(\theta_t; e^g h_t) \\ +\beta\lambda \mathrm{E}_{(\theta^0)} \max\{V_1(\theta_t; e^g h_t), V_1(\theta^0 + c; e^g h_t)\} \\ +(1-\beta)\lambda \mathrm{E}_{(\theta^1)} \max\{V_1(\theta_t; e^g h_t), V_1(\theta^1; e^g h_t)\}. \end{aligned} \tag{5}$$

Given the pay rate $\theta_t$, the employed worker leaves a firm at the rate

$$\psi_1(\theta_t) = \delta + \sigma + \beta\lambda(1 - F_0(\theta_t)) + (1-\beta)\lambda(1 - F_1(\theta_t)),$$

which is the separation rate of an employed worker with high-education. Similarly, the employed worker with high-education accepts any outsider offer which is higher than her current pay rate, while the unemployed worker with high-education accepts any outsider offer which is no less than the reservation pay rate described in the following proposition.

**Proposition 2.4.** For the unemployed workers with high-education, the reservation pay rate, denoted by $\theta_r^1$, can be described by the following equation

$$\frac{\delta\left(e^g b - \theta_r^1\right)}{e^g - 1} = b + \lambda\left[\int_{\theta_r^1}^{\overline{\theta}^0} \frac{\beta\,(1 - F_0(\theta))}{1 - e^g\,(1 - \psi_1(\theta))} \mathrm{d}\theta + \int_{\theta_r^1}^{\overline{\theta}^1} \frac{(1-\beta)\,(1 - F_1(\theta))}{1 - e^g\,(1 - \psi_1(\theta))} \mathrm{d}\theta\right]. \tag{6}$$

Furthermore, the optimal job search implies that an unemployed worker with high-education accepts job offer $\theta^1$ if and only if $\theta^1 \geq \theta_r^1$.

**Proposition 2.5.** When the fraction of the unskilled jobs is greater than $\beta$, there exists a cross-skill equilibrium that the high-educated workers choose either the unskilled job or the skilled job; when the fraction of the unskilled jobs is no greater than $\beta$, there exists a separating equilibrium that the high-educated workers only choose the skilled jobs, where $\beta$ satisfies

$$\exp\left(\frac{\delta(\delta + \sigma + \beta\lambda)}{(\beta\lambda + \delta)(1 - \delta) - \delta\sigma}\right) = \frac{(1-\beta)\lambda + \delta}{(1-\delta)[(1-\beta)\lambda + \delta] - \delta\sigma}.$$

**Proposition 2.6.** The reservation pay rate of the unemployed workers with high-education $\theta_r^1$ which satisfies Eq.(6) is unique.

Obviously, the term on the left-hand side of Eq.(6) is linear in $\theta_r^1$ with slope $-\dfrac{\delta}{e^g - 1}$ and the straight line intercepts the $x$-axis at $\theta_1^r = e^g b$. The terms on the right-hand side of Eq.(6) describe a curve which is continuous and strictly decreasing with the first derivative

$$-\frac{\lambda[1 - \beta F_0(\theta_r^1) - (1-\beta)F_1(\theta_r^1)]}{1 - e^g\,(1 - \psi_1(\theta_r^1))} = -\frac{\delta}{e^g - 1} < 0,$$

which means the straight line is tangent to the curve at $\theta_r^1$. It is clear that the terms on the right-hand side are positive strictly. Therefore, the straight and the curve must have a unique intersection at some $\theta_r^1 < e^g b$ described in Figure 3 which shows that $\theta_r^1$ is unique.

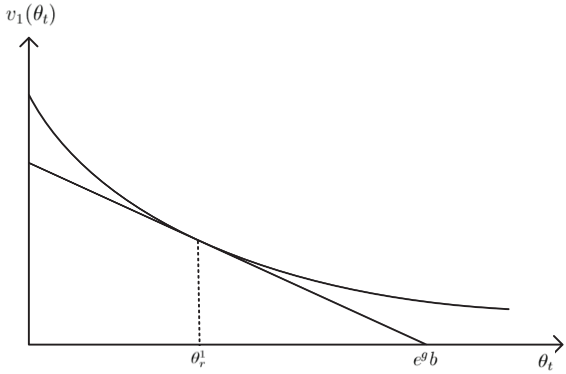



**Fig. 3.** Reservation pay rate of unemployed workers with high-education.

### 2.2. The firm's problem

The firm's optimization problem can be reduced to choosing an optimal pay rate to maximize the steady state profit. Let $N_j(h)$ denote the probability of the unemployed workers of type $j$ whose human capital is $h$, $G_j(\theta, h)$ denote the probability of employed workers with human capital $h$ and pay rate $\theta$, and $\gamma_j$ denote the unemployment rate of the workers with type $j$, $j = 0, 1$.

For a firm offering unskilled job contract $\theta^0$ to the low-educated workers with human capital $(1+g)^\tau$, the steady state profit

$$\begin{aligned}\pi_{00} =& \alpha\lambda\alpha\gamma_0 \sum_{\tau=0}^{\infty}\left[N_0\left((1+g)^\tau\right)\sum_{s=0}^{\infty}(1-\psi_0(\theta^0))^s(p-\theta^0)(1+g)^{s+\tau}\right] \\ &+\alpha\lambda\alpha(1-\gamma_0)\sum_{\tau=0}^{\infty}\left[\int_{\underline{\theta}^0}^{\theta^0} G_0(\theta,(1+g)^\tau)\mathrm{d}\theta \sum_{s=0}^{\infty}(1-\psi_0(\theta^0))^s(p-\theta^0)(1+g)^{s+\tau}\right] \\ =& \frac{\lambda\alpha^2(p-\theta^0)}{1-(1-\psi_0(\theta^0))(1+g)}\left[\gamma_0\sum_{\tau=0}^{\infty}N_0\left((1+g)^\tau\right)(1+g)^\tau\right. \\ &\left. +(1-\gamma_0)\sum_{\tau=0}^{\infty}\int_{\underline{\theta}^0}^{\theta^0} G_0(\theta,(1+g)^\tau)(1+g)^\tau\mathrm{d}\theta\right].\end{aligned} \tag{7}$$

Therefore, the steady state flow profit equals the hiring rate of the firm, multiplied by the expected profit of each hire. The first term in the above equation is the flow profit due to attracting unemployed workers with low-education whose human capital is $(1+g)^\tau$, where $\tau$ is the experience. The second term is the flow profit due to attracting employed workers with low-education whose human capital is $(1+g)^\tau$ and her current pay rate is no greater than $\theta^0$.

If the firm offers unskilled job contract $\theta^0$ to the high-educated workers with human capital $e^{g\tau}$, the steady state profit is written as follows,

$$
\begin{aligned}
\pi_{01} &= (1-\alpha)^2\lambda\gamma_1 \sum_{\tau=0}^{\infty}\left[N_1\left(e^{g\tau}\right)\sum_{s=0}^{\infty}(1-\psi_1(\theta^0))^s(p-\theta^0)e^{g(s+\tau)}\right] \\
&\quad + (1-\alpha)^2\lambda(1-\gamma_1)\sum_{\tau=0}^{\infty}\left[\int_{\underline{\theta}^1}^{\theta^0} G_1(\theta, e^{g\tau})\mathrm{d}\theta \sum_{s=0}^{\infty}(1-\psi_1(\theta^0))^s(p-\theta^0-c)e^{g(s+\tau)}\right] \\
&= \frac{\lambda(1-\alpha)^2(p-\theta^0-c)}{1-(1-\psi_1(\theta^0))e^g}\left[\gamma_1\sum_{\tau=0}^{\infty}N_1\left(e^{g\tau}\right)e^{g\tau} + (1-\gamma_1)\sum_{\tau=0}^{\infty}\int_{\underline{\theta}^1}^{\theta^0} G_1(\theta, e^{g\tau})e^{g\tau}\mathrm{d}\theta\right].
\end{aligned} \tag{8}
$$

The firm offering unskilled jobs can attract both types of workers, thus the optimization problem can be described as follows,

$$
\max_{\theta^0}\ (\pi_{00}+\pi_{01})
$$

$$
\text{s.t.}\begin{cases}
V_0(\theta_t;h_t) = \theta_t h_t + (1+g)\left[\sigma V_{u0}(h_t) + (1-\delta-\sigma)V_0(\theta_t;h_t)\right. \\
\qquad\qquad \left. + \beta\lambda\displaystyle\int_{\theta_t}^{\overline{\theta}^0}(V_0(\theta^0;h_t)-V_0(\theta_t;h_t))\mathrm{d}F_0(\theta^0)\right] \\
\delta V_{u0}(h_t) = bh_t + \beta\lambda\displaystyle\int_{\theta_r^0}^{\overline{\theta}^0}(V_0(\theta^0;h_t)-V_{u0}(h_t))\mathrm{d}F_0(\theta^0) \\
V_1(\theta_t;h_t) = \theta_t h_t + e^g\left[\sigma V_{u1}(h_t) + (1-\delta-\sigma)V_1(\theta_t;h_t)\right. \\
\qquad\qquad + \beta\lambda\displaystyle\int_{\theta_t}^{\overline{\theta}^0}(V_1(\theta^0;h_t)-V_1(\theta_t;h_t))\mathrm{d}F_0(\theta^0) \\
\qquad\qquad \left. + (1-\beta)\lambda\displaystyle\int_{\theta_t}^{\overline{\theta}^1}(V_1(\theta^1;h_t)-V_1(\theta_t;h_t))\mathrm{d}F_1(\theta^1)\right] \\
\delta V_{u1}(h_t) = bh_t + \lambda\left[\beta\displaystyle\int_{\theta_r^1}^{\overline{\theta}^0}(V_1(\theta^0;h_t)-V_{u1}(h_t))\mathrm{d}F_0(\theta^0)\right. \\
\qquad\qquad \left. + (1-\beta)\displaystyle\int_{\theta_r^1}^{\overline{\theta}^1}(V_1(\theta^1;h_t)-V_{u1}(h_t))\mathrm{d}F_1(\theta^1)\right].
\end{cases} \tag{9}
$$

If a firm offers a skilled job contract $\theta^1$ to the high-educated workers with human capital $e^{g\tau}$, the steady state profit

$$
\begin{aligned}
\pi_{11} &= \lambda(1-\alpha)\gamma_1 \sum_{\tau=0}^{\infty}\left[N_1\left(e^{g\tau}\right)\sum_{s=0}^{\infty}(1-\psi_1(\theta^1))^s(p-\theta^1)e^{g(s+\tau)}\right] \\
&\quad + \lambda(1-\alpha)(1-\gamma_1)\sum_{\tau=0}^{\infty}\left[\int_{\underline{\theta}^1}^{\theta^1} G_1(\theta, e^{g\tau})\mathrm{d}\theta \sum_{s=0}^{\infty}(1-\psi_1(\theta^1))^s(p-\theta^1)e^{g(s+\tau)}\right] \\
&= \frac{\lambda(1-\alpha)(p-\theta^1)}{1-(1-\psi_1(\theta^1))e^g}\left[\gamma_1\sum_{\tau=0}^{\infty}N_1(e^{g\tau})e^{g\tau} + (1-\gamma_1)\sum_{\tau=0}^{\infty}\int_{\underline{\theta}^1}^{\theta^1} G_1(\theta, e^{g\tau})e^{g\tau}\mathrm{d}\theta\right].
\end{aligned} \tag{10}
$$

The firm offering skilled jobs only attracts high-educated workers, then the optimization problem can be described as follows,

$$\max_{\theta^1} \ \pi_{11}$$

$$\text{s.t.} \begin{cases} V_1(\theta_t;h_t) = \theta_t h_t + e^g \Big[\sigma V_{u1}(h_t) + (1-\delta-\sigma)V_1(\theta_t;h_t) \\ \qquad + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (V_1(\theta^0;h_t) - V_1(\theta_t;h_t))\mathrm{d}F_0(\theta^0) \\ \qquad + (1-\beta)\lambda \int_{\theta_t}^{\overline{\theta}^1} (V_1(\theta^1;h_t) - V_1(\theta_t;h_t))\mathrm{d}F_1(\theta^1)\Big] \\ \delta V_{u1}(h_t) = bh_t + \lambda \Big[\beta \int_{\theta_r^1}^{\overline{\theta}^0} (V_1(\theta^0;h_t) - V_{u1}(h_t))\mathrm{d}F_0(\theta^0) \\ \qquad + (1-\beta) \int_{\theta_r^1}^{\overline{\theta}^1} (V_1(\theta^1;h_t) - V_{u1}(h_t))\mathrm{d}F_1(\theta^1)\Big] \end{cases} \tag{11}$$

Note that the objective of each firm is to maximize the steady state flow profit, that is, the unskilled firms choose $\theta^0$ to maximize $\pi_0 = \pi_{00} + \pi_{01}$, while the skilled firms choose $\theta^1$ to maximize $\pi_1 = \pi_{11}$.

## 3. MARKET EQUILIBRIUM

In this section, we formally characterize a market equilibrium, and then solve the unemployment rate $\gamma_j$, distribution functions $N_j(\cdot)$ and $G_j(\cdot)$, where $j = 0, 1$.

**Definition 3.1.** A market equilibrium is a set $\{\theta_r^0, \theta_r^1, \gamma_0, \gamma_1, N_0(\cdot), N_1(\cdot), G_0(\cdot), G_1(\cdot), F_0(\cdot), F_1(\cdot)\}$ that satisfies the following requirements:

1) $\theta_r^0$ is the optimal reservation pay rate per unit of human capital of any unemployed low-educated worker, and $\theta_r^1$ is the optimal reservation pay rate of any unemployed high-educated worker, given in Propositions 2.1 and 2.4, respectively;

2) $\gamma_0, N_0(\cdot), G_0(\cdot)$ are consistent with the steady state turnover and pay rate distribution $F_0(\cdot)$; likewise, $\gamma_1, N_1(\cdot), G_1(\cdot)$ are consistent with steady state turnover and pay rate distributions $F_0(\cdot)$ and $F_1(\cdot)$;

3) the constant profit conditions are satisfied, i. e.,

$$\begin{aligned} &\pi_{00} + \pi_{01} = \pi_0^* > 0, && \text{for all } \theta^0 \text{ where } \mathrm{d}F_0(\theta^0) > 0, \\ &\pi_{00} + \pi_{01} \le \pi_0^*, && \text{for all } \theta^0 \text{ where } \mathrm{d}F_0(\theta^0) = 0, \\ &\pi_{11} = \pi_1^* > 0, && \text{for all } \theta^1 \text{ where } \mathrm{d}F_0(\theta^1) > 0 \text{ or } \mathrm{d}F_1(\theta^1) > 0, \\ &\pi_{11} \le \pi_1^*, && \text{for all } \theta^1 \text{ where } \mathrm{d}F_0(\theta^1) = 0 \text{ and } \mathrm{d}F_1(\theta^1) = 0, \end{aligned}$$

where $\pi_0^*$ is the total optimal profit of the firms offering the unskilled jobs and $\pi_1^*$ is the total optimal profit of the firms offering the skilled jobs in a equilibrium market. The constant profit conditions imply that all equilibrium offers of the same type $i$ enjoy the same profit $\pi_i^*$ with the workers' optimal strategy.

In Subsection 3.1, we first use the steady state turnover arguments to solve for $\gamma_j$, $j = 0, 1$. And then in Subsection 3.2, we determine the distribution functions $N_j(\cdot)$ and $G_j(\cdot)$. Last, we find $F_i$ in Subsection 3.3 so that the above constant profit conditions are satisfied, $i = 0, 1$.

### 3.1. Solve $\gamma_0$, $N_0(\cdot)$ and $G_0(\cdot)$ for the low-educated workers

To solve $\gamma_0$, this subsection first considers the steady state turnover in the pool of unemployed workers whose number is $\gamma_0\alpha$. The total outflow from this pool is $(\beta\lambda + \delta)\gamma_0\alpha$, which either finds a job or leaves the market. While the inflow in the pool is composed of the new entrants and the employed workers who are displaced into the unemployment, which is $\delta\alpha + \sigma(1 - \gamma_0)\alpha$. Equating the outflow with the inflow, the equilibrium unemployment rate of the low-educated workers is

$$\gamma_0 = \frac{\sigma + \delta}{\delta + \sigma + \beta\lambda}. \tag{12}$$

To solve $N_0(h)$, we next consider the steady state turnover in the pool of the unemployed workers with human capital $h$. When $h = 1$, the outflow $(\beta\lambda + \delta)\gamma_0\alpha N_0(1)$ consists of the workers who either find an unskilled job or leave the market. And the inflow is composed only of the new entrants which is $\delta\alpha$, as the human capital of the workers who are laid off from the employment is at least $(1 + g)$. Setting the outflow equal to the inflow yields

$$N_0(1) \quad = \frac{\delta}{\gamma_0(\beta\lambda + \delta)}. \tag{13}$$

When $h \in \{(1 + g)^m\}_{m=1}^{\infty}$, the steady-state turnover requires

$$(\beta\lambda + \delta)\gamma_0\alpha N_0(h) = \sigma(1 - \gamma_0)\alpha \int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, h)\mathrm{d}\theta, \tag{14}$$

where the left hand side describes the outflow of the workers with human capital $h$ who find a new unskilled job or leave the labor market, and the right hand side describes the inflow of the employed workers with human capital $h$ and current offer $\theta$ who are displaced into the unemployment. By Eq.(14), we derive

$$N_0(h) \quad = \frac{\sigma(1 - \gamma_0)}{(\beta\lambda + \delta)\gamma_0} \int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, h)\mathrm{d}\theta. \tag{15}$$

Therefore, we have

$$N_0(h) = \begin{cases} \dfrac{\delta}{\gamma_0(\beta\lambda + \delta)}, & \text{if } h = 1 \\ \dfrac{\sigma(1 - \gamma_0)}{(\beta\lambda + \delta)\gamma_0} \displaystyle\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, h)\mathrm{d}\theta, & \text{if } h \in \{(1 + g)^m\}_{m=1}^{\infty}. \end{cases}$$

To solve $G_0(\theta, h)$, we finally consider the steady state turnover in the pool of the employed workers with human capital $h$, where $h \in \{(1 + g)^m\}_{m=1}^{\infty}$ and the current pay rate $\theta$ is no greater than $\theta^0$. The workers with human capital $h$ will leave this pool for sure, regardless of whether they stay or not. (If they stay, their human capital becomes $(1+g)h$. If they are displaced into unemployment or leave the market, their human capital does not change.) Thus the total outflow is

$$(1-\gamma_0)\alpha \int_{\theta_r^0}^{\theta^0} G_0(\theta, h)\mathrm{d}\theta.$$

The workers with human capital $(1+g)^{-1}h$ who were employed or unemployed will join this pool group if they find a job offering pay rate which is no greater than $\theta^0$. The inflow of these workers is

$$(1-\gamma_0)\alpha\left[1-\psi_0(\theta^0)\right]\int_{\theta_r^0}^{\theta^0} G_0\left(\theta, \frac{h}{1+g}\right)\mathrm{d}\theta + \gamma_0\alpha N_0\left(h\right)\beta\lambda F_0(\theta^0).$$

Equating the outflow with the inflow implies

$$(1-\gamma_0)\int_{\theta_r^0}^{\theta^0} G_0(\theta, h)\mathrm{d}\theta = (1-\gamma_0)\left[1-\psi_0(\theta^0)\right]\int_{\theta_r^0}^{\theta^0} G_0\left(\theta, \frac{h}{1+g}\right)\mathrm{d}\theta + \gamma_0 N_0\left(h\right)\beta\lambda F_0(\theta^0). \tag{16}$$

**Proposition 3.2.** For the low-educated workers, the steady state turnover in a market equilibrium implies

$$N_0(h) = \frac{\delta\sigma\beta\lambda}{\gamma_0(\beta\lambda+\delta)^2}q_0^m,$$

and $G_0(\theta, h)$ satisfies

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta, h)\mathrm{d}\theta = \frac{\delta\beta\lambda F_0(\theta^0)(\beta\lambda+\delta-\beta\lambda\sigma)}{(1-\gamma_0)(\beta\lambda+\delta)\left[\sigma(1-\psi_0(\theta^0)) + (1-F_0(\theta^0))(\beta\lambda+\delta)\right]} \times\left\{(1-F_0(\theta^0))\left[1-\psi_0(\theta^0)\right]^m + \frac{\sigma}{\beta\lambda+\delta}q_0^{m+1}\right\},$$

for all $\theta^0 \in [\theta_r^0, \overline{\theta}^0]$ and $h \in \{(1+g)^m\}_{m=0}^{\infty}$, where $q_0 = \frac{(1-\delta-\sigma)(\beta\lambda+\delta)}{\beta\lambda(1-\sigma)+\delta} < 1$.

### 3.2. Solve $\gamma_1$, $N_1(\cdot)$ and $G_1(\cdot)$ for the high-educated workers

In this subsection, the inflow and the outflow of the high-educated workers are similar to those of the low-educated workers.

To solve $\gamma_1$, we first consider the steady state turnover in the pool of the unemployed workers with high-education whose number is $\gamma_1(1-\alpha)$. Equating the outflow $(\delta+\lambda)\gamma_1(1-\alpha)$ with the inflow $\delta(1-\alpha)+\sigma(1-\gamma_1)(1-\alpha)$, the equilibrium unemployment rate of the high-educated workers is

$$\gamma_1 = \frac{\delta+\sigma}{\delta+\sigma+\lambda}. \tag{17}$$

To solve $N_1(h)$, we next consider the steady state turnover in the pool of the unemployed workers with human capital $h$. Setting the outflow equal to the inflow implies that $N_1(1)$ satisfies

$$N_1(1) = \frac{\delta}{\gamma_1(\delta+\lambda)}. \tag{18}$$

When $h \in \{e^{ng}\}_{n=1}^{\infty}$, the steady state turnover requires

$$(\delta+\lambda)\gamma_1(1-\alpha)N_1(e^{ng}) = \sigma(1-\gamma_1)(1-\alpha)\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{ng})\mathrm{d}\theta. \tag{19}$$

Solving for $N_1(e^{ng})$ implies

$$N_1(e^{ng}) \ = \frac{\sigma(1-\gamma_1)}{(\delta+\lambda)\gamma_1}\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{ng})\mathrm{d}\theta. \tag{20}$$

Therefore, we obtain

$$N_1(h) = \begin{cases} \dfrac{\delta}{\gamma_1(\delta+\lambda)}, & \text{if } h = 1 \\ \dfrac{\sigma(1-\gamma_1)}{(\delta+\lambda)\gamma_1}\displaystyle\int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta, e^{ng})\mathrm{d}\theta, & \text{if } h \in \{e^{ng}\}_{n=1}^{\infty}. \end{cases}$$

To solve $G_1(h)$, we finally consider the steady state turnover in the pool of the employed workers with human capital $h$ where $h \in \{e^{ng}\}_{n=1}^{\infty}$ and the current pay rate $\theta$ is no greater than $\theta^1$. Equating the outflow with the inflow yields

$$\begin{aligned}(1-\gamma_1)\int_{\theta_r^1}^{\theta^1} G_1(\theta, h)\mathrm{d}\theta = \ & (1-\gamma_1)\left(1-\psi_1(\theta^1)\right)\int_{\theta_r^1}^{\theta^1} G_1(\theta, \frac{h}{e^g})\mathrm{d}\theta \\ & +\gamma_1 N_1(h)\lambda\left(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)\right).\end{aligned} \tag{21}$$

By utilizing the same method like the above subsection to consider the low-educated workers, we obtain the following proposition.

**Proposition 3.3.** For the high-educated workers, the steady state turnover in a market equilibrium implies:

$$N_1(e^{ng}) = \frac{\delta\sigma\lambda}{\gamma_1(\lambda+\delta)^2}q_1^n, \tag{22}$$

and $G_1(h)$ satisfies

$$\begin{aligned}\int_{\theta_r^1}^{\theta^1} G_1(\theta, e^{ng})\mathrm{d}\theta = \ & \frac{\delta\lambda(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))(\hat{\lambda}-\sigma\lambda)}{(1-\gamma_1)\hat{\lambda}\left[\sigma(1-\psi_1(\theta^1))+(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1))\hat{\lambda}\right]} \\ & \times\left[\left(1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right)(1-\psi_1(\theta^1))^n+\frac{\sigma}{\gamma+\delta}q_1^{n+1}\right],\end{aligned} \tag{23}$$

for all $\theta \in [\theta_r^1, \overline{\theta}^1]$ and $h \in \{e^{ng}\}_{n=1}^{\infty}$, where $q_1 = \dfrac{(1-\delta-\sigma)(\lambda+\delta)}{\lambda+\delta-\sigma\lambda} < 1$ and $\hat{\lambda} = \lambda+\delta$.

### 3.3. The distributions $F_0(\theta^0)$ and $F_1(\theta^1)$ of human capital contract

Using Eqs.(48) and (49) to substitute out $N_0((1+g)^\tau)$ and $\int_{\underline{\theta}^0}^{\theta^0} G_0(\theta,(1+g)^\tau)\mathrm{d}\theta$ in Eq.(7), $\pi_{00}$ can be written as

$$\pi_{00}(\theta^0)=\frac{\beta\lambda^2\alpha^2(p-\theta^0)\delta}{[1-(1-\psi_0(\theta^0))(1+g)]\,(\beta\lambda+\delta)\,[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)]}$$
$$\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\theta^0))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)(1-F_0(\theta^0))}{1-(1-\psi_0(\theta^0))(1+g)}\right],$$

where

$$\psi_0(\theta^0)=\delta+\sigma+\beta\lambda(1-F_0(\theta^0)).$$

Moreover, substitute $N_1(e^{g\tau})$ and $\int_{\underline{\theta}^1}^{\theta^0} G_1(\theta,e^{g\tau})\mathrm{d}\theta$ into (8) to get the following formula

$$\pi_{01}(\theta^0)=\frac{\lambda^2(1-\alpha)^2(p-\theta^0)\delta}{[1-(1-\psi_1(\theta^0))e^g]\,(\lambda+\delta)\left[\sigma(1-\psi_1(\theta^0))+(1-\hat{F})(\lambda+\delta)\right]}$$
$$\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta\hat{F}\right]}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\hat{F}(1-\hat{F})}{1-(1-\psi_1(\theta^0))e^g}\right],$$

where

$$\psi_1(\theta^0)=\delta+\sigma+\beta\lambda(1-F_0(\theta^0))+(1-\beta)\lambda(1-F_1(\theta^0)).$$

Note that $\pi_0(\theta^0)=\pi_{00}(\theta^0)+\pi_{01}(\theta^0)$ for all $\theta^0\in[\underline{\theta}^0,\overline{\theta}^0]$. Based on the constant profit conditions, we have $\pi_0(\theta^0)=\pi_0^*$ for all $\theta^0\in[\underline{\theta}^0,\overline{\theta}^0]$. To obtain the equilibrium profit for the firms offering unskilled jobs $\pi_0^*$, set $\theta^0=\underline{\theta}^0$, we obtain

$$\pi_0^*=\frac{\beta\lambda^2\alpha^2\delta\sigma(p-\underline{\theta}^0)}{[1-(1-\delta-\sigma-\beta\lambda)(1+g)](\beta\lambda+\delta)^2[1-q_0(1+g)]}+\frac{\lambda^2(1-\alpha)^2\delta\sigma(p-\underline{\theta}^0)}{[1-(1-\delta-\sigma-\lambda)e^g](\lambda+\delta)^2(1-q_1e^g)}. \tag{24}$$

To solve for $F_0(\cdot)$ and $F_1(\cdot)$, we divide $[\underline{\theta}^0,\overline{\theta}^0]$ into two intervals $[\underline{\theta}^0,\underline{\theta}^1]$ and $(\underline{\theta}^1,\overline{\theta}^0]$. In interval $[\underline{\theta}^0,\underline{\theta}^1]$, there are only the unskilled jobs, while in interval $(\underline{\theta}^1,\overline{\theta}^0]$, there are both the unskilled jobs and the skilled jobs. And then we should determine $\underline{\theta}^1$. Assume that $\theta^0 = \underline{\theta}^1$,

$$\begin{aligned}\pi_0(\underline{\theta}^1) =& \frac{\beta\lambda^2\alpha^2(p-\underline{\theta}^1)\delta}{\left[1-(1-\psi_0(\underline{\theta}^1))(1+g)\right](\beta\lambda+\delta)\left[\sigma(1-\psi_0(\underline{\theta}^1))+(1-F_0(\underline{\theta}^1))(\beta\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\underline{\theta}^1))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\underline{\theta}^1)(1-F_0(\underline{\theta}^1))}{1-(1-\psi_0(\underline{\theta}^1))(1+g)}\right]\\ &+\frac{\lambda^2(1-\alpha)^2(p-\underline{\theta}^1)\delta}{\left[1-(1-\psi_1(\underline{\theta}^1))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\underline{\theta}^1))+(1-\beta F_0(\underline{\theta}^1))(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)(1-\sigma-\delta\beta F_0(\underline{\theta}^1))}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\beta F_0(\underline{\theta}^1)(1-\beta F_0(\underline{\theta}^1))}{1-(1-\psi_1(\underline{\theta}^1))e^g}\right],\end{aligned} \tag{25}$$

where

$$\psi_0(\underline{\theta}^1) = \delta+\sigma+\beta\lambda(1-F_0(\underline{\theta}^1)),$$

$$\psi_1(\underline{\theta}^1) = \delta+\sigma+\lambda-\beta\lambda F_0(\underline{\theta}^1),$$

and

$$F_0(\underline{\theta}^1) = \frac{1}{\beta} - \frac{\delta[1-e^g(1-\delta-\sigma)]}{\beta\lambda(e^g-1-\delta e^g)}.$$

As $\pi_0(\underline{\theta}^1) = \pi_0^*$, we determine $\underline{\theta}^1$. When $\theta^0 \in [\underline{\theta}^0, \underline{\theta}^1]$,

$$\begin{aligned}\pi_0(\theta^0) =& \frac{\beta\lambda^2\alpha^2(p-\theta^0)\delta}{[1-(1-\psi_0(\theta^0))(1+g)](\beta\lambda+\delta)[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\theta^0))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)(1-F_0(\theta^0))}{1-(1-\psi_0(\theta^0))(1+g)}\right]\\ &+\frac{\lambda^2(1-\alpha)^2(p-\theta^0)\delta}{[1-(1-\psi_1(\theta^0))e^g](\lambda+\delta)[\sigma(1-\psi_1(\theta^0))+(1-\beta F_0(\theta^0))(\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)(1-\sigma-\delta\beta F_0(\theta^0))}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\beta F_0(\theta^0)(1-\beta F_0(\theta^0))}{1-(1-\psi_1(\theta^0))e^g}\right],\end{aligned} \tag{26}$$

where

$$\psi_0(\theta^0) = \delta+\sigma+\beta\lambda(1-F_0(\theta^0)),$$

and

$$\psi_1(\theta^0) = \delta+\sigma+\lambda-\beta\lambda F_0(\theta^0).$$

Thus we can get the expression for $F_0(\theta^0)$ by Eqs.(24) and (26), where $\theta^0 \in [\underline{\theta}^0, \underline{\theta}^1]$. When $\theta^0 \in (\underline{\theta}^1, \overline{\theta}^0]$,

$$\begin{aligned}\pi_0(\theta^0)= &\frac{\beta\lambda^2\alpha^2(p-\theta^0)\delta}{[1-(1-\psi_0(\theta^0))(1+g)]\,(\beta\lambda+\delta)\,[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)]}\\ &\times\left[\frac{\sigma(\beta\lambda+\delta+\sigma)(1-\sigma-\delta F_0(\theta^0))}{[1-q_0(1+g)](\beta\lambda+\delta)}+\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)(1-F_0(\theta^0))}{1-(1-\psi_0(\theta^0))(1+g)}\right]\\ &+\frac{\lambda^2(1-\alpha)^2(p-\theta^0)\delta}{[1-(1-\psi_1(\theta^0))e^g]\,(\lambda+\delta)\left[\sigma(1-\psi_1(\theta^0))+(1-\hat{F})(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left(1-\sigma-\delta\hat{F}\right)}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\hat{F}(1-\hat{F})}{1-(1-\psi_1(\theta^0))e^g}\right],\end{aligned}\tag{27}$$

where

$$\psi_0(\theta^0)=\delta+\sigma+\beta\lambda(1-F_0(\theta^0)),\quad \hat{F}=\beta F_0(\theta^0)+(1-\beta)F_1(\theta^0)$$

and

$$\psi_1(\theta^0)=\delta+\sigma+\beta\lambda(1-F_0(\theta^0))+(1-\beta)\lambda(1-F_1(\theta^0)).$$

Setting $\theta^0=\overline{\theta}^0$ yields

$$\begin{aligned}\pi_0(\overline{\theta}^0)\;=&\frac{\beta\lambda^2\alpha^2(p-\overline{\theta}^0)\delta(\beta\lambda+\delta+\sigma)}{[1-(1-\delta-\sigma)(1+g)]\,(\beta\lambda+\delta)^2[1-q_0(1+g)]}\\ &+\frac{\lambda^2(1-\alpha)^2(p-\overline{\theta}^0)\delta}{\left[1-(1-\psi_1(\overline{\theta}^0))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\overline{\theta}^0))+(1-\beta)(1-F_1(\overline{\theta}^0))(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta+(1-\beta)F_1(\overline{\theta}^0))\right]}{(1-q_1e^g)(\lambda+\delta)}\right.\\ &\left.+\frac{(\lambda+\delta-\sigma\lambda)(\beta+(1-\beta)F_1(\overline{\theta}^0))(1-\beta)(1-F_1(\overline{\theta}^0))}{1-(1-\psi_1(\overline{\theta}^0))e^g}\right],\end{aligned}\tag{28}$$

where

$$\psi_1(\overline{\theta}^0)=\delta+\sigma+(1-\beta)\lambda(1-F_1(\overline{\theta}^0)).$$

Using Eqs.(22) and (23) to substitute out $N_1(e^{g\tau})$ and $\int_{\underline{\theta}^1}^{\theta^1} G_1(\theta,e^{g\tau})\mathrm{d}\theta$ in Eq.(10), we can get the following formula

$$\begin{aligned}\pi_{11}(\theta^1)= &\frac{\lambda^2(1-\alpha)(p-\theta^1)\delta}{[1-(1-\psi_1(\theta^1))e^g]\,(\lambda+\delta)\left[\sigma(1-\psi_1(\theta^1))+(1-\hat{F}^1)(\lambda+\delta)\right]}\\ &\times\left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta\hat{F}^1\right]}{(1-q_1e^g)(\lambda+\delta)}+\frac{(\lambda+\delta-\sigma\lambda)\hat{F}^1(1-\hat{F}^1)}{1-(1-\psi_1(\theta^1))e^g}\right],\end{aligned}$$

where

$$\psi_1(\theta^1) = \delta + \sigma + \beta\lambda(1 - F_0(\theta^1)) + (1-\beta)\lambda(1 - F_1(\theta^1)).$$

Note that $\pi_1(\theta^1) = \pi_{11}(\theta^1)$ for all $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$. Based on the constant profit conditions, we have $\pi_1(\theta^1) = \pi_1^*$ for all $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$. To obtain the equilibrium profit for the firms offering skilled jobs, set $\theta^1 = \underline{\theta}^1$, we obtain

$$\begin{aligned} \pi_1^* &= \frac{\lambda^2(1-\alpha)(p-\underline{\theta}^1)\delta}{\left[1-(1-\psi_1(\underline{\theta}^1))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\underline{\theta}^1)) + (1-\beta F_0(\underline{\theta}^1))(\lambda+\delta)\right]} \\ &\quad \times \left[\frac{\sigma(\lambda+\delta+\sigma)(1-\sigma-\delta\beta F_0(\underline{\theta}^1))}{(1-q_1e^g)(\lambda+\delta)} + \frac{(\lambda+\delta-\sigma\lambda)\beta F_0(\underline{\theta}^1)(1-\beta F_0(\underline{\theta}^1))}{1-(1-\psi_1(\underline{\theta}^1))e^g}\right], \end{aligned} \tag{29}$$

where

$$\psi_1(\underline{\theta}^1) = \delta + \sigma + \lambda - \beta\lambda F_0(\underline{\theta}^1),$$

and

$$F_0(\underline{\theta}^1) = \frac{1}{\beta} - \frac{\delta[1 - e^g(1-\delta-\sigma)]}{\beta\lambda(e^g - 1 - \delta e^g)}.$$

Given $\theta^1 = \overline{\theta}^1$,

$$\pi_1(\overline{\theta}^1) = \frac{\lambda^2(1-\alpha)(p-\overline{\theta}^1)\delta(\lambda+\delta+\sigma)}{[1-(1-\delta-\sigma)e^g](\delta+\lambda)^2(1-q_1e^g)}.$$

As $\pi_1(\overline{\theta}^1) = \pi_1^*$, we can obtain $\overline{\theta}^1$. Divide $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$ into two intervals $[\underline{\theta}^1, \overline{\theta}^0]$ and $(\overline{\theta}^0, \overline{\theta}^1]$, and then we should determine $\overline{\theta}^0$. Setting $\theta^1 = \overline{\theta}^0$ yields

$$\begin{aligned} \pi_1(\overline{\theta}^0) &= \frac{\lambda^2(1-\alpha)(p-\overline{\theta}^0)\delta}{\left[1-(1-\psi_1(\overline{\theta}^0))e^g\right](\lambda+\delta)\left[\sigma(1-\psi_1(\overline{\theta}^0))+(1-\beta)(1-F_1(\overline{\theta}^0))(\lambda+\delta)\right]} \\ &\quad \times \left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta(\beta+(1-\beta)F_1(\overline{\theta}^0))\right]}{(1-q_1e^g)(\lambda+\delta)}\right. \\ &\quad \left. + \frac{(\lambda+\delta-\sigma\lambda)(\beta+(1-\beta)F_1(\overline{\theta}^0))(1-\beta)(1-F_1(\overline{\theta}^0))}{1-(1-\psi_1(\overline{\theta}^0))e^g}\right], \end{aligned} \tag{30}$$

where

$$\psi_1(\overline{\theta}^0) = \delta + \sigma + (1-\beta)\lambda(1 - F_1(\overline{\theta}^0)).$$

We can get the expressions for $\overline{\theta}^0$ and $F_1(\overline{\theta}^0)$, as Eqs.(28) and (30) are two equations about $\overline{\theta}^0$ and $F_1(\overline{\theta}^0)$. When $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^0]$, there exists

$$\begin{aligned} \pi_1(\theta^1) =& \frac{\lambda^2(1-\alpha)(p-\theta^1)\delta}{[1-(1-\psi_1(\theta^1))e^g](\lambda+\delta)\left[\sigma(1-\psi_1(\theta^1)) + (1-\hat{F}^1)(\lambda+\delta)\right]} \\ &\times \left[\frac{\sigma(\lambda+\delta+\sigma)\left[1-\sigma-\delta\hat{F}^1\right]}{(1-q_1e^g)(\lambda+\delta)} + \frac{(\lambda+\delta-\sigma\lambda)(\hat{F}^1)(1-\hat{F}^1)}{1-(1-\psi_1(\theta^1))e^g}\right], \end{aligned} \tag{31}$$

where

$$\psi_1(\theta^0) = \delta + \sigma + \beta\lambda(1 - F_0(\theta^0)) + (1 - \beta)\lambda(1 - F_1(\theta^0)).$$

We obtain the expression for $F_0(\theta^1)$ and $F_1(\theta^1)$ by Eqs.(24), (27), (29) and (31), when $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^0]$. Moreover, when $\theta^1 \in (\overline{\theta}^0, \overline{\theta}^1]$,

$$\begin{aligned}\pi_1(\theta^1) = & \frac{\lambda^2(1-\alpha)(p-\theta^1)\delta}{[1-(1-\psi_1(\theta^1))e^g]\,(\lambda+\delta)\,[\sigma(1-\psi_1(\theta^1))+(1-\beta)(1-F_1(\theta^1))(\lambda+\delta)]} \\ & \times \left[\frac{\sigma(\lambda+\delta+\sigma)\,\left[1-\sigma-\delta(\beta+(1-\beta)F_1(\theta^1))\right]}{(1-q_1e^g)(\lambda+\delta)} \right. \\ & \left. + \frac{(\lambda+\delta-\sigma\lambda)(\beta+(1-\beta)F_1(\theta^1))(1-\beta)(1-F_1(\theta^1))}{1-(1-\psi_1(\theta^1))e^g}\right],\end{aligned} \tag{32}$$

where

$$\psi_1(\theta^1) = \delta + \sigma + (1 - \beta)\lambda(1 - F_1(\theta^1)).$$

We can get the expression for $F_1(\theta^1)$ by Eqs. (29) and (32), when $\theta^1 \in (\overline{\theta}^0, \overline{\theta}^1]$.

## 4. HUMAN CAPITAL AND EQUILIBRIUM PAY RATE

In this section, we first research the effect of the low-educated workers' human capital on equilibrium pay rate. Next, we consider the effect of the high-educated workers' human capital on the equilibrium pay rate. Similar to the findings of Burdett et al. [5], there is positive correlation between the human capital of workers and the equilibrium pay rates. Last, we discuss differences between these two kinds of effects.

### 4.1. The effect of low-educated workers' human capital on equilibrium pay rate

Denote $\displaystyle\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta = \frac{\int_{\theta_r^0}^{\theta^0} G_0(\theta,h)\mathrm{d}\theta}{\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta,h)\mathrm{d}\theta}$, which implies the probability of the low-educated workers' pay rate is less than $\theta^0$ conditional on human capital $h = (1+g)^m$. By Eq.(49), we obtain that

$$\begin{aligned}\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta = & \frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)}{\sigma(1-\psi_0(\theta^0))+(\beta\lambda+\delta)(1-F_0(\theta^0))}\left\{(1-F_0(\theta^0))\left(\frac{1-\psi_0(\theta^0)}{q_0}\right)^m\right. \\ & \left. + \frac{\sigma(1-\delta-\sigma)}{\beta\lambda+\delta-\beta\lambda\sigma}\right\}.\end{aligned} \tag{33}$$

Notice that $\frac{(\beta\lambda+\delta-\beta\lambda\sigma)F_0(\theta^0)}{\sigma(1-\psi_0(\theta^0))+(\beta\lambda+\delta)(1-F_0(\theta^0))} > 0$, $1 - F_0(\theta^0) > 0$ and $\frac{\sigma(1-\delta-\sigma)}{\beta\lambda+\delta-\beta\lambda\sigma} > 0$. Since $0 < 1 - \psi_0(\theta^0) < 1$ and $0 < q_0 < 1$, we can get $\frac{1-\psi_0(\theta^0)}{q_0} > 0$. On the other hand,

$\frac{1-\psi_0(\theta^0)}{q_0} = \frac{\left[1-\delta-\sigma-\beta\lambda(1-F_0(\theta^0))\right](\beta\lambda+\delta-\beta\lambda\sigma)}{(1-\delta-\sigma)(\beta\lambda+\delta)} < 1$, therefore, $0 < \frac{1-\psi_0(\theta^0)}{q_0} < 1$. Thus, it can prove that the conditional probability is decreasing in $m$. That is to say, a low-educated worker with higher human capital is more likely to have a higher pay rate.

Specially, when $m = 0$, Eq.(33) can be written as

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta|1)\mathrm{d}\theta = F_0(\theta^0).$$

It shows that for the new entrants with low-education, their pay rates are randomly drawn from $F_0(\theta^0)$. Further, when $m \to \infty$,

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta|\infty)\mathrm{d}\theta = \frac{\sigma(1-\delta-\sigma)F_0(\theta^0)}{\sigma(1-\psi_0(\theta^0)) + (\beta\lambda+\delta)(1-F_0(\theta^0))},$$

which implies that $\int_{\theta_r^0}^{\theta^0} G_0(\theta|\infty)\mathrm{d}\theta$ is non-degenerate.

**Proposition 4.1.** The probability of the low-educated workers' equilibrium pay rate which is less than $\theta^0$ conditional on human capital $h$, $\displaystyle\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta$, is decreasing in $h$.

### 4.2. The effect of high-educated workers' human capital on equilibrium pay rate

Note that $\displaystyle\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta = \int_{\theta_r^1}^{\theta^1} G_1(\theta,h)\mathrm{d}\theta \Big/ \int_{\theta_r^1}^{\overline{\theta}^1} G_1(\theta,h)\mathrm{d}\theta$, which implies the probability of the high-educated workers' pay rate is less than $\theta^1$ condition on human capital $h$. By Eq.(23), we obtain that

$$\begin{aligned}\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta = &\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))}{\sigma(1-\psi_1(\theta^1)) + (\lambda+\delta)\left[1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right]} \\ &\times\left\{\left[1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)\right]\left(\frac{1-\psi_1(\theta^1)}{q_1}\right)^n + \frac{\sigma(1-\delta-\sigma)}{\lambda+\delta-\sigma\lambda}\right\}.\end{aligned} \tag{34}$$

Note that $\frac{(\lambda+\delta-\sigma\lambda)(\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1))}{\sigma(1-\psi_1(\theta^1))+(\lambda+\delta)[1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1)]} > 0$, $1-\beta F_0(\theta^1)-(1-\beta)F_1(\theta^1) > 0$, $\frac{\sigma(1-\delta-\sigma)}{\lambda+\delta-\sigma\lambda} > 0$ and $0 < \frac{1-\psi_1(\theta^1)}{q_1} < 1$, which implies that the conditional probability is decreasing in $n$. That is to say, a high-educated worker with higher human capital is more likely to have a higher pay rate.

Specially, when $n = 0$, Eq.(34) can be written as

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta|1)\mathrm{d}\theta = \beta F_0(\theta^1) + (1-\beta)F_1(\theta^1).$$

For the new entrants with high-education, their pay rates are randomly drawn from $\beta F_0(\theta^1) + (1-\beta)F_1(\theta^1)$. When $n \to \infty$,

$$\int_{\theta_r^1}^{\theta^1} G_1(\theta|\infty)\mathrm{d}\theta = \frac{\sigma(1-\delta-\sigma)\left[\beta F_0(\theta^1)+(1-\beta)F_1(\theta^1)\right]}{\sigma(1-\psi_1(\theta^1)) + (\lambda+\delta)\left[1-\beta F_0(\theta^1))-(1-\beta)F_1(\theta^1))\right]},$$

which implies $\int_{\theta_r^1}^{\theta^1} G_1(\theta|\infty)\mathrm{d}\theta$ is non-degenerate.

**Proposition 4.2.** The probability of the high-educated workers' equilibrium pay rate which is less than $\theta^1$ conditional on human capital $h$, $\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta$, is decreasing in $h$.

### 4.3. Comparative analysis

We compare the effect of the low-educated workers' tenure $m$ on conditional probability $\int_{\theta_r^0}^{\theta^0} G_0(\theta|h)\mathrm{d}\theta$ with the effect of the high-educated workers' tenure $n$ on conditional probability $\int_{\theta_r^1}^{\theta^1} G_1(\theta|h)\mathrm{d}\theta$. Assume that $m = n = \tau$ and $\theta^0 = \theta^1 = \theta'$. When $\tau = 0$,

$$\int_{\theta_r^0}^{\theta'} G_0(\theta|1)\mathrm{d}\theta = F_0(\theta'),$$

and

$$\int_{\theta_r^1}^{\theta'} G_1(\theta|1)\mathrm{d}\theta = \beta F_0(\theta') + (1-\beta)F_1(\theta').$$

Since

$$F_0(\theta') \geq \beta F_0(\theta') + (1-\beta)F_1(\theta'),$$

we can get

$$\int_{\theta_r^0}^{\theta'} G_0(\theta|1)\mathrm{d}\theta \geq \int_{\theta_r^1}^{\theta'} G_1(\theta|1)\mathrm{d}\theta,$$

which indicates that for the new entrants, the high-educated workers have relatively higher probability to enjoy a higher pay rate. As

$$\frac{1-\psi_0(\theta')}{q_0} = \frac{[1-\delta-\sigma-\beta\lambda(1-F_0(\theta'))]\,(\beta\lambda+\delta-\beta\lambda\sigma)}{(1-\delta-\sigma)(\beta\lambda+\delta)},$$

and

$$\frac{1-\psi_1(\theta')}{q_1} = \frac{[1-\delta-\sigma-\beta\lambda(1-F_0(\theta'))-(1-\beta)\lambda(1-F_1(\theta'))]\,(\lambda+\delta-\sigma\lambda)}{(1-\delta-\sigma)(\lambda+\delta)},$$

it is easily to get

$$0 < \frac{1-\psi_1(\theta')}{q_1} < \frac{1-\psi_0(\theta')}{q_0} < 1.$$

From Eqs.(33) and (34), we can get that with the increase of tenure, the conditional probability of the high-educated workers falls faster than that of the low-educated workers which presents that with same tenure, the high-educated workers are more likely to own higher pay rates.

## 5. NUMERICAL EXAMPLE

In this section, we perform a numerical example in a more general situation of the model, so as to show how the profit of the firms offering the unskilled jobs to the low-educated workers $\pi_{00}$, the profit of the firms offering the unskilled jobs to the high-educated workers $\pi_{01}$, the total profit of the firms offering the unskilled jobs $\pi_0$, the profit of the firms offering the skilled jobs $\pi_1$ and the supports of the pay rate change with respect to the proportion of the low-educated workers. Following Burdett [5] who considered a year as the reference time unit and assume workers have a 40 year expected working lifetime, we set $\delta = 0.025$. Following Jolivet et al. [15] who estimated American turnover parameters, we set $\sigma = 0.055$ and $\lambda = 0.15$. The 3-year panel of individual worker data are from the Panel Study of Income Dynamics (PSID) for the analysis of the U.S. labor market in the 1990s. In addition, we set $\beta = 0.727$,[1] which represents the proportion of unskilled jobs in the U.S. labor market [12, 16].

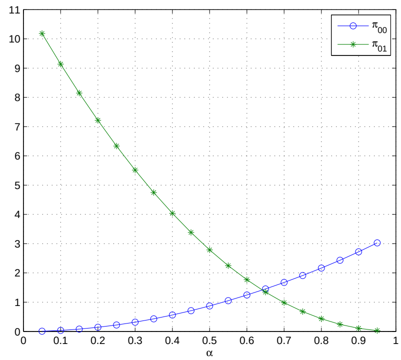
In the numerical example, we group over 500 separate occupations in the census into two main categories based on the job description and the average educational attainment[2]. Skilled jobs include includes lawyers, physicians, managers, accountants, engineers, social workers and teachers. And unskilled jobs include waiters, salespersons, cashiers, construction laborers, automotive mechanics and drivers. In addition, we interpret low-education as the average education is below 12 years, and interpret high-education as some college, bachelor's degree, master's degree, professional degree and doctorate degree. By solving the market equilibrium[3], we investigate the effects of human capital on market profit in the United States.

From Table 1 and Figure 4, it is apparent that $\pi_{00}$ is increasing along with the proportion of low-educated workers $\alpha$, while $\pi_{01}$ is decreasing with the proportion. In the real life, the firms offering unskilled jobs are more easily to employ the low-educated workers and more hardly to find the high-educated workers and that is why $\pi_{00}$ is increasing with $\alpha$, while $\pi_{01}$ is decreasing. In addition, $\pi_{01}$ is greater than $\pi_{00}$ if $\alpha < 0.65$ and $\pi_{00}$ is greater than $\pi_{01}$ if $\alpha \geq 0.65$ which indicate that the profit of the firms offering the unskilled jobs to the high-educated workers is greater than that of offering the unskilled jobs to the low-educated workers when there are more high-educated workers as it is more easy to employ the high-educated workers to gain more profit. Further, $\pi_0$ decreases with $\alpha$ if $\alpha \leq 0.77$, while $\pi_0$ increases with $\alpha$ if $\alpha > 0.77$ as in this situation there are many low-educated workers and the profit of the firms offering the unskilled job to the low-educated workers is very large so that the total profit of the firms offering the unskilled job increases with the proportion of the low-educated workers.

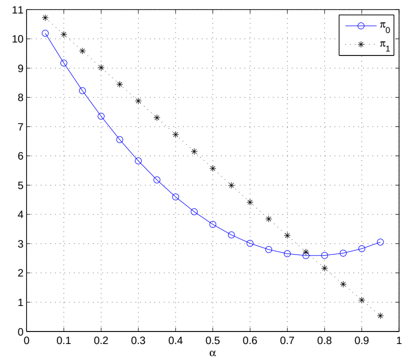
[1] Mattoo et al. [16] investigated the occupational placement of immigrants in the U.S. labor market using the 1990s census data. The data extract from the Census samples were made through IPUMS (www.ipums.org).

[2] Educational attainments were obtained by computing the average years of education in each profession, with all US-born and foreignborn people (males and females) included.

[3] The numerical results are computed by applying the Matlab 7.10.0.499. The information on CPU and memory of computer are AMD A6-3420M APU with Radeon(tm) HD Graphics 1.5 GHz and 4.00 GB, respectively. The average running time of the computations is $4.3601 \times 10^{-4}$ seconds.



(a) The effects of $\alpha$ on $\pi_{00}$ and $\pi_{01}$



(b) The effects of $\alpha$ on $\pi_0$ and $\pi_1$

**Fig. 4.** The influence tendency of different proportion of low-educated workers.

| $\alpha$ | $\pi_{00}$ | $\pi_{01}$ | $\pi_0$ | $\pi_1$ | $[\underline{\theta}^0, \overline{\theta}^0]$ | $[\underline{\theta}^1, \overline{\theta}^1]$ |
|---|---|---|---|---|---|---|
| 0.05 | 0.0088 | 10.1829 | 10.1917 | 10.7220 | [0.3, 0.9580] | [0.7047, 0.9608] |
| 0.10 | 0.0353 | 9.1379 | 9.1732 | 10.1542 | [0.3, 0.9376] | [0.7048, 0.9608] |
| 0.15 | 0.0794 | 8.1484 | 8.2278 | 9.5869 | [0.3, 0.9364] | [0.7049, 0.9609] |
| 0.20 | 0.1411 | 7.2144 | 7.3555 | 9.0168 | [0.3, 0.9356] | [0.7051, 0.9609] |
| 0.25 | 0.2204 | 6.3361 | 6.5565 | 8.4475 | [0.3, 0.9360] | [0.7053, 0.9609] |
| 0.30 | 0.3170 | 5.5136 | 5.8306 | 7.8763 | [0.3, 0.9361] | [0.7056, 0.9610] |
| 0.35 | 0.4309 | 4.7471 | 5.1780 | 7.3038 | [0.3, 0.9363] | [0.7060, 0.9610] |
| 0.40 | 0.5617 | 4.0368 | 4.5985 | 6.7282 | [0.3, 0.9364] | [0.7066, 0.9611] |
| 0.45 | 0.7089 | 3.3833 | 4.0922 | 6.1507 | [0.3, 0.9365] | [0.7074, 0.9612] |
| 0.50 | 0.8722 | 2.7868 | 3.6590 | 5.5724 | [0.3, 0.9367] | [0.7084, 0.9613] |
| 0.55 | 1.0510 | 2.2481 | 3.2991 | 4.9946 | [0.3, 0.9370] | [0.7096, 0.9615] |
| 0.60 | 1.2448 | 1.7675 | 3.0123 | 4.4182 | [0.3, 0.9373] | [0.7110, 0.9617] |
| 0.65 | 1.4528 | 1.3459 | 2.7987 | 3.8445 | [0.3, 0.9377] | [0.7126, 0.9619] |
| 0.70 | 1.6755 | 0.9828 | 2.6583 | 3.2770 | [0.3, 0.9380] | [0.7142, 0.9621] |
| 0.75 | 1.9120 | 0.6790 | 2.5910 | 2.7146 | [0.3, 0.9384] | [0.7159, 0.9623] |
| 0.80 | 2.1647 | 0.4323 | 2.5970 | 2.1609 | [0.3, 0.9387] | [0.7173, 0.9625] |
| 0.85 | 2.4342 | 0.2419 | 2.6761 | 1.6144 | [0.3, 0.9389] | [0.7184, 0.9627] |
| 0.90 | 2.7213 | 0.1071 | 2.8284 | 1.0732 | [0.3, 0.9391] | [0.7192, 0.9628] |
| 0.95 | 3.0266 | 0.0273 | 3.0539 | 0.5356 | [0.3, 0.9392] | [0.7197, 0.9628] |

**Tab. 1.** The effects of different proportion of the low-educated workers.

For the skilled jobs, the profit decreases with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than that of the firms offering the unskilled jobs until there is only a few of high-educated workers. This implies that the profit of firms offering the skilled jobs increases with the increasing number of the high-educated workers in the United States, and profit dispersion between the skilled jobs and the unskilled jobs reaches its maximum when the proportion of low-educated workers is approximately from 35% to 50%. In the real life, the labor market in the United States has seen a substantial increase in profit dispersion since the early 1990s [12]. From the US Census [16], the proportion of low-educated workers is 47.5% in the 1990s. Therefore, we find that our equilibrium search model fits the data well. Given $\underline{\theta}^0$, with the decreasing number of the high-educated workers, $\underline{\theta}^1$ decreases, while there is no distinct tendency in $\overline{\theta}^0$ and $\overline{\theta}^1$. That is, along with the number of the high-educated workers decreasing, the reservation pay rate of the high-educated workers increases.

## 6. CONCLUSION

In this paper, we construct and analyze an equilibrium search model in a labor market where firms post wage-human capital contracts and risk neutral workers search for better job opportunities whether employed or unemployed. There are heterogeneous firms (unskilled or skilled) and workers (low-educated or high-educated), and high-educated workers may accept unskilled jobs for which they are over-qualified. In addition, the structure proportion of the offered jobs affects the equilibrium, which shows there exists a threshold that can distinguish whether the equilibrium is separating or cross-skill. The cross-skill equilibrium solution implies the workers with higher human capital are more likely to earn higher pay rates and that the high-educated workers are more likely to own higher pay rates than the low-educated workers with the same tenure are likely to.

Numerical simulations show the profit of the firms offering unskilled jobs to low-educated workers is increasing with the proportion of low-educated workers, while the profit of the firms offering unskilled jobs to high-educated workers is decreasing with the proportion. Moreover, the profit of the firms offering the unskilled jobs to the high-educated workers is greater than the profit of the firms offering the unskilled jobs to the low-educated workers when there are more high-educated workers. The total profit of the firms offering the unskilled jobs decrease with the increasing number of the low-educated workers until the great majority of workers are low-educated worker. The profit of the firms offering skilled jobs decreases with the increasing number of the low-educated workers. Moreover, the profit of the firms offering the skilled jobs is greater than the profit of offering the unskilled jobs until there is only very few high-educated workers. Along with the number of the high-educated workers decreasing, the reservation pay rate of the high-educated workers increases. One of the most interesting conclusions is the growth rate of human capital is an endogenous variable which is determined by death shock, job destruction shock, the fraction of the unskilled jobs and the arrival rate of jobs.

## APPENDIX

P r o o f of Proposition 2.1. It is obvious that a worker's income, whether the worker is unemployed or employed, is always proportional to $h_t$, that is, there exists a number $v_{u0}$ and a function $v_0(\theta_t)$ such that $V_{u0}(h_t) = v_{u0}h_t$ and $V_0(\theta_t; h_t) = v_0(\theta_t)h_t$, respectively. In this way, Eq.(1) can be written as

$$\delta v_{u0} = b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta) - v_{u0})\mathrm{d}F_0(\theta), \tag{35}$$

while Eq.(2) can be written as

$$v_0(\theta_t) = \theta_t + (1+g)\left[\sigma v_{u0} + (1-\delta-\sigma)v_0(\theta_t) + \beta\lambda \int_{\theta_t}^{\overline{\theta}^0} (v_0(\theta) - v_0(\theta_t))\mathrm{d}F_0(\theta)\right]. \tag{36}$$

Since $\theta_r^0$ is the reservation pay rate per unit of human capital of the unemployed workers with low-education, there is no difference between accepting the offer $\theta_r^0$ and keeping unemployment, i. e., $V_0(\theta_r^0; h_t) = V_{u0}(h_t)$. Therefore, we get $v_0(\theta_r^0) = v_{u0}$. Let $\theta_t = \theta_r^0$ in Eq.(36). Thus,

$$v_{u0} = \theta_r^0 + (1+g)\left[(1-\delta)v_{u0} + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta) - v_{u0})\mathrm{d}F_0(\theta)\right]. \tag{37}$$

Note that $\displaystyle\int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta)) - v_{u0})\mathrm{d}F_0(\theta) = \frac{\delta v_{u0} - b}{\beta\lambda}$ by (35), and then (37) can be written as

$$g v_{u0} = b(1+g) - \theta_r^0. \tag{38}$$

On the other hand, differentiating (36) with respect to $\theta_t$ yields

$$\frac{\mathrm{d}v_0(\theta_t)}{\mathrm{d}\theta_t} = \frac{1}{1-(1+g)\,(1-\psi_0(\theta_t))}. \tag{39}$$

Therefore, Eq.(35) can be written as

$$\begin{aligned} \delta v_{u0} &= b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} (v_0(\theta) - v_{u0})\mathrm{d}F_0(\theta) \\ &= b + \beta\lambda \left[\int_{\theta_r^0}^{\overline{\theta}^0} (1 - F_0(\theta))\frac{\mathrm{d}v_0(\theta)}{\mathrm{d}\theta}\mathrm{d}\theta\right] && \text{(integrate by parts)} \\ &= b + \beta\lambda \int_{\theta_r^0}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)(1-\psi_0(\theta))}\mathrm{d}\theta && \text{(by(39))} \end{aligned} \tag{40}$$

Substituting (38) into (40) yields Eq.(3). Moreover, the optimal job search implies that any unemployed worker with low-education accepts job offer $\theta^0$ if and only if $\theta^0 \geq \theta_r^0$. This completes the proof. □

P r o o f of Proposition 2.2. Calculating the derivative of Eq.(3) with $\theta_r^0$, so as to get

$$F_0(\theta_r^0) = 1 - \frac{\delta[1-(1+g)(1-\delta-\sigma)]}{\beta\lambda(g-\delta-g\delta)}. \tag{41}$$

All the unskilled jobs offer $\theta^0 \geq \theta_r^0$, $\theta^0 \in [\underline{\theta}^0, \overline{\theta}^0]$, otherwise there is no worker accepts the offer. Thus in the market equilibrium, $F_0(\theta_r^0) = 0$, and from Eq.(41), we obtain

$$g = \frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}. \tag{42}$$

Therefore, $g$ is determined by $\delta$, $\sigma$, $\beta$ and $\lambda$. It is easily to prove that the partial derivatives of $g$ have the following properties: $\dfrac{\partial g}{\partial \delta} > 0$, $\dfrac{\partial g}{\partial \sigma} > 0$, $\dfrac{\partial g}{\partial \beta} < 0$ and $\dfrac{\partial g}{\partial \lambda} < 0$. □

P r o o f of Proposition 2.3. From Eq.(3), we can get

$$\theta_r^0 = -\frac{g\beta\lambda}{\delta}\int_{\theta_r^0}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\psi_0(\theta)\right)}\mathrm{d}\theta - \frac{gb}{\delta} + (1+g)b.$$

Let

$$T(x) = -\frac{g\beta\lambda}{\delta}\int_{x}^{\overline{\theta}^0} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\psi_0(\theta)\right)}\mathrm{d}\theta - \frac{gb}{\delta} + (1+g)b.$$

$\forall x_1, x_2 \in [\underline{\theta}^0, \overline{\theta}^0]$,

$$|T(x_1) - T(x_2)| = \left|\frac{g\beta\lambda}{\delta}\int_{x_1}^{x_2} \frac{1-F_0(\theta)}{1-(1+g)\left(1-\delta-\sigma-\beta\lambda(1-F_0(\theta)\right))}\mathrm{d}\theta\right|.$$

$\exists \mu \in [x_1, x_2] \in (\underline{\theta}^0, \overline{\theta}^0)$,

$$|T(x_1) - T(x_2)| = \frac{g\beta\lambda}{\delta}\left|\frac{(x_2-x_1)(1-F_0(\mu))}{1-(1+g)(1-\delta-\sigma-\beta\lambda(1-F_0(\mu)))}\right|.$$

Since

$$g = \frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma},$$

it is easy to find that

$$0 < k = \frac{g\beta\lambda(1-F_0(\epsilon))}{\delta\left[1-(1+g)(1-\delta-\sigma-\beta\lambda(1-F_0(\epsilon)))\right]} < 1.$$

So that

$$|T(x_1) - T(x_2)| < k\,|x_2 - x_1|\,.$$

By using the contraction mapping principle [1], we can prove that there exists a unique solution $\theta_r^0 \in [\underline{\theta}^0, \overline{\theta}^0]$. □

P r o o f of Proposition 2.5. Calculating the derivative of Eq.(6) with $\theta_r^1$, so as to get

$$\beta F_0(\theta_r^1) + (1-\beta)F_1(\theta_r^1) = 1 - \frac{\delta[1-e^g(1-\delta-\sigma)]}{\lambda(e^g-1-\delta e^g)}. \tag{43}$$

All the skilled jobs offer $\theta^1 \geq \theta_r^1$, $\theta^1 \in [\underline{\theta}^1, \overline{\theta}^1]$, otherwise there is no worker accepts the offer. Thus in the market equilibrium, $F_1(\theta_r^1) = 0$, and Eq.(43) can be written as

$$F_0(\theta_r^1) = \frac{1}{\beta} - \frac{\delta[1-e^g(1-\delta-\sigma)]}{\beta\lambda(e^g-1-\delta e^g)}. \tag{44}$$

For the unskilled jobs, if $\overline{\theta}^0 \geq \theta_r^1$ which means that the high-educated workers choose either the unskilled job or the skilled job, we can get $F_0(\theta_r^1) < 1$, and by Eq.(44), there exists

$$e^g < \frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

Since

$$g = \frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma},$$

it is easy to get

$$\exp\left(\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}\right) < \frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

Therefore, when the fraction of the unskilled jobs is greater than $\beta$, the high-educated workers choose either the unskilled job or the skilled job.

If $\overline{\theta}^0 < \theta_r^1$ which implies that the high-educated workers only choose the skilled jobs, we can get $F_0(\theta_r^1) = 1$, and by Eq.(44), there exists

$$e^g = \frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

Since

$$g = \frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma},$$

it is easy to get

$$\exp\left(\frac{\delta(\delta+\sigma+\beta\lambda)}{(\beta\lambda+\delta)(1-\delta)-\delta\sigma}\right) = \frac{(1-\beta)\lambda+\delta}{[(1-\beta)\lambda+\delta](1-\delta)-\delta\sigma}.$$

When the fraction of the unskilled jobs is no greater than $\beta$, the high-educated workers only choose the skilled jobs. This completes the proof. □

P r o o f of Proposition 2.6. From Eq.(6), we obtain

$$\theta_r^1 = e^g b - \frac{(e^g-1)b}{\delta} - \frac{(e^g-1)\lambda}{\delta}\left[\int_{\theta_r^1}^{\overline{\theta}^0} \frac{\beta\,(1-F_0(\theta))}{1-e^g\,(1-\psi_1(\theta))}\mathrm{d}\theta + \int_{\theta_r^1}^{\overline{\theta}^1} \frac{(1-\beta)\,(1-F_1(\theta))}{1-e^g\,(1-\psi_1(\theta))}\mathrm{d}\theta\right]. \tag{45}$$

Let

$$T(x) = e^g b - \frac{(e^g - 1)b}{\delta} - \frac{(e^g - 1)\lambda}{\delta}\left[\int_x^{\overline{\theta}^0} \frac{\beta\,(1 - F_0(\theta))}{1 - e^g\,(1 - \psi_1(\theta))}\mathrm{d}\theta + \int_x^{\overline{\theta}^1} \frac{(1-\beta)\,(1 - F_1(\theta))}{1 - e^g\,(1 - \psi_1(\theta))}\mathrm{d}\theta\right]. \tag{46}$$

$\forall x_1, x_2 \in [\underline{\theta}^0, \overline{\theta}^0]$,

$$|T(x_1) - T(x_2)| = \left|\frac{(e^g - 1)\lambda}{\delta}\left[\int_{x_1}^{x_2} \frac{1 - \beta F_0(\theta) - (1-\beta)F_1(\theta)}{1 - e^g(1 - \psi_1(\theta))}\mathrm{d}\theta\right]\right|$$

$\exists \mu \in [x_1, x_2] \in (\underline{\theta}^0, \overline{\theta}^0)$,

$$\begin{aligned}
|T(x_1) - T(x_2)| &= \frac{(e^g - 1)\lambda}{\delta}\int_{x_1}^{x_2} \frac{1 - \beta F_0(\theta) - (1-\beta)F_1(\theta)}{1 - e^g\left[1 - \delta - \sigma - \lambda + \beta F_0(\theta) + (1-\beta)F_1(\theta)\right]}\mathrm{d}\theta \\
&= \left|\frac{(e^g - 1)\lambda}{\delta}\left[(x_2 - x_1)\frac{1 - \beta F_0(\mu) - (1-\beta)F_1(\mu)}{1 - e^g(1 - \psi_1(\mu))}\right]\right| \\
&< \frac{(e^g - 1)\lambda}{\delta[1 - e^g(1 - \delta - \sigma - \lambda)]}\left|x_2 - x_1\right|.
\end{aligned}$$

It is easy to find that $0 < \dfrac{(1-\sigma)(e^g - 1)\lambda}{\delta[1 - e^g(1 - \delta - \sigma - \lambda)]} < 1$. By using the contraction mapping principle [1], we can prove that there exists a unique solution $\theta_r^1 \in [\underline{\theta}^0, \overline{\theta}^0]$. □

Proof of Proposition 3.2. Let $h = 1$ and $\theta^0 = \overline{\theta}^0$ in Eq.(16). Thus,

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, 1)\mathrm{d}\theta = \frac{\gamma_0\beta\lambda}{1 - \gamma_0}N_0(1),$$

and using Eq.(13) to substitute out $N_0(1)$ obtains

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, 1)\mathrm{d}\theta = \frac{\delta\beta\lambda}{(1 - \gamma_0)(\beta\lambda + \delta)}.$$

Let $h \in \{(1+g)^m\}_{m=1}^{\infty}$ and $\theta^0 = \overline{\theta}^0$ in Eq.(16) once again. Therefore,

$$\begin{aligned}
(1-\gamma_0)\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^m)\mathrm{d}\theta = {} & (1-\gamma_0)\left[1 - \psi_0(\overline{\theta}^0)\right]\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^{m-1})\mathrm{d}\theta \\
& + \gamma_0 N_0\left((1+g)^m\right)\beta\lambda,
\end{aligned}$$

and using Eq.(15) to substitute out $N_0(\cdot)$ yields

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^m)\mathrm{d}\theta = q_0\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta, (1+g)^{m-1})\mathrm{d}\theta,$$

where

$$q_0 = \frac{(1-\delta-\sigma)(\beta\lambda+\delta)}{\beta\lambda+\delta-\sigma\beta\lambda} < 1.$$

Therefore, when $h \in \{(1+g)^m\}_{m=0}^{\infty}$,

$$\int_{\theta_r^0}^{\overline{\theta}^0} G_0(\theta,(1+g)^m)\mathrm{d}\theta = \frac{\delta\beta\lambda}{(1-\gamma_0)(\beta\lambda+\delta)} q_0^m. \tag{47}$$

Using Eq.(47) in Eq.(15) obtains

$$N_0((1+g)^m) = \frac{\delta\sigma\beta\lambda}{\gamma_0(\beta\lambda+\delta)^2} q_0^m. \tag{48}$$

Given $\theta_0$, if $h=1$, Eq.(16) can be written as

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta,1)\mathrm{d}\theta = \frac{\delta\beta\lambda F_0(\theta^0)}{(1-\gamma_0)(\beta\lambda+\delta)},$$

and if $h \in \{(1+g)^m\}_{m=1}^{\infty}$, Eq.(16) can be written as

$$\int_{\theta_r^0}^{\theta^0} G_0(\theta,(1+g)^m)\mathrm{d}\theta = \left[1-\psi_0(\theta^0)\right]\int_{\theta_r^0}^{\theta^0} G_0(\theta,(1+g)^{m-1})\mathrm{d}\theta + \frac{\sigma\delta\beta^2\lambda^2 F_0(\theta^0)}{(1-\gamma_0)(\beta\lambda+\delta)^2} q_0^m.$$

Then we can get that $G_0(\theta,h)$ satisfies

$$\begin{aligned}\int_{\theta_r^0}^{\theta^0} G_0(\theta,h)\mathrm{d}\theta = {} & \frac{\delta\beta\lambda F_0(\theta^0)(\beta\lambda+\delta-\lambda\sigma)}{(1-\gamma_0)(\beta\lambda+\delta)\left[\sigma(1-\psi_0(\theta^0))+(1-F_0(\theta^0))(\beta\lambda+\delta)\right]} \\ & \times\left\{(1-F_0(\theta^0))\left[1-\psi_0(\theta^0)\right]^m + \frac{\sigma}{\beta\lambda+\delta} q_0^{m+1}\right\}.\end{aligned} \tag{49}$$

This completes the proof. □

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