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PINNING LAG SYNCHRONIZATION BETWEEN TWO DYNAMICAL NETWORKS WITH NON-DERIVATIVE AND DERIVATIVE COUPLINGS

Zhi-wei Li, Zhe-yong Qiu and Wei-gang Sun

In this paper, we study lag synchronization between two dynamical networks with non-derivative and derivative couplings via pinning control. We design two types of pinning control schemes, including linear and adaptive feedback controllers. With the corresponding control algorithms, we obtain two theorems on the lag synchronization based on Schur complement and Barbalat's lemma. In addition, we obtain the domain for the linear feedback gains. Finally, we provide two numerical examples to show the efficiency of the proposed schemes and apply random and high-degree chosen pinning schemes.

Keywords: lag synchronization, pinning control, derivative coupling

Classification: 34D06,05C82

1. INTRODUCTION

During the past few decades, complex dynamical networks have received considerable attention and been extensively applied in the fields of sciences and humanities. Many real systems in nature can be described by complex dynamical networks, including internet, World Wide Web, electrical power grids, social networks and so forth [1, 2, 9, 17, 10]. Presently synchronization of dynamical networks has been widely studied [6, 7, 21, 24, 29, 30]. Generally speaking, synchronization appearing inside a network is referred as inner synchronization. On the criteria for inner synchronization [18], the master stability function and linear matrix inequality are two ways. Apart from the inner synchronization, we call the synchronization happening between two or more dynamical networks as outer synchronization, which is more complicated because of the diverse interconnected actions between two networks. An example of outer synchronization is the outer and inner doors simultaneously open or close in subway systems when the trains arrive at the platform.

When the synchronization of dynamical networks does not appear under some appropriate node dynamics and topological structures, many controlling (e. g., the adaptive and pinning control) methods have been designed to achieve synchronization [12, 23, 25, 26, 28] and many references cited therein. When the control is employed for every node

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in realizing the synchronization, it would take a great cost. To reduce the control cost, we use the pinning control method, namely, controlling a small fraction of nodes. Chen et al. used a single controller to synchronize the network [4]. The synchronization of delayed and non-delayed complex dynamical networks via pinning control was investigated in [12].

In the above mentioned works, they all studied the complete inner or outer synchronization. Recently lag synchronization [15, 16, 19] between two networks has attracted increasing attention, which is referred as the coincidence of the state vectors, where one of the networks follows another network with time delay. Pinning lag synchronization between two dynamical networks was studied in [22], where the linear and adaptive feedback control schemes were provided. Shi et al. studied the lag outer synchronization of complex networks with noise coupling and showed the effect of noise on the synchronization [20]. Since the complexity of network structures, it needs more information to characterize the network, e. g., considering the effect of the velocity in nodes to other nodes. Xu et al. pioneered in studying the synchronization in a complex dynamical network with non-derivative and derivative couplings [26]. Afterwards, Deng et al. used the pinning control to realize synchronization inside a network with non-derivative and derivative couplings [5]. To the best of our knowledge, few theoretical results involve the lag synchronization between two networks with non-derivative and derivative couplings by the pinning control.

Inspired by the above discussions, we use the pinning control schemes to study the lag synchronization between two dynamical networks with non-derivative and derivative couplings. With the proposed linear and adaptive feedback controllers, we obtain two theorems on the lag synchronization in the sense of Lyapunov stability. For the linear feedback control, we also obtain the domain of feedback gains. Numerical simulation results show that the proposed control algorithms are effective.

The outline of the rest of this paper is organized as follows. The network models and some necessary preliminaries are described in Section 2. Two criteria on the lag synchronization are given in Section 3. Section 4 provides the numerical examples to verify the obtained theoretical results. Conclusions are drawn in Section 5.

Throughout this paper, some notations are first introduced. A^{-1} denotes the inverse of a matrix. ^{T} is the transpose of a vector or a matrix. If A is a symmetric matrix, $\lambda_{\max}(A)$ denotes its largest eigenvalue. I_n is an identity matrix of order n. The symbol $\otimes$ denotes the Kronecker product, $A > 0 (< 0)$ means that the matrix A is positive (negative) definite.

2. MODEL DESCRIPTION AND PRELIMINARIES

In this section, we introduce the network model and provide some mathematical preliminaries. On the basis of network model, we propose the model between two dynamical networks with non-derivative and derivative couplings, which is given by

$$\begin{aligned}
\dot{x}_i(t) &= f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}x_j(t) + c\sum_{j=1}^{N} b_{ij}\dot{x}_j(t), \\
&\quad i = 1, 2, \ldots, N, \\
\dot{y}_i(t) &= f(y_i(t)) + c\sum_{j=1}^{N} a_{ij}y_j(t) + c\sum_{j=1}^{N} b_{ij}\dot{y}_j(t) + u_i, \\
&\quad i = 1, 2, \ldots, N,
\end{aligned} \tag{1}$$

where $x_i(t), y_i(t) \in R^n$ denote the node state variables in networks X and Y. The node dynamical function $f : R^n \to R^n$ is a continuously differentiable vector-valued function, $c > 0$ is the coupling strength. A and B are outer coupling configuration matrices representing the topological structures and coupling strength among nodes. They satisfy the zero-sum rows and the elements of A and B are defined as follows: if there is a link between node i and node $j (j \neq i)$, then $a_{ij} = a_{ji} > 0$ and $b_{ij} = b_{ji} > 0$, otherwise, $a_{ij} = a_{ji} = 0, b_{ij} = b_{ji} = 0 (j \neq i)$. In this paper, we assume that A and B are irreducible, which shows that the networks are connected and have no isolated nodes. $u_i (i = 1, 2, \ldots, N)$ are the controllers to be designed.

Remark 2.1. In the above mentioned network model, the derivative of state variables $\dot{x}_i(t)$, $\dot{y}_i(t)$ are regarded as the velocity of transmitting information. In interconnected networks, the velocity of transmitting information in one network may affect its dynamics or the synchronized behavior of another network. Throughout the rest of this paper, the definition, assumption and lemmas are presented.

Definition 2.2. For a positive time delay τ, the lag synchronization of networks (1) is achieved if

$$\lim_{t \to \infty} \|y_i(t) - x_i(t - \tau)\| = 0, \forall\, x, y \in R^n, \qquad i = 1, 2, \ldots, N.$$

Assumption 2.3. Suppose that there exists a positive constant $L > 0$ such that the nonlinear function $f(x)$ holds:

$$(y - x)^T (f(y) - f(x)) \leq L(y - x)^T (y - x), \quad \forall\, x, y \in R^n.$$

It has been verified that many typical benchmark chaotic systems, such as Lorenz system, Chen system, Lü system and unified chaotic system satisfy Assumption 2.3.

Lemma 2.4. (Schur Complement [3]) The following liner matrix inequality

$$\begin{pmatrix} A(x) & B(x) \\ B^T(x) & C(x) \end{pmatrix} > 0$$

where $A(x) = (A(x))^T,\ C(x) = (C(x))^T$, if and only if

$$\begin{aligned} &A(x) < 0 \ \text{ and } \ C(x) - B(x)^T A(x)^{-1} B(x) < 0; \\ &C(x) < 0 \ \text{ and } \ A(x) - B(x) C(x)^{-1} B(x)^T < 0. \end{aligned}$$

Lemma 2.5. (Barbalat Lemma [11]) If the function $\phi(t)$ is uniformly continuous, and $\lim_{t \to \infty} \int_0^\tau |\phi(\tau)| \, \mathrm{d}\tau$ is bounded, then $\phi(t) \to 0$ when $t \to \infty$.

3. PINNING LAG SYNCHRONIZATION BETWEEN TWO COUPLED NETWORKS

3.1. Lag synchronization via linear feedback control

In this subsection, we design the liner feedback control to achieve lag synchronization. Suppose the first $m(1 \leq m < N)$ nodes are controlled in network Y. Then, the pinning linear feedback controllers are designed as follows:

$$\begin{cases} u_i = -cd_i e_i(t), \quad i = 1, 2, \ldots, m, \\ u_i = 0, \quad i = m+1, \ldots, N, \end{cases}$$

where $e_i(t) = y_i(t) - x_i(t-\tau)$, $d_i(i = 1, 2, \ldots, m) > 0$ are feedback gains. Using the designed controllers, we obtain the lag synchronization error of networks (1), that is,

$$\begin{array}{rcl} \dot{e}_i(t) & = & f(y_i(t)) - f(x_i(t-\tau)) + c\sum_{j=1}^{N} a_{ij} e_j(t) + c\sum_{j=1}^{N} b_{ij} \dot{e}_j(t) - cd_i e_i(t), \\ & & i = 1, 2, \ldots, m, \\ \dot{e}_i(t) & = & f(y_i(t)) - f(x_i(t-\tau)) + c\sum_{j=1}^{N} a_{ij} e_j(t) + c\sum_{j=1}^{N} b_{ij} \dot{e}_j(t), \\ & & i = m+1, m+2, \ldots, N. \end{array} \tag{2}$$

Then we rewrite (2) in a compact form,

$$\dot{e}(t) = [(I_N - cB) \otimes I_n]^{-1} [F(Y) - F(X)_\tau + c(A - D) \otimes I_n e(t)], \tag{3}$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$, $F(X)_\tau = (f^T(x_1(t-\tau)), f^T(x_2(t-\tau)), \ldots$ $\ldots, f^T(x_N(t-\tau)))^T$, $F(Y) = (f^T(y_1(t)), f^T(y_2(t)), \ldots, f^T(y_N(t)))^T$, $D = \mathrm{diag}(d_1, d_2, \ldots, d_m, 0, 0, \ldots, 0)$.

Theorem 3.1. Suppose that Assumption 1 holds and $A = \begin{pmatrix} A_1 & A_3 \\ A_3^T & A_2 \end{pmatrix}$ is a symmetric matrix. The lag synchronization of the networks (1) is realized if

$$\begin{array}{l} \lambda_{\max}(A_2) < -L/c, \\ d_i > \lambda_{\max}(Q_1 - Q_3 Q_2^{-1} Q_3^T)/c, i = 1, 2, \ldots, m, \end{array}$$

where $Q_1 = LI_m + cA_1, Q_2 = LI_{N-m} + cA_2, Q_3 = cA_3$.

Proof. Construct a Lyapunov function as follows,

$$V(t) = \frac{1}{2} e^T(t)[(I_N - cB) \otimes I_n] e(t).$$

The derivative of $V(t)$ along Eq. (3) yields

$$\begin{array}{rcl} \dot{V}(t) & = & e^T(t)[(I_N - cB) \otimes I_n]\dot{e}(t) \\ & = & e^T(t)[F(Y) - F(X)_\tau + c(A - D) \otimes I_n e(t)] \\ & \leq & e^T(t)[(LI_N + c(A - D)) \otimes I_n] e(t). \end{array}$$

Using $A = \begin{pmatrix} A_1 & A_3 \\ A_3^T & A_2 \end{pmatrix}$, then

$$Q = LI_N + c(A - D) = \begin{pmatrix} Q_1 - cD_1 & Q_3 \\ Q_3^T & Q_2 \end{pmatrix},$$

where $D_1 = \operatorname{diag}(d_1, d_2, \dots, d_m)$. According to $\lambda_{\max}(A_2) < -L/c$, we have $Q_2 < 0$. Because $d_i > \lambda_{\max}(Q_1 - Q_3 Q_2^{-1} Q_3^T)/c$, we obtain $Q_1 - cD_1 - Q_3 Q_2^{-1} Q_3^T < 0$. By Lemma 2.4, we have $Q < 0$ and $Q \otimes I_n < 0$. Then $\dot{V}(t) \le e^T(t)(Q \otimes I_n)e(t) = -e^T(t)Me(t) \le 0$. Further,

$$0 \le \lambda_{\min}(M)\|e(t)\|^2 \le e^T(t)Me(t) \le -\dot{V}(t),$$

where $\lambda_{\min}(M) > 0$ is the minimal eigenvalue of matrix M.

Since $V(t) > 0$, then

$$\int_0^t \lambda_{\min}(M)\|e(s)\|^2 \,\mathrm{d}s \le -\int_0^t \dot{V}(s)\,\mathrm{d}s = V(0) - V(t) \le V(0) < +\infty.$$

By lemma 2.5, we obtain $\lim_{t\to\infty} \|e(t)\|^2 = 0$, showing that the lag synchronization of networks (1) is achieved. □

Remark 3.2. From the pinning condition, we see that the feedback gains vary with the number of pinned nodes. In addition, the feedback gains may be much bigger than the needed values. In the following subsection, by avoiding the large values of feedback gains, we apply the adaptive feedback control to achieve the lag synchronization.

3.2. Lag synchronization via adaptive feedback control

In this subsection, we design the pinning adaptive feedback controllers to realize the lag synchronization, which is given by

$$\begin{cases} u_i = -cd_i(t)e_i(t), \ i = 1, 2, \dots, m, \\ \dot{d}_i(t) = \xi_i e_i^T(t)e_i(t), \ i = 1, 2, \dots, m, \\ u_i = 0, \quad i = m+1, \dots, N, \end{cases}$$

where ξ_i are positive constants. Then the synchronization errors read as

$$\begin{aligned} \dot{e}_i(t) &= f(y_i(t)) - f(x_i(t-\tau)) + c\textstyle\sum_{j=1}^N a_{ij}e_j(t) + c\sum_{j=1}^N b_{ij}\dot{e}_j(t) - cd_i(t)e_i(t), \\ &\quad i = 1, 2, \dots, m, \\ \dot{d}_i(t) &= \xi_i e_i^T(t)e_i(t), \ i = 1, 2, \dots, m, \\ \dot{e}_i(t) &= f(y_i(t)) - f(x_i(t-\tau)) + c\textstyle\sum_{j=1}^N a_{ij}e_j(t) + c\sum_{j=1}^N b_{ij}\dot{e}_j(t), \\ &\quad i = m+1, m+2, \dots, N. \end{aligned} \tag{4}$$

Theorem 3.3. Suppose that Assumption 1 holds. If $\lambda_{\max}(A_2) < -L/c$, then the lag synchronization under the designed adaptive feedback controllers is achieved.

P r o o f. Define a Lyapunov function as

$$V(t) = \frac{1}{2}e^T(t)[(I_N - cB) \otimes I_n]e(t) + \frac{1}{2}\sum_{i=1}^{m} \frac{c}{\xi_i}(d_i(t) - d_i^*)^2,$$

where $d_i^*(i = 1, 2 \cdots, m)$ are positive constants to be determined below.

The derivative of $V(t)$ with respect to Eq. (4) gives

$$\begin{aligned} \dot{V}(t) &= e^T(t)[(I_N - cB) \otimes I_n]\dot{e}(t) + \sum_{i=1}^{m} c(d_i(t) - d_i^*)e_i^T(t)e_i(t) \\ &\leq e^T[(LI_N + c(A - D^*)) \otimes I_n]e(t), \end{aligned}$$

where $D^* = \mathrm{diag}(d_1^*, d_2^*, \ldots, d_m^*, 0, \ldots, 0)$. If $\lambda_{\max}(A_2) < -L/c$, we obtain $Q_2^* < 0$. By Lemma 2.4, we choose the suitable positive constants $d_i^* > \lambda_{\max}(Q_1^* - Q_3^* {Q_2^*}^{-1} Q_3^{*T})/c, i = 1, 2, \ldots, m$, which satisfies $Q^* = LI_N + c(A - D^*) < 0$. By the similar proof of Theorem 3.1, we obtain $\lim_{t\to\infty} \|e(t)\| = 0$, which implies that the lag synchronization of networks (1) is achieved. □

4. NUMERICAL SIMULATIONS

In this section, we present numerical examples to verify the theoretical results obtained in the previous section. The node dynamics follows the Chua's circuits, which is described as,

$$\begin{cases} \dot{x}_{i1}(t) = \alpha(x_{i2}(t) - x_{i1}(t)) - \phi(x_{i1}(t)), \\ \dot{x}_{i2}(t) = x_{i1}(t) - x_{i2}(t) + x_{i3}(t), \\ \dot{x}_{i3}(t) = -\beta x_{i3}(t), \end{cases}$$

where $\phi(x_{i1}(t)) = bx_{i1}(t) + \frac{1}{2}(a - b)(|x_{i1}(t) + 1| - |x_{i1}(t) - 1|)$. When $a = -1.27$, $b = -0.68$, $\alpha = 10$, $\beta = 14.87$, it has a chaos attractor. According to the result in [22], we choose a positive constant $L = 11.435$ such that Assumption 2.3 holds.

In the numerical simulations, the initial values are randomly given in the interval $(-7, 7)$, $\tau = 0.03$. To measure the extent to which the lag synchronization, we introduce $e_{i1}(t) = |y_{i1}(t) - x_{i1}(t - \tau)|$, $e_{i2}(t) = |y_{i2}(t) - x_{i2}(t - \tau)|$, $e_{i3}(t) = |y_{i3}(t) - x_{i3}(t - \tau)|$, $i = 1, 2, \ldots, N, t \in [0, +\infty)$.

4.1. Linear feedback control

In this subsection, we apply the liner feedback control. The controlled networks become

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + c\textstyle\sum_{j=1}^{12} a_{ij}x_j(t) + c\sum_{j=1}^{12} b_{ij}\dot{x}_j(t), \quad i = 1, 2, \ldots, 12, \\ \dot{y}_i(t) &= f(y_i(t)) + c\textstyle\sum_{j=1}^{12} a_{ij}y_j(t) + c\sum_{j=1}^{12} b_{ij}\dot{y}_j(t) + u_i, \quad i = 1, 2, \ldots, 12, \end{aligned} \tag{5}$$

and the feedback controllers are

$$\begin{cases} u_i = -cd_ie_i(t), \quad i = 1, 2, \\ u_i = 0, \quad i = 3, 4, \cdots, 12. \end{cases}$$

For the coupling matrices, we set $A = B$ and A is chosen as

$$A = \begin{bmatrix} -3 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & -6 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -4 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -5 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & -6 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & -4 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}.$$

For $L = 11.435$, we obtain $\lambda_{\max}(A_2) = -0.6170$ and $\lambda_{\max}(Q_1 - Q_3 Q_2^{-1} Q_3^T) = 6.3961$, then we choose the coupling strength $c = 18.54$ and feedback gains $d_i = 2.5 (i = 1, 2)$, satisfying the pinning condition. The first two nodes are chosen as the pinned nodes. In the process of simulations, Figures $1-3$ show the errors of lag synchronization of two dynamical networks (5), which shows that the lag synchronization is realized under the linear feedback controllers.

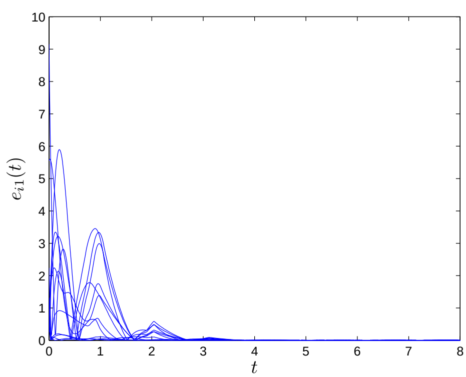

Fig. 1. Lag synchronization errors $e_{i1}(t)(i = 1, 2, \ldots, 12)$ with linear feedback control and $m = 2, \tau = 0.03$.

4.2. Adaptive feedback control

The dynamical networks with adaptive feedback controllers become

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + c\sum_{j=1}^{100} a_{ij} x_j(t) + c\sum_{j=1}^{100} b_{ij}\dot{x}_j(t), \ i = 1, 2, \ldots, 100, \\ \dot{y}_i(t) &= f(y_i(t)) + c\sum_{j=1}^{100} a_{ij} y_j(t) + c\sum_{j=1}^{100} b_{ij}\dot{y}_j(t) + u_i, i = 1, 2, \ldots, 100, \end{aligned} \tag{6}$$

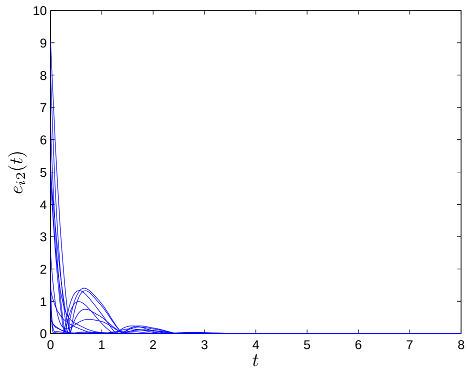

Fig. 2. Lag synchronization errors $e_{i2}(t)(i = 1, 2, \ldots, 12)$ with linear feedback control and $m = 2, \tau = 0.03$.

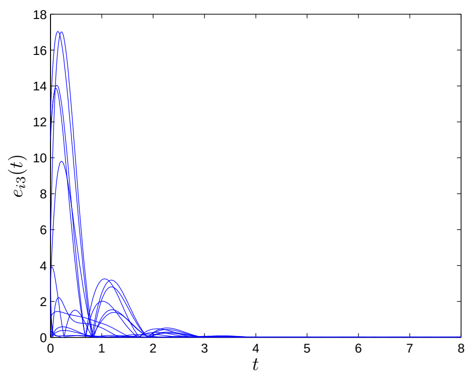

Fig. 3. Lag synchronization errors $e_{i3}(t)(i = 1, 2, \ldots, 12)$ with linear feedback control and $m = 2, \tau = 0.03$.

and the adaptive feedback controllers are

$$\begin{cases} u_i = -cd_i(t)e_i(t), \ \ i = 1, 2, 3, 4, \\ \dot{d}_i(t) = \xi_i e_i^T(t)e_i(t), \ \ i = 1, 2, 3, 4, \\ u_i = 0, \quad i = 5, \ldots, 100. \end{cases}$$

For this numerical example, we choose the coupling matrix $A = B$ as a scale-free network [1] with $m_0 = m = 8$. By simple calculations, $\lambda_{\max}(A_2) = -0.8588$, then the coupling strength satisfies $c > 13.315$, and we choose the coupling strength $c = 20$ and $\xi_i = 1, i = 1, \ldots, 4$ for simulation. We rearrange the node degrees from high to low and choose four nodes with the highest degree as the pinned nodes. The errors of lag synchronization of networks (6) are shown in Figures 4−6. In addition, for the coupling strength in $13.315 < c < 20$, the lag synchronization also happens with different synchronized time. Figure 7 gives the orbits of the updated feedback strength $d_i(t)(i = 1, 2, 3, 4)$.

In the above mentioned numerical examples, when the coupling strength c is fixed, we consider the effect of lag time τ on the synchronization. We randomly choose the values of τ in $(0.01, 2)$ for simulation and see that the lag synchronization also happens, which shows that the designed feedback controllers are robust.

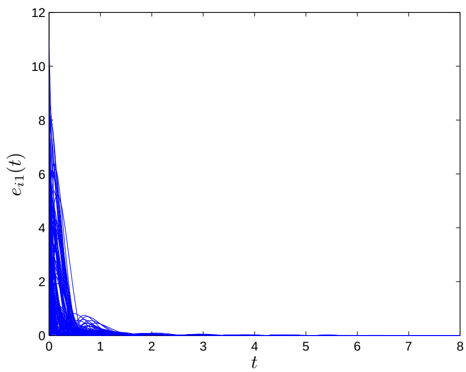

Fig. 4. Lag synchronization errors $e_{i1}(t)(i = 1, 2, \ldots, 100)$ with adaptive feedback control and $m = 4, \tau = 0.03$.

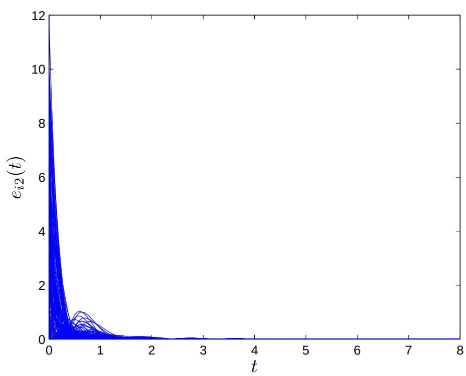

Fig. 5. Lag synchronization errors $e_{i2}(t)(i = 1, 2, \ldots, 100)$ with adaptive feedback control and $m = 4, \tau = 0.03$.

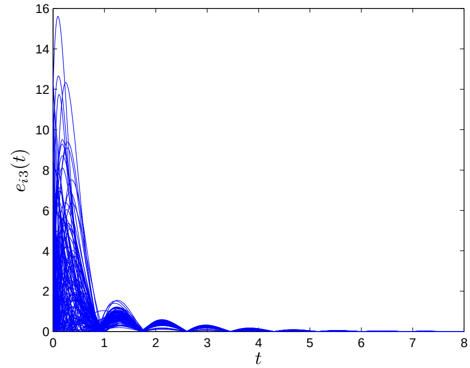

Fig. 6. Lag synchronization errors $e_{i3}(t)(i = 1, 2, \ldots, 100)$ with adaptive feedback control and $m = 4, \tau = 0.03$.

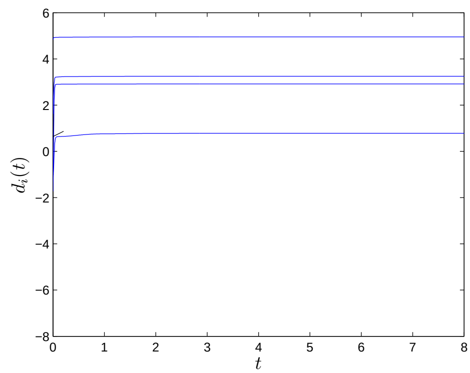

Fig. 7. Evolution of feedback gains $d_i(t)$ with $\xi_i = 1, i = 1, 2, 3, 4$.

5. CONCLUSIONS

In the present study, we have studied the lag synchronization between two dynamical networks with non-derivative and derivative couplings. By designing the pinning linear and adaptive feedback controllers, we have obtained the pinning conditions on the lag synchronization by Schur complement and Barbalat's lemma. The number of pinned nodes is related to the node dynamics and the coupling strength. For the linear feedback control, we have derived the domain of feedback gains. In the numerical examples, we have provided two numerical examples to show the efficiency of the designed controllers. Compared to the linear feedback control, the adaptive feedback control is better for realizing the lag synchronization. In the future, we will study the consensus dynamics of multi-agents systems [8, 13, 14] by pinning and optimal control.

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