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ON EUROPEAN OPTION PRICING UNDER PARTIAL INFORMATION

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Abstract. We consider a European option pricing problem under a partial information market, i.e., only the security's price can be observed, the rate of return and the noise source in the market cannot be observed. To make the problem tractable, we focus on gap option which is a generalized form of the classical European option. By using the stochastic analysis and filtering technique, we derive a Black-Scholes formula for gap option pricing with dividends under partial information. Finally, we apply filtering technique to solve a utility maximization problem under partial information through transforming the problem under partial information into the classical problem.

Keywords: option pricing; European option; partial information; backward stochastic differential equation

MSC 2010: 93E11, 60H10, 91B24

1. Introduction

Option pricing is one of the most important problems which has been widely studied. The Black-Scholes-Merton model [2], [15] for valuing European call and put options on an investment asset was published in 1973. It assumes the volatility of the asset is a constant and the price of the asset changes smoothly with no jumps. Since neither of these conditions is satisfied for exchange rates, individual stocks or stock indices, and no empirical evidence in financial industry shows that the geometric Brownian motion is suitable, one major extension of the Black-Scholes option pricing model is to overcome the drawbacks of the model. One renowned model is the well-known constant elasticity of variance (for short, CEV) diffusion

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model. This model was initially studied by Cox [3]. Then, Cox and Ross [4] designed and developed it to incorporate the negative correlation between underlying asset price change and volatility change. The CEV diffusion has been applied to exotic options as well as standard options by many researchers. However, it is found in Ballestra and Pacelli [1] that the CEV model does not offer a correct description of equity prices, whereas Emanuel and Macbeth [7] state that the CEV model with stationary parameters does not appear to be able to explain the mispricing of call options by the Black-Scholes model. Therefore, many researchers study the option pricing problem from other perspectives. We refer interested readers to Duffie [5], Karatzas [8], [9] and Merton [16] for detailed reviews of the option pricing and its extensions.

The common feature of previous studies for the option pricing problem is that they all assume investors can observe the drift process and the Brownian motion appearing in the stochastic differential equation for the security prices. However, it is more realistic to assume that investors have only partial information since prices and interest rates are published and available to the public, but drifts and paths of Brownian motions are merely mathematical tools for model creation, but certainly not observable. Therefore, we shall call this situation the case of partial information to distinguish it from the case of full information.

Under partial information, the utility maximization problem was for the first time considered by Lakner [10]. Later, Lakner [11] discussed the optimal trading strategy problem with partial information. Recently, Wu and Wang [18] studied an option pricing problem under partial information by using the convex analysis and the backward stochastic differential equation techniques. To our best knowledge, there are no studies to present a general result for European-type option pricing problem under partial information.

To fill this research gap, we study the gap option pricing problem with dividends under partial information, using stochastic analysis and filtering techniques. We find that the European-type option pricing problem is tractable. By using filtering technique, we derive a Black-Scholes formula for the gap option pricing problem with dividends under partial information. Further, we solve a utility maximization problem under partial information through transforming the problem under partial information into the classical problem by using the filtering technique.

2. Model formulation

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ be a complete probability space, where $t \in [0, T]$. We consider a market which consists of one risky and one risk-free asset with price processes S_t

and B_t, respectively. The price process B_t of risk-free asset satisfies

$$\mathrm{d}B_t = r_t B_t \,\mathrm{d}t \quad \text{if } B_t > 0, \tag{1}$$

$$\mathrm{d}B_t = -R_t B_t \,\mathrm{d}t \quad \text{if } B_t < 0, \tag{2}$$

with an initial condition $B_0 = b_0$ where b_0 is a constant, r_t and R_t are the lending and borrowing interest rates at time t, respectively. The dynamics of the stock's price process S_t is determined by the stochastic differential equations

$$\mathrm{d}S_t = \mu_t S_t \,\mathrm{d}t + \sigma_t S_t \,\mathrm{d}W_t, \quad S_0 = s_0 > 0, \tag{3}$$

where s_0 is a constant, μ_t and σ_t are the expected return and the volatility rate at time t, respectively. Since the expected return of the stock μ_t may not be observed directly, we suppose that it could be described as

$$\mathrm{d}\mu_t = a\mu_t \,\mathrm{d}t + b\,\mathrm{d}W_t + c\,\mathrm{d}V_t, \quad \mu_0 = \eta,$$

where $\{W_t\}_{t\geqslant 0}$ and $\{V_t\}_{t\geqslant 0}$ are independent 1-dimensional Brownian motions on the space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$. Further, we assume that a, b and c are constants, r_t, R_t, and σ_t are deterministic bounded functions and σ_t has a bounded inverse function.

Note that μ_t should be driven by another Wiener process which is different from the Wiener process W. Since μ_t is the expected return of the stock and S_t is driven by W_t, Wiener process W should affect both μ_t and S_t. Thus, we assume that μ_t is driven by both W_t and V_t.

For investors, both the stock price and the interest rate are published and available. However, drifts and all information of white noises $\{W_t\}_{t\geqslant 0}$ and $\{V_t\}_{t\geqslant 0}$ are certainly not observable. Thus we let

$$\mathcal{F}_t := \sigma\{W_s, V_s;\ 0 \leqslant s \leqslant t\}, \quad \mathcal{G}_t := \sigma\{S_u;\ 0 \leqslant u \leqslant t\},$$

and suppose that S_t is $\mathcal{G}_t$-adapted and both $\{W_t\}_{t\geqslant 0}$ and $\{V_t\}_{t\geqslant 0}$ are $\mathcal{F}$-adapted. Since drifts and all information of white noises are not observable, only $\mathcal{G}_t$-adapted processes are observable. Further, the decisions of investors mainly depend on the information of $\mathcal{G}_t$. Therefore, in order to get some ideas of the nature of partial information, we assume that only $\mathcal{G}_t$-adapted processes are observable, which implies investors cannot directly observe the drift process $\{\mu_t\}_{t\geqslant 0}$.

We assume that X_t represents the wealth of an agent at time t and the initial wealth $X_0 = x_0 > 0$ is a deterministic constant. Further, we define a trading strategy π_t for an agent acting in the market, i.e., the amount of money invested in the stock at time t, where π_t is a measurable, $\mathcal{G}_t$-adapted process such that

$E[\int_0^T \pi_t^2 \,\mathrm{d}t] < \infty$. The negative values of both π_t and $X_t - \pi_t$ are allowed, which indicates that the stock can be sold out and the agent could get a loan from a bank, respectively. We denote the dividend rate at time t by $d(t, S_t)$ where dividends of stock are assumed to be paid continuously and $d(t, S_t)$ is a bounded function.

Under the above assumptions, the wealth process $\{X_t;\ t \in [0, T]\}$ is assumed to evolve according to the dynamics

$$\mathrm{d}X_t = [r_t(X_t - \pi_t)^+ + (\mu_t + d(t, S_t))\pi_t - R_t(X_t - \pi_t)^-]\,\mathrm{d}t + \sigma_t \pi_t \,\mathrm{d}W_t, \quad X_0 = x_0,$$

which is equivalent to

$$\mathrm{d}X_t = [r_t X_t + (\mu_t + d(t, S_t) - r_t)\pi_t - (R_t - r_t)(X_t - \pi_t)^-]\,\mathrm{d}t + \sigma_t \pi_t \,\mathrm{d}W_t, \quad X_0 = x_0,$$

where $x^- = (|x| - x)/2$, $x^+ = (|x| + x)/2$, for all $x \in \mathbb{R}$. The explanation for the wealth process X_t is that, for any time t, if investors buy a stock with their own capital, i.e., $\pi_t \leqslant X_t$, then investors can get revenue from both the stock and the riskless asset, i.e., $r_t(X_t - \pi_t)^+ + (\mu_t + d(t, S_t))\pi_t$; otherwise, investors can buy stocks with borrowed money, i.e., $\pi_t \geqslant X_t$, then investors should pay for the cost of borrowed money, i.e., $R_t(X_t - \pi_t)^-$, and get revenue from the stock, i.e., $(\mu_t + d(t, S_t))\pi_t$.

Gap option is one of the most widely used options in the real world. For a striking price K and a predetermined price ξ which is a constant and irrelevant to K, the investor who has a gap call option can get $S_t - \xi$ when stock price is higher than K at time T. Otherwise, there is no revenue for investors. The situation of a gap put option is in the opposite. The value of gap option at expire time T is defined as

$$V(T) = \begin{cases} (S_T - \xi) I_{\{S_T > K\}}, & \text{call option}, \\ (\xi - S_T) I_{\{S_T \leqslant K\}}, & \text{put option}. \end{cases}$$

To derive the price of a gap option in the next section, the following lemma will be employed.

Lemma 2.1. *Consider a continuous system variable $x_t \in \mathbb{R}$ and a continuously observable variable $Z_t \in \mathbb{R}$ which satisfy*

$$\begin{cases} \mathrm{d}x_t = F(t) x_t \,\mathrm{d}t + G(t)\,\mathrm{d}W_t + C(t)\,\mathrm{d}U_t, \\ \mathrm{d}Z_t = x_t Z_t \,\mathrm{d}t + D(t) Z_t \,\mathrm{d}W_t \quad \forall\, t \in [0, T], \end{cases}$$

where $F(t)$, $G(t)$, $C(t)$, $D(t)$, $D^{-1}(t) \in \mathbb{R}$ are bounded, $\{U_t\}_{0 \leqslant t \leqslant T}$ and $\{W_t\}_{0 \leqslant t \leqslant T}$ are two independent 1-dimensional Brownian motions. If $Ex_0^4 < \infty$, x_0 and Z_0 are independent of the Wiener processes U and W, and the conditional distribution

$P(x_0 \leqslant x \mid Z_0)$ is Gaussian, $N(\widehat{x}_0, \Pi_0)$, where $\mathcal{Z}_t := \sigma\{Z_u;\ 0 \leqslant u \leqslant t\}$, then the solution of the filtering problem $\widehat{x}_t = E[x_t|\mathcal{Z}_t]$ satisfies the equations

$$\begin{cases} \mathrm{d}\widehat{x}_t = F(t)\widehat{x}_t\,\mathrm{d}t + \dfrac{G(t)D(t)+\Pi_t}{D^2(t)}\Big(\dfrac{\mathrm{d}Z_t}{Z_t} - \widehat{x}_t\,\mathrm{d}t\Big), \\ \widehat{x}_0 = E[x_0] \quad \forall\, t \in [0,T], \end{cases}$$

where $\Pi_t = E[(x_t - \widehat{x}_t)^2|\mathcal{Z}_t]$ satisfies the Ricatti equation

$$\begin{cases} \dfrac{\mathrm{d}\Pi_t}{\mathrm{d}t} = -\dfrac{\Pi_t^2}{D^2(t)} + 2\Big(F(t) - \dfrac{G(t)}{D(t)}\Big)\Pi_t + C^2(t), \\ \Pi_0 = E[(x_0 - \widehat{x}_0)^2] \quad \forall\, t \in [0,T]. \end{cases}$$

P r o o f. The proof is analogous to that of Theorem 12.2 in [13]. So we omit it here. □

3. Gap option pricing under partial information

To derive the price of a gap option, we first should discuss whether the contingent claim replication is possible. For an arbitrary contingent claim ζ, we consider the backward stochastic differential equation (for short, BSDE)

$$\begin{cases} \mathrm{d}X_t = [r_tX_t + (\mu_t + d(t,S_t) - r_t)\pi_t - (R_t - r_t)(X_t - \pi_t)^-]\,\mathrm{d}t + \sigma_t\pi_t\,\mathrm{d}W_t, \\ X_T = \zeta, \end{cases} \tag{4}$$

where ζ is an arbitrary F_T-measurable process such that $E|\zeta^2| < \infty$. Let $Z_t = \sigma_t\pi_t$ and

$$h(t,X_t,Z_t) = r_tX_t + (\mu_t + d(t,S_t) - r_t)\sigma_t^{-1}Z_t - (R_t - r_t)(X_t - \sigma_t^{-1}Z_t)^-, \tag{5}$$

then

$$\begin{cases} \mathrm{d}X_t = h(t,X_t,Z_t)\,\mathrm{d}t + Z_t\,\mathrm{d}W_t, \\ X_T = \zeta. \end{cases} \tag{6}$$

Lemma 3.1. *For any* ζ, *the BSDE* (6) *admits a unique adapted solution* $(X(\cdot), Z(\cdot))$.

Proof. By Theorem 4.1 of [14], we have that (6) admits a unique adapted solution if there exists a constant L which satisfies the inequalities

$$
\begin{cases} |h(t, X_t, Z_t) - h(t, \overline{X}_t, \overline{Z}_t)| \leqslant L(|X_t - \overline{X}_t| + |Z_t - \overline{Z}_t|), \\ |h(t, 0, 0)| \leqslant L, \quad t \in [0, T]. \end{cases} \tag{7}
$$

It follows from (5) that

$$
\begin{aligned}
|h(t, X_t, Z_t) - h(t, \overline{X}_t, \overline{Z}_t)| &= |r_t(X_t - \overline{X}_t) + (\mu_t + d(t, S_t) - r_t)\sigma_t^{-1}(Z_t - \overline{Z}_t) \\
&\quad - (R_t - r_t)((X_t - \sigma_t^{-1} Z_t)^- - (\overline{X}_t - \sigma_t^{-1}\overline{Z}_t)^-)| \\
&= \Big|\frac{1}{2}(R_t + r_t)(X_t - \overline{X}_t) + \Big(\mu_t + d(t, S_t) - \frac{1}{2}(R_t + r_t)\Big)\sigma_t^{-1}(Z_t - \overline{Z}_t) \\
&\quad - \frac{1}{2}(R_t - r_t)(|X_t - \sigma_t^{-1} Z_t| - |\overline{X}_t - \sigma_t^{-1}\overline{Z}_t|)\Big| \\
&\leqslant \Big|\frac{1}{2}(R_t + r_t)(X_t - \overline{X}_t) + \Big(\mu_t + d(t, S_t) - \frac{1}{2}(R_t + r_t)\Big)\sigma_t^{-1}(Z_t - \overline{Z}_t)\Big| \\
&\quad + \frac{1}{2}|(R_t - r_t)||(X_t - \overline{X}_t) + \sigma_t^{-1}(Z_t - \overline{Z}_t)| \\
&\leqslant \frac{1}{2}(|(R_t + r_t)| + |(R_t - r_t)|)|X_t - \overline{X}_t| + |\sigma_t^{-1}|\Big(|\mu_t + d(t, S_t) \\
&\quad - \frac{1}{2}(R_t + r_t)| + \frac{1}{2}|(R_t - r_t)|\Big)|Z_t - \overline{Z}_t| \\
&\leqslant R_t|X_t - \overline{X}_t| + |\sigma_t^{-1}|(|\mu_t| + d(t, S_t) + R_t)|Z_t - \overline{Z}_t|
\end{aligned}
$$

and

$$
|h(t, 0, 0)| = 0.
$$

Let $L = \max\{R_t, |\sigma_t^{-1}|(|\mu_t| + d(t, S_t) + R_t)\}$. Since σ_t^{-1}, μ_t and $d(t, S_t)$ are bounded and λ, ψ are constants, (7) holds. Therefore, BSDE (6) admits a unique adapted solution $(X(\cdot), Z(\cdot))$. This completes the proof. $\square$

Lemma 3.1 implies that BSDE(4) has a unique adapted solution $(X(\cdot), \sigma(\cdot)\eta(\cdot))$. As σ_t is a deterministic bounded function, $(X(\cdot), \eta(\cdot))$ also is an adapted solution. Since the contingent claim ζ is arbitrary, the market is complete and the price of the gap call option satisfies the BSDE

$$
\begin{cases} \mathrm{d}X_t = [r_t X_t + (\mu_t + d(t, S_t) - r_t)\pi_t - (R_t - r_t)(X_t - \pi_t)^-]\,\mathrm{d}t + \sigma_t \pi_t\,\mathrm{d}W_t, \\ X_T = (S_T - \xi) I_{\{S_T > K\}}. \end{cases} \tag{8}
$$

Since neither $\{W_t\}_{t\in[0,T]}$ nor $\{V_t\}_{t\in[0,T]}$ are observable, we cannot observe $\{\mu_t\}_{t\in[0,T]}$ directly. However, we could employ the filtering technique to observe $\{\mu_t\}_{t\in[0,T]}$ through the information of $\{S_t\}_{t\in[0,T]}$. Then we have the following result.

Theorem 3.1. *Under partial information, the price process of a gap call option* $\{X_t;\ t\in[0,T]\}$ *is*

$$X_t = \operatorname{ess\,sup}\{X_t^\delta;\ r_t \leqslant \delta_t \leqslant R_t\},$$

where

$$X_t^\delta = E_Q\Big[\exp\Big\{-\int_t^T \delta_s\,\mathrm{d}s\Big\}(S_T-\xi)I_{\{S_T>K\}}|\mathcal{G}_t\Big]$$

and $\mathbb{Q}$ *is a risk neutral probability measure which satisfies*

$$\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}} = \exp\Big\{\int_0^T -\frac{(\widehat{\mu}_t + d(t,S_t)-\delta_t)^2}{2\sigma_t^2}\,\mathrm{d}t - \int_0^T \frac{\widehat{\mu}_t + d(t,S_t)-\delta_t}{\sigma_t}\,\mathrm{d}\overline{W}_t\Big\}$$

and

$$\overline{W}_t = W_t + \int_0^t \frac{\mu_s-\widehat{\mu}_s}{\sigma_s}\,\mathrm{d}s.$$

Proof. Let

$$\widehat{\mu}_t = E[\mu_t|\mathcal{G}_t], \quad \gamma_t = E[(\mu_t-\widehat{\mu}_t)^2|\mathcal{G}_t].$$

By Theorem 7.17 of [12], there exists a standard Brownian motion $\{\overline{W}_t\}_{t\in[0,T]}$ on the space $(\Omega,\mathcal{G},\mathcal{G}_t,\mathbb{P})$ which satisfies

$$\mathrm{d}S_t = \widehat{\mu}_t S_t\,\mathrm{d}t + \sigma_t S_t\,\mathrm{d}\overline{W}_t. \tag{9}$$

From Lemma 2.1 we have

$$\begin{cases} \mathrm{d}\widehat{\mu}_t = a\widehat{\mu}_t\,\mathrm{d}t + \dfrac{b\sigma_t+\gamma_t}{\sigma_t^2}\Big(\dfrac{\mathrm{d}S_t}{S_t} - \widehat{\mu}_t\,\mathrm{d}t\Big), \\ \widehat{\mu}_0 = E[\mu_0] \quad \forall\, t\in[0,T], \end{cases} \tag{10}$$

and γ_t is the unique solution of the Riccati equation

$$\begin{cases} \dfrac{\mathrm{d}\gamma_t}{\mathrm{d}t} = 2a\gamma_t + b^2 + c^2 - \Big(b+\dfrac{\gamma_t}{\sigma_t}\Big)^2, \\ \gamma_0 = E[(\mu_0-\widehat{\mu}_0)^2] \quad \forall\, t\in[0,T]. \end{cases} \tag{11}$$

Solving (11), we have

$$\gamma_t = \frac{\beta_1 - L\beta_2\exp((\beta_2-\beta_1)\sigma_t^{-2}t)}{1-L\exp((\beta_2-\beta_1)\sigma_t^{-2}t)}, \tag{12}$$

where

$$\begin{aligned}
\beta_1 &= \Big(a - \frac{b}{\sigma_t}\Big)\sigma_t^2 - \sigma_t\sqrt{\Big(a - \frac{b}{\sigma_t}\Big)\sigma_t^2 + c^2},\\
\beta_2 &= \Big(a - \frac{b}{\sigma_t}\Big)\sigma_t^2 + \sigma_t\sqrt{\Big(a - \frac{b}{\sigma_t}\Big)\sigma_t^2 + c^2},\\
L &= \frac{\gamma_0 - \beta_1}{\gamma_0 - \beta_2}.
\end{aligned}$$

It follows from (10) and (12) that

$$\widehat{\mu}_t = \widehat{\mu}_0 \mathrm{e}^{at} + \int_0^t \mathrm{e}^{a(t-s)}\Big(b + \frac{\gamma_s}{\sigma_s}\Big)\,\mathrm{d}\overline{W}_s.$$

By (1), (8), and (9), the price of a gap call option satisfies

$$\begin{cases} \mathrm{d}X_t = [r_t X_t + (\widehat{\mu}_t + d(t,S_t) - r_t)\pi_t - (R_t - r_t)(X_t - \pi_t)^-]\,\mathrm{d}t + \sigma_t\pi_t\,\mathrm{d}\overline{W}_t,\\ X_T = (S_T - \xi)I_{\{S_T > K\}}. \end{cases}$$

Let

$$\begin{gathered} Y_t = \sigma_t \pi_t, \quad \theta_t = \sigma_t^{-1}(\widehat{\mu}_t + d(t,S_t) - r_t),\\ b(t,X,Y) = -[r_t X_t + \theta_t Y_t - (R_t - r_t)(X_t - \pi_t)^-]. \end{gathered}$$

Then we have

$$\begin{cases} -\mathrm{d}X_t = -[r_t X_t + \theta_t Y_t - (R_t - r_t)(X_t - \pi_t)^-]\,\mathrm{d}t - Y_t\,\mathrm{d}\overline{W}_t,\\ X_T = (S_T - \xi)I_{\{S_T > K\}} \quad \forall\, t \in [0,T]. \end{cases} \tag{13}$$

Since (13) is a nonlinear BSDE, by using the method of variational formulation of the price system in [6], we have

$$b(t,X_t,Y_t) = \sup\{b^\delta(t,X_t,Y_t);\ r_t \leqslant \delta_t \leqslant R_t\}, \tag{14}$$

where

$$b^\delta(t,X_t,Y_t) = -\delta_t X_t - \theta_t Y_t - \frac{r_t - \delta_t}{\sigma_t} Y_t = -\delta_t X_t - \sigma_t^{-1}(\widehat{\mu}_t + d(t,S_t) - \delta_t)Y_t.$$

By Proposition 3.6 and the BSDE comparison theorem in [6], the solution of BSDE (13) is

$$X_t = \operatorname{ess\,sup}\{X_t^\delta;\ r_t \leqslant \delta_t \leqslant R_t\}, \tag{15}$$

where X_t^δ satisfies the equation

$$\begin{cases} -\mathrm{d}X_t^\delta = -[\delta_t X_t^\delta + \sigma_t^{-1}(\widehat{\mu}_t + d(t,S_t) - \delta_t)Y_t^\delta]\,\mathrm{d}t - Y_t^\delta\,\mathrm{d}\overline{W}_t, \\ X_T^\delta = (S_T - \xi)I_{\{S_T > K\}} \quad \forall\, t \in [0,T]. \end{cases} \tag{16}$$

The adjoint process of (16) is

$$\begin{cases} \mathrm{d}\Gamma_s^t = -\Gamma_s^t[\delta_s\,\mathrm{d}s + \sigma_s^{-1}(\widehat{\mu}_s + d(t,S_s) - \delta_s)\,\mathrm{d}\overline{W}_s], \\ \Gamma_t^t = 1 \quad \forall\, s \in [t,T]. \end{cases}$$

Thus

$$\Gamma_s^t = \exp\left\{\int_t^s -\delta_v - \frac{(\widehat{\mu}_v + d(v,S_v) - \delta_v)^2}{2\sigma_v^2}\,\mathrm{d}v - \int_t^s \frac{\widehat{\mu}_v + d(v,S_v) - \delta_v}{\sigma_v}\,\mathrm{d}\overline{W}_v\right\}.$$

Further, we have

$$\begin{aligned} X_t^\delta = E[\Gamma_T^t X_T^\delta | \mathcal{G}_t] = E\bigg[\exp\bigg\{\int_t^T -\delta_s - \frac{(\widehat{\mu}_s + d(s,S_s) - \delta_s)^2}{2\sigma_s^2}\,\mathrm{d}s \\ - \int_t^T \frac{\widehat{\mu}_s + d(s,S_s) - \delta_s}{\sigma_s}\,\mathrm{d}\overline{W}_s\bigg\}(S_T - \xi)I_{\{S_T > K\}} | \mathcal{G}_t\bigg]. \end{aligned} \tag{17}$$

Since $d(t,S_t), \delta_t, \sigma_t$ and σ_t^{-1} are bounded, it is easy to verify that $\widehat{\mu}_t$ is a Gaussian process and satisfies

$$\sup_{0\leqslant t\leqslant T} E\widehat{\mu}_t < \infty, \qquad \sup_{0\leqslant t\leqslant T} E\widehat{\mu}_t^2 < \infty, \qquad \sup_{0\leqslant t\leqslant T} D\widehat{\mu}_t < \infty.$$

Further, we have

$$E\left[\exp\left\{\int_0^T -\frac{(\widehat{\mu}_t + d(t,S_t) - \delta_t)^2}{2\sigma_t^2}\,\mathrm{d}t - \int_0^T \frac{\widehat{\mu}_t + d(t,S_t) - \delta_t}{\sigma_t}\,\mathrm{d}\overline{W}_t\right\}\right] = 1.$$

It follows from the Girsanov theorem (see, for example, Theorem 8.26 of [17]) that

$$\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}} = \exp\left\{\int_0^T -\frac{(\widehat{\mu}_t + d(t,S_t) - \delta_t)^2}{2\sigma_t^2}\,\mathrm{d}t - \int_0^T \frac{\widehat{\mu}_t + d(t,S_t) - \delta_t}{\sigma_t}\,\mathrm{d}\overline{W}_t\right\},$$

where $\mathbb{Q}$ is a risk neutral probability measure, and

$$\widehat{W}_t = \overline{W}_t + \int_0^t \frac{\widehat{\mu}_s + d(t,S_s) - \delta_s}{\sigma_s}\,\mathrm{d}s \tag{18}$$

is a standard Brownian motion under the probability measure $\mathbb{Q}$. From (9) and (16), we have

$$\mathrm{d}S_t = (\delta_t - d(t,S_t))S_t\,\mathrm{d}t + \sigma_t S_t\,\mathrm{d}\widehat{W}_t,$$

which is equivalent to

$$S_T = S_t \exp\left\{\left[\delta_t - d(t,S_t) - \frac{1}{2}\sigma_t^2\right](T-t) + \sigma_t(\widehat{W}_T - \widehat{W}_t)\right\},$$

and then

$$\begin{cases} -\mathrm{d}X_t^\delta = -[\delta_t X_t^\delta]\,\mathrm{d}t - Y_t^\delta\,\mathrm{d}\widehat{W}_t, \\ X_T^\delta = (S_T - \xi)I_{\{S_T>K\}}. \end{cases}$$

The adjoint process of the above BSDE is

$$\begin{cases} \mathrm{d}\Gamma_s^t = -\Gamma_s^t\delta_s\,\mathrm{d}s, \\ \Gamma_t^t = 1 \quad \forall\, s\in[t,T]. \end{cases}$$

This gives

$$\Gamma_s^t = \exp\left\{\int_t^s -\delta_v\,\mathrm{d}v\right\}.$$

Then we have

$$X_t^\delta = E_Q[\Gamma_T^t X_T^\delta|\mathcal{G}_t] = E_Q\left[\exp\left\{-\int_t^T \delta_s\,\mathrm{d}s\right\}(S_T-\xi)I_{\{S_T>K\}}|\mathcal{G}_t\right]. \tag{19}$$

This completes the proof. □

Corollary 3.1. *When all parameters are constants, the price of a gap call option with dividends under partial information is*

$$X_t = \mathrm{e}^{-d(T-t)}S_t N(d_1^R(S_t)) - \xi\mathrm{e}^{-R(T-t)}N(d_0^R(S_t)),$$

where $d(t,S_t)=d$, $\delta_t=\delta$, $\sigma_t=\sigma$ *are constants and*

$$\begin{aligned} d_0^\delta(S_t) &= \frac{1}{\sigma\sqrt{T-t}}\ln\left(\frac{S_t}{K\mathrm{e}^{-(\delta-d)(T-t)}}\right) - \frac{1}{2}\sigma\sqrt{T-t}, \\ d_1^\delta(S_t) &= d_0^\delta(S_t) + \sigma\sqrt{T-t}, \\ \{S_T>K\} &= \left\{-\frac{\widehat{W}_T-\widehat{W}_t}{\sqrt{T-t}} < d_0^\delta(S_t)\right\}, \\ N(x) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^x \mathrm{e}^{-z^2/2}\,\mathrm{d}z. \end{aligned}$$

P r o o f. From (19), we have

$$
\begin{aligned}
X_t^\delta &= E_{\mathbb{Q}}\Big[\exp\Big\{-\int_t^T \delta\,\mathrm{d}s\Big\}(S_T-\xi)I_{\{S_T>K\}}|\mathcal{G}_t\Big]\\
&= E_{\mathbb{Q}}[\exp\{-\delta(T-t)\}(S_T-\xi)I_{\{S_T>K\}}|\mathcal{G}_t]\\
&= E_{\mathbb{Q}}[\mathrm{e}^{-\delta(T-t)}S_T I_{\{S_T>K\}}|\mathcal{G}_t] - \xi\mathrm{e}^{-\delta(T-t)}E_{\mathbb{Q}}[I_{\{S_T>K\}}]\\
&= -\xi\mathrm{e}^{-\delta(T-t)}\mathbb{Q}(I_{\{S_T>K\}}) + \mathrm{e}^{-d(T-t)}S_t E_{\mathbb{Q}}[\mathrm{e}^{\sigma(\widehat{W}_T-\widehat{W}_t)-1/2\sigma^2(T-t)}I_{\{S_T>K\}}]\\
&= -\xi\mathrm{e}^{-\delta(T-t)}N(d_0^\delta(S_t))\\
&\quad+\mathrm{e}^{-d(T-t)}S_t\int_{-\infty}^{\infty}\mathrm{e}^{-\sigma\sqrt{T-t}x-1/2\sigma^2(T-t)}I_{\{x<d_0^\beta(S_t)\}}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-x^2/2}\,\mathrm{d}x\\
&= -\xi\mathrm{e}^{-\delta(T-t)}N(d_0^\delta(S_t)) + \mathrm{e}^{-d(T-t)}S_t\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-1/2(x+\sigma\sqrt{T-t})^2}I_{\{x<d_0^\beta(S_t)\}}\,\mathrm{d}x\\
&= -\xi\mathrm{e}^{-\delta(T-t)}N(d_0^\delta(S_t)) + \mathrm{e}^{-d(T-t)}S_t\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-1/2y^2}I_{\{y<d_1^\beta(S_t)\}}\,\mathrm{d}y\\
&= -\xi\mathrm{e}^{-\delta(T-t)}N(d_0^\delta(S_t)) + \mathrm{e}^{-d(T-t)}S_t\int_{-\infty}^{d_1^\beta(S_t)}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-1/2y^2}\,\mathrm{d}y\\
&= \mathrm{e}^{-d(T-t)}S_tN(d_1^\delta(S_t)) - \xi\mathrm{e}^{-\delta(T-t)}N(d_0^\delta(S_t))
\end{aligned}
$$

and

$$\frac{\partial X_t^\delta}{\partial\delta} = \xi(T-t)\mathrm{e}^{-(\delta-\alpha)(T-t)}N(d_0^\delta(S_t)) > 0, \tag{20}$$

which implies that X_t^δ increases with respect to δ. By (15) and (20), the option price at time t is

$$X_t = \mathrm{e}^{-d(T-t)}S_tN(d_1^R(S_t)) - \xi\mathrm{e}^{-R(T-t)}N(d_0^R(S_t)). \tag{21}$$

This completes the proof. □

If $d=0$ and $R_t=r_t=r$, then (21) reduces to the classical gap call option pricing formula, i.e.,

$$X_t = S_tN(d_1(S_t)) - \xi\mathrm{e}^{-r(T-t)}N(d_0(S_t)),$$

where

$$
\begin{aligned}
d_0(S_t) &= \frac{1}{\sigma\sqrt{T-t}}\Big\{\ln\frac{S_t}{K} + \Big(r-\frac{1}{2}\sigma^2\Big)(T-t)\Big\},\\
d_1(S_t) &= d_0(S_t) + \sigma\sqrt{T-t}.
\end{aligned}
$$

In the same way, we can get the gap put option pricing formula under partial information either.

4. Optimal trading strategy under partial information

In this section, we consider the optimal trading strategy under partial information. Let x_0 be the initial wealth at time 0 of an agent. The investor wants to select a portfolio in order to maximize the expected utility on the finite time interval $[0,T]$.

Definition 4.1. A trading strategy $\pi = \{\pi(t) = (\pi_1(t),\ldots,\pi_N(t));\ 0 \leqslant t \leqslant T\}$ is an N-dimensional, measurable, $\mathcal{G}$-adapted process such that

$$E\int_0^T \pi_t^2 \,\mathrm{d}t < \infty,$$

where $\mathcal{G}$ is the augmented filtration generated by the price process S_t.

Definition 4.2. A trading strategy π is called admissible if $X_t^{x_0,\pi} \geqslant 0$, a.s. $t \in [0,T]$.

Definition 4.3. A function $U\colon\ [0,\infty) \to \mathcal{R} \cup \{-\infty\}$ is called a utility function if it is continuous, strictly increasing, strictly concave on its domain, continuously differentiable on $[0,\infty)$ with derivative function $U'(\cdot)$ satisfying the relation $\lim\limits_{x\to\infty} U'(x) = 0$.

Our optimization problem is to maximize the expected utility from the terminal wealth, i.e.,

$$\max_{\{\pi_t\}_{0\leqslant t\leqslant T}} E[U(X_T^{x_0,\pi})] \tag{22}$$

over all admissible trading strategies. To find an optimal solution for the above problem, we can use both the information of the stock price and the optimal estimation of μ_t which was obtained by using the filtering technique.

From the preceding section, we have

$$\begin{cases} \mathrm{d}X_t^{x_0,\pi} = [r_t X_t^{x_0,\pi} + (\widehat{\mu}_t + d(t,S_t) - r_t)\pi_t \\ \qquad\qquad -(R_t - r_t)(X_t^{x_0,\pi} - \pi_t)^-]\,\mathrm{d}t + \sigma_t\pi_t\,\mathrm{d}\overline{W}_t, \\ \mathrm{d}S_t = \widehat{\mu}_t S_t\,\mathrm{d}t + \sigma_t S_t\,\mathrm{d}\overline{W}_t, \\ \mathrm{d}\widehat{\mu}_t = a\widehat{\mu}_t\,\mathrm{d}t + \dfrac{b\sigma_t + \gamma_t}{\sigma_t}\,\mathrm{d}\overline{W}_t \quad \forall\, t \in [0,T], \end{cases} \tag{23}$$

where $\{\overline{W}_t\}_{t\in[0,T]}$ and $\{\widehat{\mu}_t\}_{t\in[0,T]}$ are observable.

In an uncertain market, besides the expected profit, investors focus more on risk or potential loss. In view of this, we consider the Constant Relative Risk Aversion (CRRA) utility function which is defined as

$$U(x) = \begin{cases} \dfrac{1}{\nu}x^\nu & \text{if } \nu < 0, \\ \ln x & \text{if } \nu = 0. \end{cases}$$

First, we consider the case of $\nu < 0$, then the utility function is

$$U(x) = \frac{1}{\nu} x^{\nu}. \tag{24}$$

By using the Itô formula, we have

$$\begin{aligned} E\Big(\frac{1}{\nu}(X_T^{x_0,\pi})^{\nu}\Big) &= \frac{1}{\nu}x^{\nu} + \frac{1}{2} E\int_0^T (\nu-1)(X_t^{x_0,\pi})^{\nu-2}\sigma_t^2\pi_t^2\,\mathrm{d}t \\ &\quad + E\int_0^T (X_t^{x_0,\pi})^{\nu-1}[r_t X_t^{x_0,\pi} + (\widehat{\mu}_t + d(t,S_t) - r_t)\pi_t \\ &\quad - (R_t - r_t)(X_t^{x_0,\pi} - \pi_t)^-]\,\mathrm{d}t \\ &= \frac{1}{\nu}x^{\nu} + E\int_0^T (X_t^{x_0,\pi})^{\nu} r_t\,\mathrm{d}t \\ &\quad - E\int_0^T \frac{1}{(X_t^{x_0,\pi})^{2-\nu}}\Big[X_t^{x_0,\pi}[(r_t - \widehat{\mu}_t - d(t,S_t))\pi_t \\ &\quad + (R_t - r_t)(X_t^{x_0,\pi} - \pi_t)^-] + \frac{1}{2}(1-\nu)\sigma_t^2\pi_t^2\Big]\,\mathrm{d}t. \end{aligned} \tag{25}$$

Theorem 4.1. *Under partial information, the optimal trading strategy for* (22) *and* (24) *is*

$$\pi_t = \frac{X_t^{x_0,\pi}}{\sigma_t^2(\nu-1)}\Big[(r_t - \mu_t - d(t,S_t)) + (R_t - r_t)\frac{1-\operatorname{sgn}(X_t^{x_0,\pi} - \pi_t)}{2}\Big],$$

where $\operatorname{sgn}(\cdot)$ *is the sign function.*

P r o o f. In order to maximize the expected utility from the terminal wealth, we should maximize (25), i.e., minimize the following term:

$$M(\pi_t) := X_t^{x_0,\pi}[(r_t - \widehat{\mu}_t - d(t,S_t))\pi_t + (R_t - r_t)(X_t^{x_0,\pi} - \pi_t)^-] + \frac{1}{2}(1-\nu)\sigma_t^2\pi_t^2.$$

Let $\mathrm{d}M(\pi_t)/\mathrm{d}\pi_t = 0$. Then we have

$$X_t^{x_0,\pi}\Big[(r_t - \widehat{\mu}_t - d(t,S_t)) + (R_t - r_t)\frac{1-\operatorname{sgn}(X_t^{x_0,\pi} - \pi_t)}{2}\Big] + (1-\nu)\sigma_t^2\pi_t = 0,$$

which implies

$$\pi_t = \frac{X_t^{x_0,\pi}}{\sigma_t^2(\nu-1)}\Big[(r_t - \widehat{\mu}_t - d(t,S_t)) + (R_t - r_t)\frac{1-\operatorname{sgn}(X_t^{x_0,\pi} - \pi_t)}{2}\Big].$$

This completes the proof. $\square$

Corollary 4.1. *If both the borrowing and lending interest rates are identical, then the optimal trading strategy under partial information is*

$$\pi_t = \frac{X_t^{x_0,\pi}}{\sigma_t^2(\nu-1)}(r_t - \widehat{\mu}_t - d(t,S_t)).$$

Next, we consider the case of $\nu = 0$. In this case, the utility function is also called the logarithmic utility function, i.e.,

$$U(x) = \ln x. \tag{26}$$

By using the Itô formula, we have

$$\begin{aligned} E(\ln(X_T^{x_0,\pi})) = {} & \ln x_0 + E\int_0^T r_t \,\mathrm{d}t - E\int_0^T \frac{1}{(X_t^{x_0,\pi})^2}\Big[X_t^{x_0,\pi}[(r_t - \widehat{\mu}_t - d(t,S_t))\pi_t \\ & + (R_t - r_t)(X_t^{x_0,\pi} - \pi_t)^-] + \frac{1}{2}\sigma_t^2\pi_t^2\Big]\,\mathrm{d}t. \end{aligned}$$

Corollary 4.2. *Under partial information, the optimal trading strategy for* (22) *and* (26) *is*

$$\pi_t = \frac{X_t^{x_0,\pi}}{\sigma_t^2}\Big[(\widehat{\mu}_t + d(t,S_t) - r_t) - (R_t - r_t)\frac{1-\operatorname{sgn}(X_t^{x_0,\pi} - \pi_t)}{2}\Big],$$

where $\operatorname{sgn}(\cdot)$ *is the sign function. If both the borrowing and lending interest rates are identical, then the optimal trading strategy under partial information is*

$$\pi_t = \frac{X_t^{x_0,\pi}}{\sigma_t^2}(\widehat{\mu}_t + d(t,S_t) - r_t).$$

P r o o f. The proof is analogous to that of Theorem 4.1. So we omit it here. □

For both utility functions, the optimal trading strategies heavily depend on the filtered estimation of μ_t, i.e., $\widehat{\mu}_t$. Therefore, in the sequel, we compare the difference of the optimal expected utility between full information and partial information. To make the problem easy to analyze, we assume that all parameters are constant and both the borrowing and lending interest rates are identical, i.e., $r_t = R_t = r$, $\sigma_t = \sigma$, and $d(t,S_t) = 0$. The terminal wealth under full information is given as

$$\mathrm{d}\widetilde{X}_t^{x_0,\pi} = [r\widetilde{X}_t^{x_0,\pi} + (\mu_t - r)\pi_t]\,\mathrm{d}t + \sigma\pi_t\,\mathrm{d}W_t,$$

and the optimal trading strategy is

$$\pi_t = \frac{\widetilde{X}_t^{x_0,\pi}}{\sigma^2}(\mu_t - r).$$

Therefore, the optimal terminal wealth under full information is

$$\mathrm{d}\widetilde{X}_t^{x_0} = \left[r + \frac{(\mu_t - r)^2}{\sigma^2}\right]\widetilde{X}_t^{x_0}\,\mathrm{d}t + \frac{\mu_t - r}{\sigma}\widetilde{X}_t^{x_0}\,\mathrm{d}W_t.$$

Similarly, the optimal terminal wealth under partial information is

$$\mathrm{d}X_t^{x_0} = \left[r + \frac{(\widehat{\mu}_t - r)^2}{\sigma^2}\right]X_t^{x_0}\,\mathrm{d}t + \frac{\widehat{\mu}_t - r}{\sigma}X_t^{x_0}\,\mathrm{d}\overline{W}_t.$$

Under the logarithmic utility function, the difference of the optimal expected utility between full information and partial information is

$$|E\ln(\widetilde{X}_T^{x_0}) - E\ln(X_T^{x_0})| = \frac{1}{2\sigma^2}\left|E\int_0^T(\mu_s - r)^2\,\mathrm{d}s - E\int_0^T(\widehat{\mu}_s - r)^2\,\mathrm{d}s\right|. \tag{27}$$

For μ_t, the unique solution of SDE (3) is

$$\mu_t = \eta\mathrm{e}^{at} + \int_0^t b\mathrm{e}^{a(t-s)}\,\mathrm{d}W_s + \int_0^t c\mathrm{e}^{a(t-s)}\,\mathrm{d}V_s. \tag{28}$$

It is easy to verify that μ_t is Gaussian on the space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$. Then we have $E\mu_t = \widehat{\mu}_0\mathrm{e}^{at}$ and

$$E\mu_t^2 = (\gamma_0 + \widehat{\mu}_0^2)\mathrm{e}^{2at} + \int_0^t(b^2 + c^2)\mathrm{e}^{2a(t-s)}\,\mathrm{d}s,$$

where $\widehat{\mu}_0 = E(\mu_0) = E(\eta)$ and $\gamma_0 = E[(\mu_0 - \widehat{\mu}_0)^2] = E[(\mu_0 - E(\mu_0)^2]$.

For $\widehat{\mu}_t$, it follows from (9) and (10) that

$$\mathrm{d}\widehat{\mu}_t = a\widehat{\mu}_t\,\mathrm{d}t + \frac{b\sigma + \gamma_t}{\sigma}\,\mathrm{d}\overline{W}_t,$$

which implies that

$$\widehat{\mu}_t = \widehat{\mu}_0\mathrm{e}^{at} + \int_0^t\left(b + \frac{\gamma_s}{\sigma}\right)\mathrm{e}^{a(t-s)}\,\mathrm{d}\overline{W}_s. \tag{29}$$

Then, from (11) and (18), we have

$$\begin{aligned}\widehat{\mu}_t = \widehat{\mu}_0\mathrm{e}^{\int_0^t a-\sigma^{-1}(b+\gamma_s/\sigma)\,\mathrm{d}s} &+ \int_0^t\frac{r}{\sigma}\left(b + \frac{\gamma_s}{\sigma}\right)\mathrm{e}^{\int_s^t a-\sigma^{-1}(b+\gamma_v/\sigma)\,\mathrm{d}v}\,\mathrm{d}s\\ &+ \int_0^t\left(b + \frac{\gamma_s}{\sigma}\right)\mathrm{e}^{\int_s^t a-\sigma^{-1}(b+\gamma_v/\sigma)\,\mathrm{d}v}\,\mathrm{d}\widehat{W}_s,\end{aligned} \tag{30}$$

where

$$\gamma_t = \frac{\beta_1 - \beta_2 L\mathrm{e}^{(\beta_2-\beta_1)t/\sigma^2}}{1 - L\mathrm{e}^{(\beta_2-\beta_1)t/\sigma^2}}, \quad \beta_1 = \sigma(a\sigma - b - \sqrt{a\sigma^2 - b\sigma + c^2}),$$
$$\beta_2 = \sigma(a\sigma - b + \sqrt{a\sigma^2 - b\sigma + c^2}),$$
$$L = \frac{\gamma_0 - \beta_1}{\gamma_0 - \beta_2} \quad \text{and} \quad \overline{W}_t = W_t + \int_0^t \frac{\mu_s - \widehat{\mu}_s}{\sigma}\,\mathrm{d}s = \widehat{W}_t - \int_0^t \frac{\widehat{\mu}_s - r}{\sigma}\,\mathrm{d}s.$$

By (28) and (30), the difference of the optimal expected utility between full information and partial information (27) can be further written as

$$|E\ln(\widetilde{X}_T^{x_0}) - E\ln(X_T^{x_0})| = \frac{1}{8a^2\sigma^2}\bigg|(b^2 + c^2 + 2a\gamma_0)(\mathrm{e}^{2aT} - 1) - 2a(b^2 + c^2)T - 4a^2\int_0^T\int_0^s \Big(b + \frac{\gamma_s}{\sigma}\Big)^2 \mathrm{e}^{2a(s-v)}\,\mathrm{d}v\,\mathrm{d}s\bigg|.$$

Obviously, the difference is decreasing with respect to both σ and γ_0.

5. Conclusions

This paper addresses a gap option pricing problem with dividends under partial information. Since investors could not observe complete information in financial markets, it is more realistic to consider how to price financial derivatives under partial information. By using the filtering technique, a Black-Scholes formula for pricing a gap option is derived. Although we only focus on the gap option, this method could be applied to other European-type options. In the last part of this paper, we solve a utility maximization problem of an investor who wants to maximize the expected utility from the terminal value of his portfolio on the finite time interval $[0, T]$ under partial information. By using the filtering technique, the problem under partial information can be transformed into the classical problem.

Future studies can go one step further by adapting the proposed method to the valuation of American-type options. Moreover, it would be more interesting to generalize the model to discuss the portfolio selection problem under partial information.

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