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IMPROVED INTERVAL DEA MODELS WITH COMMON WEIGHT

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The traditional data envelopment analysis (DEA) model can evaluate the relative efficiencies of a set of decision making units (DMUs) with exact values. But it cannot handle imprecise data. Imprecise data, for example, can be expressed in the form of the interval data or mixtures of interval data and exact data. In order to solve this problem, this study proposes three new interval DEA models from different points of view. Two examples are presented to illustrate and validate these models.

Keywords: data envelopment analysis (DEA), interval data, interval DEA model, common weight

Classification: 90B50

1. INTRODUCTION

Data envelopment analysis (DEA), as a very useful management and decision tool, is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) with multiple inputs and multiple outputs [3]. It has been widely applied in various performance evaluation cases, such as the performance evaluation of R&D [4], evaluating and selecting investments in industrial robots [2], assessing computer numerical control machines [14], measuring production and marketing efficiency [15], evaluating the preferential voting system [1], the performance of medical centers [12], and so on. The original DEA models assume that all the data of inputs and outputs are known exactly. However, this assumption may not always be true. Due to the existence of uncertainty, the data may be given in the form of the interval data. Therefore, how to assess the efficiencies of DMUs with interval data is still worth researching.

The problem of the evaluation of DMUs with interval data has attracted attentions of some scholars. For example, Cooper et al. [5] may be the first to study the DEA models with imprecise data, and their model is called imprecise DEA (IDEA). It can be transformed into a linear programming problem from a nonlinear programming problem through a series of variable alternations and scale transformations. Kim et al. [11] also proposed a method through variable alternation and analogous scale transformation. But they did not consider the situation of interval data. Lee et al. [13] pointed out that IDEA model was complicated due to great numbers of variable alternations and data

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transformations. In the IDEA model, the numbers of decision variables increase from $(m+s)$ to $(m+s)\times n$, where m, s and n represent the numbers of inputs, outputs and DMUs, respectively. This may lead to a rapid increase in computation burden. Aiming at this problem, Despotis and Smirlis [6] developed two approaches for dealing with imprecise data. Their approaches were linear and can be used to obtain the lower and upper bound of the efficiency of each DMU. Based on this idea, Haghighat and Khorram [7] studied the problem of numbers of efficient DMUs. Jahanshahloo et al. [8, 9, 10] further extended this idea considering return to scale, sensitivity and stability analysis and so on. However, Wang et al. [16] pointed out that the efficiencies calculated by the models of Despotis and Smirlis were lack of the comparability. The reason is that two different production frontiers have been adopted when efficiency was measured. In order to deal with such an uncertain situation, they developed a new pair of DEA models to obtain the interval efficiency of each DMU. Then the interval efficiencies of all DMUs were ranked by a minimax regret-based approach.

It should be pointed out that the above models on dealing with interval data have a defect that two sets of weights are used to obtain the interval efficiency of each DMU, which may be unreasonable. The purpose of this paper is to solve this problem. Three different new models from different variations are suggested. Each model can obtain the efficiencies of the interval data with only a set of weights. The rest of the paper is organized as follows. Section 2 introduces the interval DEA models. Section 3 presents three new methods from different variations. Two illustrative examples are presented in Section 4. Conclusions and further remarks are given in Section 5.

2. INTERVAL DEA MODELS

Assume there are n DMUs to be evaluated. Each DMU has s different outputs and m different inputs, denoted as y_{rj} and x_{ij} , respectively. Due to the uncertainty, only their bounded interval $[x_{ij}^l, x_{ij}^u]$ and $[y_{rj}^l, y_{rj}^u]$, with $x_{ij}^l > 0$ and $y_{rj}^l > 0$, are known. In order to measure the efficiencies of the DMUs with interval data, Wang et al. [16] proposed the following two linear formulations to generate the bounded interval $[\theta_d^l, \theta_d^u]$.

$$\begin{aligned} &\max \theta_d^l = \sum_{r=1}^{s} \mu_{rd} y_{rd}^l \\ &s.t. \sum_{i=1}^{m} \omega_{id} x_{id}^l - \sum_{r=1}^{s} \mu_{rd} y_{rd}^u \geq 0, \quad j = 1, 2, \ldots, n \\ &\sum_{i=1}^{m} \omega_{id} x_{id}^u = 1 \\ &\omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall id, rd \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\max \theta_d^u = \sum_{r=1}^{s} \mu_{rd} y_{rd}^u \\ &s.t. \sum_{i=1}^{m} \omega_{id} x_{id}^l - \sum_{r=1}^{s} \mu_{rd} y_{rd}^u \geq 0, \quad j = 1, 2, \ldots, n \end{aligned} \tag{2}$$

$$\sum_{i=1}^{m} \omega_{id} x_{id}^{l} = 1$$
$$\omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall\, id, rd.$$

In the above two models, DMU_d is under evaluation, ω_{id} and μ_{rd} are the weights assigned to the inputs and outputs respectively. θ_d^l is the lower efficiency for DMU_d, θ_d^u is the upper efficiency. ε is the non-Archimedean infinitesimal.

3. NEW INTERVAL DEA MODELS

The main defect of the above models is that θ_d^l and θ_d^u of DMU_d are calculated by two different evaluation criterions. In other words, two sets of weights are used to obtain the lower and upper efficiencies for each DMU. Thus, the range of interval efficiency of each DMU will be larger than the reality. In order to overcome this problem, three new improved interval DEA models are proposed in this section.

3.1. New interval DEA models considering preference

If the decision maker considers that the lower bound of interval efficiency is more important, it should be calculated firstly by model (1). Then the upper bound of interval efficiency can be solved by model (3) as follows

$$\begin{aligned}
&\max E_d^u = \sum_{r=1}^{s} \mu_{rd} y_{rd}^{u} \\
&s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} - \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} \geq 0, \quad j = 1, 2, \ldots, n \\
&\theta_d^l * \sum_{r=1}^{s} \mu_{rd} y_{rd}^{l} - \sum_{i=1}^{m} \omega_{id} x_{id}^{u} = 0 \\
&\sum_{i=1}^{m} \omega_{id} x_{id}^{l} = 1 \\
&\omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall\, id, rd.
\end{aligned} \tag{3}$$

Note that in the model (3), θ_d^l is the lower bound of interval efficiency of model (1). This model aims to maximize the upper bound of interval efficiency when the efficiency of lower bound has been determined. Using these models preferring lower bound (model (1) and model (3)), bounded interval $[\theta_d^l, E_d^u]$ of DMU_d can be generated.

In a similar manner, if the decision maker considers the upper bound of interval efficiency is more important, the upper bound of interval efficiency of each DMU could be obtained firstly by model (2). Then the lower bound of interval efficiency can be solved by model (4) as follows:

$$
\begin{aligned}
&\max E_d^l = \sum_{r=1}^{s} \mu_{rd} y_{rd}^l \\
&s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^l - \sum_{r=1}^{s} \mu_{rd} y_{rj}^u \geq 0, \quad j = 1, 2, \ldots, n \\
&\theta_d^u * \sum_{r=1}^{s} \mu_{rd} y_{rd}^u - \sum_{i=1}^{m} \omega_{id} x_{id}^l = 0 \\
&\sum_{i=1}^{m} \omega_{id} x_{id}^u = 1 \\
&\omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall\, id, rd.
\end{aligned}
\tag{4}
$$

In model (4), θ_d^u is the upper bound of interval efficiency of DMU_d, which can be obtained by model (2). This model is to maximize the lower bound of interval efficiency when upper bound has obtained the maximum efficiency. Using this preference models (model (2) and model (4)), bounded interval $[E_d^l, \theta_d^u]$ of DMU_d can be generated. With regard to these models, we have the following theorems.

Theorem 3.1. If $\theta_d^{l^*}$ and $E_d^{u^*}$ are optimum objective function values of models (1) and (3), respectively, then $\theta_d^{l^*} \leq E_d^{u^*}$.

Proof. Comparing models (1) and (3), we know that the optimal weights of these two models are the same. Assume that ω_{id}^* and μ_{rd}^* are optimal solutions of the two models. For $x_{id}^u \geq x_{id}^l$ and $y_{rd}^u \geq y_{rd}^l$, then $\sum_{i=1}^{m} \omega_{id}^* x_{id}^u \geq \sum_{i=1}^{m} \omega_{id}^* x_{id}^l$ and $\sum_{r=1}^{m} \mu_{rd}^* y_{rd}^u \geq \sum_{r=1}^{s} \mu_{rd}^* y_{rd}^l$. Therefore, we have

$$
\frac{\sum_{r=1}^{s} \mu_{rd}^* y_{rd}^l}{\sum_{i=1}^{m} \omega_{id}^* x_{id}^u} \leq \frac{\sum_{r=1}^{s} \mu_{rd}^* y_{rd}^u}{\sum_{i=1}^{m} \omega_{id}^* x_{id}^l},
$$

namely, $\theta_d^{l^*} \leq E_d^{u^*}$. □

Theorem 3.2. If $E_d^{l^*}$ and $\theta_d^{l^*}$ are optimum objective function values of models (4) and (1), respectively, then $E_d^{l^*} \leq \theta_d^{l^*}$.

Proof. Comparing models (1) and (4), it is noted that the feasible regions of model (1) contains the feasible region of model (4), and two objective functions are the same. Let ω_{id}^{1*} and μ_{rd}^{1*} be optimal solutions of model (1), and ω_{id}^{2*} and μ_{rd}^{2*} be optimal solutions of model (4). Therefore, we have

$$
\frac{\sum_{r=1}^{s} \mu_{rd}^{2*} y_{rd}^l}{\sum_{i=1}^{m} \omega_{id}^{2*} x_{id}^u} \leq \frac{\sum_{r=1}^{s} \mu_{rd}^{1*} y_{rd}^l}{\sum_{i=1}^{m} \omega_{id}^{1*} x_{id}^u},
$$

namely, $E_d^{l^*} \leq \theta_d^{l^*}$. □

Theorem 3.3. If $\theta_d^{u^*}$ and $E_d^{l^*}$ are optimum objective function values of models (2) and (4), respectively, then $E_d^{l^*} \leq \theta_d^{u^*}$.

Proof. The proof is similar to that in theorem 3.1, and it is omitted here. □

Theorem 3.4. If $E_d^{u^*}$ and $\theta_d^{u^*}$ are optimum objective function values of models (3) and (2), respectively, then $E_d^{u^*} \leq \theta_d^{u^*}$.

Proof. The proof is similar to that in theorem 3.2, and it is omitted here. □

3.2. New interval DEA models without preferences

If the decision maker considers that the lower and upper bound of interval efficiency are equally important, then the above two models are not adequate. An alternative approach to measuring the lower and upper bound of interval efficiency is shown as follows

$$\begin{aligned} &\max F_d = \frac{\sum\limits_{r=1}^{s} \mu_{rd} y_{rd}^u}{\sum\limits_{i=1}^{m} \omega_{id} x_{id}^l} * \frac{\sum\limits_{r=1}^{s} \mu_{rd} y_{rd}^l}{\sum\limits_{i=1}^{m} \omega_{id} x_{id}^u} \\ &s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^l - \sum_{r=1}^{s} \mu_{rd} y_{rj}^u \geq 0, \quad j = 1, 2, \ldots, n \\ &\omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall\, id, rd. \end{aligned} \tag{5}$$

Theorem 3.5. The feasible set S of model (5) is compact and non-empty convex.

Proof. Since the feasible set S is bounded and closed in Euclidean space, then S is compact. Next we will prove that S is also convex.

It is obvious that S is non-empty. Now, assume both $(\omega'_{1d}, \ldots, \omega'_{md}, \mu'_{1d}, \ldots, \mu'_{md})$ and $(\omega^*_{1d}, \ldots, \omega^*_{md}, \mu^*_{1d}, \ldots, \mu^*_{md}) \in S$. For any $\beta \in [0, 1]$, we have $\beta\omega'_{id} + (1-\beta)\omega^*_{id} \geq \varepsilon$ and $\beta\mu'_{rd} + (1-\beta)\mu^*_{rd} \geq \varepsilon$. Then,

$$\begin{aligned} \sum_{i=1}^{m} [\beta\omega'_{id} + (1-\beta)\omega^*_{id}] x_{ij}^l &= \beta \sum_{i=1}^{m} \omega'_{id} x_{ij}^l + (1-\beta) \sum_{i=1}^{m} \omega^*_{id} x_{ij}^l \\ &\geq \beta \sum_{r=1}^{s} \mu'_{rd} y_{rj}^u + (1-\beta) \sum_{r=1}^{s} \mu^*_{rd} y_{rj}^u \\ &= \sum_{r=1}^{s} [\beta\mu'_{rd} + (1-\beta)\mu^*_{rd}] y_{rj}^l. \end{aligned} \tag{6}$$

From (6), we know that $[\beta\omega'_{id} + (1-\beta)\omega^*_{id}, \beta\mu'_{rd} + (1-\beta)\mu^*_{rd}] \in S$. Consequently S is a convex set. □

Theorem 3.6. For any DMU$_d$, $F_d^* \le \theta_d^{l^*} \times \theta_d^{u^*}$, where F_d^* s the (maximum) optimal value of model (5), $\theta_d^{l^*}$ and $\theta_d^{u^*}$ are the (maximum) optimal values of model (1) and (2), respectively.

Proof. Models (1) and (2) can be changed into the following two regular DEA models (7) and (8), respectively.

$$\begin{aligned} &\max \theta_d^l = \frac{\sum\limits_{r=1}^{s} \mu_{rd} y_{rd}^l}{\sum\limits_{i=1}^{m} \omega_{id} x_{id}^u} \\ &s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^l - \sum_{r=1}^{s} \mu_{rd} y_{rj}^u \ge 0, \quad j = 1, 2, \ldots, n \\ &\omega_{id}, \mu_{rd} \ge \varepsilon, \quad \forall id, rd \end{aligned} \tag{7}$$

and

$$\begin{aligned} &\max \theta_d^u = \frac{\sum\limits_{r=1}^{s} \mu_{rd} y_{rd}^u}{\sum\limits_{i=1}^{m} \omega_{id} x_{id}^l} \\ &s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^l - \sum_{r=1}^{s} \mu_{rd} y_{rj}^u \ge 0, \quad j = 1, 2, \ldots, n \\ &\omega_{id}, \mu_{rd} \ge \varepsilon, \quad \forall id, rd. \end{aligned} \tag{8}$$

By comparing the constraints in models (5), (7) and (8), we note that the feasible regions of three models are the same, objective functions are different. The objective function of model (5) includes the objective functions of model (7) and (8). Therefore, we have

$$\frac{\sum_{r=1}^{s} \mu_{rd}^* y_{rd}^u}{\sum_{i=1}^{m} \omega_{id}^* x_{id}^l} \le \theta_d^u \quad \text{and} \quad \frac{\sum_{r=1}^{s} \mu_{rd}^* y_{rd}^l}{\sum_{i=1}^{m} \omega_{id}^* x_{id}^u} \le \theta_d^l$$

(ω_{id}^* and μ_{rd}^* are optimal solutions of model (5)), and furthermore $F_d^* \le \theta_d^{l^*} \times \theta_d^{u^*}$.

Model (5) is a nonlinear programming, now let us describe how to calculate model (5). We know that

$$E_d^{u^*} \le \frac{\sum_{r=1}^{s} \mu_{rd} y_{rd}^u}{\sum_{i=1}^{m} \omega_{id} x_{id}^l} \le \theta_d^{u^*}$$

(see the above Theorems), $E_d^{u^*}$ and $\theta_d^{u^*}$ are the optimum objective function values of models (3) and (2), respectively. Thus, model (5) can be turned into following programming:

$$
\begin{aligned}
& \max k \times \frac{\sum\limits_{r=1}^{s} \mu_{rd} y_{rd}^{l}}{\sum\limits_{i=1}^{m} \omega_{id} x_{id}^{u}} \\
& s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} - \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} \geq 0, \quad j = 1, 2, \ldots, n \\
& \frac{\sum\limits_{r=1}^{s} \mu_{rd} y_{rd}^{u}}{\sum\limits_{i=1}^{m} \omega_{id} x_{id}^{l}} = k \\
& E_d^{u^*} \leq k \leq \theta_d^{u^*} \\
& \omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall\, id, rd.
\end{aligned} \tag{9}
$$

We point out that the DEA model (9) is equivalent to the following linear program.

$$
\begin{aligned}
& \max e_d^k = k \times \sum_{r=1}^{s} \mu_{rd} y_{rd}^{l} \\
& s.t. \sum_{i=1}^{m} \omega_{id} x_{ij}^{l} - \sum_{r=1}^{s} \mu_{rd} y_{rj}^{u} \geq 0, \quad j = 1, 2, \ldots, n \\
& \sum_{i=1}^{m} \omega_{id} x_{id}^{u} = 1 \\
& k * \sum_{i=1}^{m} \omega_{id} x_{id}^{l} - \sum_{r=1}^{s} \mu_{rd} y_{rd}^{u} = 0 \\
& E_d^{u^*} \leq k \leq \theta_d^{u^*} \\
& \omega_{id}, \mu_{rd} \geq \varepsilon, \quad \forall\, id, rd.
\end{aligned} \tag{10}
$$

In the model (10), k is treated as a parameter within $[E_d^{u^*}, \theta_d^{u^*}]$. As a result, model (10) can be solved as a parametric linear program according to searching over the possible k values within $[E_d^{u^*}, \theta_d^{u^*}]$. First, we set the initial value for k as the lower bound $E_d^{u^*}$, and calculate the corresponding linear program. Then we begin to increase k by a very small positive number $\delta = 0.0001$ (for example) for each step q (namely, $k_q = E_d^{u^*} + \delta q, q = 1, 2, \ldots$) until the upper bound $\theta_d^{u^*}$ is reached. When solving each linear program of model (10) corresponding to k_q, we will obtain an optimal objective function value and a set of weights. After solving out all k_q, we would get a set of objective function values. The maximum value and the corresponding weights are the optimal value and optimal solution of the model (10).

Set that ω_{id}^* and μ_{rd}^* are optimal solutions of model (10), which are also the optimal solutions of model (5). So in the interval DEA models without preferences, the lower bound of interval efficiency of DMU_d is

$$
\frac{\sum_{r=1}^{s} \mu_{rd}^* y_{rd}^{l}}{\sum_{i=1}^{m} \omega_{id}^* x_{id}^{u}},
$$

and the upper bound of interval efficiency is

$$\frac{\sum_{r=1}^{s} \mu_{rd}^{*} y_{rd}^{u}}{\sum_{i=1}^{m} \omega_{id}^{*} x_{id}^{l}}.$$

□

4. ILLUSTRATIONS

Two numerical examples are presented to illustrate the proposed methods. The data of the examples are in Tables 1 and 4, respectively.

4.1. A simple numerical example

A simple numerical example is shown in Table 1, which is used to illustrate the proposed method. There are 10 DMUs and each DMU has 2 inputs (X_1 and X_2) and 2 outputs (Y_1 and Y_2). The data of inputs and outputs are all given in the form of the interval.

DMU	X_1	X_2	Y_1	Y_2
1	[1,2]	[2,3]	[23,24]	[22,24]
2	[2,3]	[3,4]	[20,22]	[20,21]
3	[3,4]	[5,6]	[18,21]	[19,19]
4	[3,4]	[5,7]	[16,17]	[15,18]
5	[3,5]	[5,7]	[14,17]	[13,15]
6	[4,5]	[6,7]	[12,15]	[10,14]
7	[4,5]	[7,8]	[10,15]	[9,14]
8	[4,6]	[8,8]	[9,14]	[8,13]
9	[5,6]	[8,9]	[9,14]	[8,13]
10	[5,7]	[8,9]	[8,12]	[7,13]

Tab. 1. A simple numerical example.

The interval efficiency results of all DMUs considering preferences can be calculated by model (3) and (4). Next, we describe how to use model (5) to get interval efficiency of DMUs without preferences. Take DMU_{10} for example, through model (2) and model (3), the k values are from range of 0.125 to 0.135. Then we begin to increase k by a very small positive number $\delta = 0.0001$ for each step q (namely, $k_q = 0.125 + 0.0001 * q, q = 1, 2, \ldots$) until 0.135 is reached. After solving out all k_q, we would get a set of objective function values. The maximum value and the corresponding weights are 0.0093 and (0,0.1111,0.0093,0), respectively. By this set of weight, the lower and upper bound of interval efficiency of DMU_{10} are 0.074 and 0.125, respectively. Similarly, The interval efficiencies of other DMUs can be calculated. Table 2 reports the interval efficiency results of all DMUs from three models. From the table, it is found that the lower efficiencies of model (3) are all not less than ones of model (4). The upper efficiencies of model (4) are all not less than ones of model (3).

Table 3 shows the final efficiency results of all DMUs from three models. In this paper, the linear weighting method is used to aggregate the lower and upper efficiencies

DMU	Model 3	Model 4	Model 5
1	[0.639,1.000]	[0.639,1.000]	[0.639,1.000]
2	[0.417,0.611]	[0.417,0.611]	[0.417,0.611]
3	[0.264,0.317]	[0.250,0.350]	[0.250,0.350]
4	[0.191,0.283]	[0.179,0.300]	[0.191,0.283]
5	[0.167,0.283]	[0.167,0.283]	[0.167,0.283]
6	[0.143,0.208]	[0.143,0.208]	[0.143,0.208]
7	[0.104,0.179]	[0.104,0.179]	[0.104,0.179]
8	[0.094,0.146]	[0.094,0.146]	[0.094,0.146]
9	[0.083,0.146]	[0.083,0.146]	[0.083,0.146]
10	[0.074,0.125]	[0.065,0.135]	[0.074,0.125]

Tab. 2. The interval efficiency results of all DMUs from three models.

for obtaining the final efficiency for each DMU. If the decision maker considers that the lower/upper bound of interval efficiency is more important, a large weight should be assigned to it. Because model (3) prefers the lower bound, we give the weight 0.6 for lower efficiency and 0.4 for upper efficiency. Model (4) prefers the upper bound, and then the weight 0.6 is given to upper efficiency and 0.4 for lower efficiency. Model (5) does not have preferences, and then the weights for lower and upper efficiencies are both 0.5. From Table 3, we find that the final efficiency of each DMU obtained by model (4) is larger than its efficiency obtained by model (3) or model (5). The finall efficiency of each DMU obtained by the model (3) is smaller than the one obtained by other two models. These reveal that interval DEA models considering different preferences will obtain different efficiency results. Another finding is that the efficiency scores decrease gradually from DMU_1 to DMU_{10}. Through analyzing the data of Table 1, it is concluded that a DMU will get a higher overall efficiency if it produces more outputs with fewer inputs.

DMU	Model 3	Model 4	Model 5
1	0.783	0.856	0.820
2	0.495	0.533	0.514
3	0.285	0.310	0.300
4	0.228	0.251	0.237
5	0.213	0.237	0.225
6	0.169	0.182	0.176
7	0.134	0.149	0.141
8	0.115	0.125	0.120
9	0.108	0.121	0.114
10	0.095	0.107	0.100

Tab. 3. The efficiency results of all DMUs from three models.

MI	input		output
	Capital (10^4)	Labor (10^4)	Gross Output Value (10^4)
1	[56.4,62.2]	[67.4,74.3]	[80.7,86.6]
2	[61.4,67.0]	[68.6,74.2]	[91.8,98.5]
3	[76.2,79.8]	[76.2,80.6]	[111.7,119.6]
4	[86.2,93.7]	[78.0,84.6]	[120.6,126.1]
5	[101.7,108.3]	[80.0,87.7]	[138.1,146.3]
6	[116.4,126.8]	[80.7,88.9]	[149.8,165.3]
7	[173.2,181.6]	[81.8,89.6]	[170.2,181.3]

Tab. 4. The data of seven manufacturing industries.

4.2. Seven manufacturing industries

An example of manufacturing industries (MIs) from Wang et al. [16] is discussed in this section. The data shown in Table 4 are all not known exactly. There are seven MIs, and each MI is described by two inputs and one output. After obtaining the interval efficiency of each DMU through three proposed models, linear weighting method is used to aggregate the lower and upper efficiencies for obtaining the final efficiency. The aggregating weights of the lower and upper efficiencies are h_1 and h_2, and $h_1 + h_2 = 1$.

MI		1	2	3	4	5	6	7
Model 3	$h_1 = 0.9 \quad h_2 = 0.1$	0.824	0.87	0.901	0.857	0.875	0.847	0.879
	rank	7	4	1	5	3	6	2
	$h_1 = 0.8 \quad h_2 = 0.2$	0.838	0.885	0.912	0.869	0.887	0.864	0.893
	rank	7	4	1	5	3	6	2
	$h_1 = 0.7 \quad h_2 = 0.3$	0.853	0.899	0.9231	0.88	0.899	0.881	0.906
	rank	7	3	1	6	4	5	2
	$h_1 = 0.6 \quad h_2 = 0.4$	0.869	0.913	0.934	0.892	0.911	0.898	0.92
	rank	7	3	1	6	4	5	2
Model 5	$h_1 = 0.5 \quad h_2 = 0.5$	0.883	0.928	0.945	0.903	0.926	0.915	0.933
	rank	7	3	1	6	4	5	2
Model 4	$h_1 = 0.4 \quad h_2 = 0.6$	0.898	0.942	0.956	0.915	0.937	0.932	0.946
	rank	7	3	1	6	4	5	2
	$h_1 = 0.3 \quad h_2 = 0.7$	0.913	0.957	0.967	0.926	0.948	0.949	0.96
	rank	7	3	1	6	5	4	2
	$h_1 = 0.2 \quad h_2 = 0.8$	0.927	0.971	0.978	0.938	0.959	0.966	0.973
	rank	7	3	1	6	5	4	2
	$h_1 = 0.1 \quad h_2 = 0.9$	0.942	0.986	0.989	0.949	0.97	0.983	0.987
	rank	7	3	1	6	5	4	2

Tab. 5. The final efficiencies and ranking results of all MIs.

Table 5 shows the interval efficiency results of all MIs from three models. Because model (3) prefers the lower bound, the larger weight is given to lower bound (from 0.9 to 0.6). As regards the interval results of model (4), the larger weight is assigned to upper bound. The weight for upper bound is set from 0.6 to 0.9. Model (5) considers that the lower and upper bounds of interval efficiency are equally important, and then weights of lower and upper bounds are both 0.5. From the table, we can find that rankings of all DMUs are the same when the h_1 decreases from 0.7 to 0.4 and h_2 increases from 0.3 to 0.6. When h_1 is 0.8 or 0.9, the orders of DMU_2, DMU_4, DMU_5 and DMU_6 have changed. When h_2 is 0.7 or 0.8, or 0.9, the result is also different. The results of this table reveal that the different preferences are given to the lower or upper bound of the interval, the final ranking results are not the same. If the decision maker would like to evaluate efficiencies of MIs with interval data, the first thing is to determine which bound (lower or upper) is more important.

5. CONCLUSIONS

In many practical examples, the outputs and inputs of DMUs are not known exactly, for example, given as intervals. The existing classical DEA method is not able to rank these DMUs. In order to solve this problem, Wang et al. [16] proposed a new pair of DEA models for obtaining the interval efficiency of each DMU. However, these models have a disadvantage that two sets of weights are used to obtain the interval efficiency, which may be unreasonable. In this work we further extend these models. Three new models are constructed from different points of view. Two numerical examples are presented to illustrate and validate these models finally.

This paper has made two contributions. On the one hand, this paper explores three new models in order to obtain a common set of weights for the lower and upper bound. On the other hand, the results of the example reveal that the different preferences on the lower or upper bound of interval may have different efficiency and ranking results. The methods in this paper can also be further expanded. For example, imprecise data considered in this paper are given in the form of the interval data. In many real examples, some data are missing. How to construct the method could be studied in the future. In additional, the imprecise data may be uniformly distributed between lower and upper bound, which is not considered in this paper. How to extend the proposed models to deal with this problem is difficult but worth researching.

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