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A MATHEMATICAL MODEL FOR THE RECOVERY OF HUMAN AND ECONOMIC ACTIVITIES IN DISASTER REGIONS

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Abstract. In this paper a model for the recovery of human and economic activities in a region, which underwent a serious disaster, is proposed. The model treats the case that the disaster region has an industrial collaboration with a non-disaster region in the production system and, especially, depends upon each other in technological development. The economic growth model is based on the classical theory of R. M. Solow (1956), and the full model is described as a nonlinear system of ordinary differential equations.

Keywords: economic growth; human activity; economic activity; system of ordinary differential equations

MSC 2010: 91B62, 49J15, 35K40

1. Introduction

In this paper we discuss a design for the recovery process of human and economic activities in the disaster region Ω_1, getting support from the government and a (non-disaster) collaborative region Ω_2. In our model the basic assumptions and ideas for recovery are mentioned as follows:

(i) The recovery of economy is designed simultaneously with that of human living conditions in the disaster region, and the supply of labor force relies on it.

(ii) For a moment after the disaster, the recover of living conditions and economy of Ω_1 is supported by the public funds, $v = v(t)$, but it is temporary.

(iii) The region Ω_1 has an industrial collaboration with a (non-disaster) region Ω_2, and the region Ω_2 contributes part of its capital to the economic recovery of Ω_1.

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(iv) It is expected that the industrial relationship between Ω_1 and Ω_2 goes back to the usual one as soon as possible, and it is finally important to establish a self-reliant recovery system only in Ω_1.

2. State problems

We begin with explaining the meanings of the unknown functions. Let $s := (s_1, s_2, \ldots, s_N)$ be the environmental order parameters in Ω_1, with constraint $-1 \leqslant s_i \leqslant 1$, $i = 1, 2, \ldots, N$, and $w := (w_1, w_2)$ the vector function of capitals of Ω_1 and Ω_2. Moreover, let $A := (A_1, A_2)$ be the vector function describing technological order parameters of Ω_1 and Ω_2 and $L_1 := L_1(s)$ the labor force depending on s in Ω_1. Our model is described in three time intervals $[0, T_1]$, $[T_1, T_2]$, $[T_2, T_3]$ and their terminals T_1, T_2 and T_3 are unknown, too, and they are determined by some optimization conditions mentioned later in detail. Assume that s and w are governed by a system of ordinary differential inclusions or equations of the following form:

$$(1)\ s_i' + \partial I_{[0,\infty)}(s_i') + F_i(s) + \partial I_{(-\infty,0]}(s_i) \ni \begin{cases} \xi_{0i} v, & t \in [0, T_1], \\ \xi_{1i} w_1, & t \in [T_1, T_2],\ 1 \leqslant i \leqslant N, \\ \hat{\xi}_{1i} w_1, & t \in [T_2, T_3], \end{cases}$$

$$(2)\ w_1' + b_1 w_1 = \sigma_1 (A_1 L_1(s))^{1-\alpha} w_1^\alpha + \begin{cases} -\kappa_{12} w_1 + \eta_0 v, & t \in [0, T_1], \\ -(\kappa_{11} + \kappa_{12}) w_1 + \kappa_{21} w_2, & t \in [T_1, T_2], \\ -(\kappa_{11} + \kappa_{12}) w_1, & t \in [T_2, T_3], \end{cases}$$

$$(3)\ w_2' + b_2 w_2 = \sigma_2 (A_2 l_2)^{1-\beta} w_2^\beta + \begin{cases} -\kappa_{22} w_2, & t \in [0, T_1], \\ -(\kappa_{21} + \kappa_{22}) w_2, & t \in [T_1, T_2], \\ -\kappa_{22} w_2, & t \in [T_2, T_3], \end{cases}$$

$$(4)\ A_1' + c_1 A_1 = g_1(\kappa_{22} w_2), \quad A_2' + c_2 A_2 = g_2(\kappa_{12} w_1), \quad t \in [0, T_3],$$

with

$$(5) \qquad s_i(0) = -1, \quad 1 \leqslant i \leqslant N, \qquad w_k(0) = w_{k0}, \quad A_k(0) = A_{k0}, \quad k = 1, 2,$$

where

$$(6) \qquad L_1(s) := \frac{l_1}{2N} \sum_{i=1}^{N} (s_i + 1), \quad (\{\xi_{0i}\}, \eta_0) \in U_1,\ \{\xi_{1i}\} \in U_2,\ \{\hat{\xi}_{1i}\} \in U_3.$$

Here we denote by s_i', w_k', A_k' the time derivatives of s_i, w_k, A_k, and

▷ $F_i\colon [-1,1]^N \to \mathbb{R}$, $i = 1,2,\dots,N$, are Lipschitz continuous and non-increasing, $I_{[0,\infty)}$ and $I_{(-\infty,1]}$ are the indicator functions of the intervals $[0,\infty)$ and $(-\infty,1]$, respectively, and $\partial I_{[0,\infty)}$ and $\partial I_{(-\infty,1]}$ are their subdifferentials in $\mathbb{R}$;

▷ $0<\alpha<1$, $0<\beta<1$, $b_k>0$ (depreciation rate), $c_k>0$, $0<\sigma_k<1$ (saving rate), $l_k>0$ (normal labor force), $k=1,2$, and $\kappa_{kj}>0$ (support ratio), $k,j=1,2$, are economic constants; in economics, $(A_1L_1)^{1-\alpha}$ and $(A_2l_2)^{1-\beta}$ are called the production functions of Cobb-Douglas type;

▷ U_1, U_2, U_3 are control spaces used to determine T_1, T_2 and T_3; they are, respectively, given by

$$U_1 := \left\{(\{\xi_{0i}\},\eta_0);\ \xi_{0i}\geqslant 0,\ i=1,2,\dots,N,\ \eta_0\geqslant 0,\ \sum_{i=1}^N \xi_{0i}+\eta_0 = 1\right\},$$

$$U_2 = U_3 := \left\{\{\xi_{1i}\};\ \xi_{1i}\geqslant 0,\ i=1,2,\dots,N,\ \sum_{i=1}^N \xi_{1i} = \kappa_{11}\right\};$$

▷ $g_k\colon [0,\infty)\to[0,\infty)$, $k=1,2$, are Lipschitz continuous, bounded and non-decreasing functions such that $g_k(0)>0$, $g_k^* := \max g_k = g_k(r_k^*)$ and $g_k'>0$ on $(0,r_k^*)$; note that in real cases, each of functions g_1 and g_2 depend on both of w_1 and w_2, but we treat unusual cases as above in order to emphasize the influence of technological collaboration between Ω_1 and Ω_2.

In general, for two vectors $u=(u_1,u_2,\dots,u_M)$ and $\tilde u = (\tilde u_1,\tilde u_2,\dots,\tilde u_M)$, we simply denote by $u\leqslant\tilde u$ or $u<\tilde u$ the inequalities "$u_i\leqslant\tilde u_i$" or "$u_i<\tilde u_i$", respectively, $i=1,2,\dots,M$.

One of the characteristics of our model is the setup of check points $\{s^{(k)},w^{(k)},A^{(k)}\}$, $k=1,2$, with $s^{(k)} := (s_1^{(k)},s_2^{(k)},\dots,s_N^{(k)})$, $w^{(k)} := (w_1^{(k)},w_2^{(k)})$ and $A^{(k)} := (A_1^{(k)},A_2^{(k)})$ such that

$$\begin{cases} 0<s^{(1)}<s^{(2)}\leqslant(1,1,\dots,1),\\ 0<w^{(1)}<w^{(2)}\leqslant(r_1^*/\kappa_{12},r_2^*/\kappa_{22}), \quad 0<A^{(1)}<A^{(2)}\leqslant(g_1^*/c_1,g_2^*/c_2),\end{cases}\tag{7}$$

$$\begin{cases} (b_1+\kappa_{11}+\kappa_{12})w_1^{(1)}\leqslant\sigma_1\{A_1^{(1)}L_1(s^{(1)})\}^{1-\alpha}(w_2^{(1)})^\alpha+\kappa_{21}w_2^{(1)},\\ (b_2+\kappa_{21}+\kappa_{22})w_2^{(1)}\leqslant\sigma_2\{A_2^{(1)}l_2\}^{1-\beta}(w_2^{(1)})^\beta,\\ c_1A_1^{(1)}\leqslant g_1(\kappa_{22}w_2^{(1)}), \quad c_2A_2^{(1)}\leqslant g_2(\kappa_{12}w_1^{(1)}),\\ \sum\limits_{i=1}^N \max\limits_{s\geqslant s^{(1)}} F_i(s)+\varepsilon_0\leqslant\kappa_{11}w_1^{(1)} \quad \text{for some constant } \varepsilon_0>0,\end{cases}\tag{8}$$

and

$$\begin{cases} (b_1+\kappa_{11}+\kappa_{12})w_1^{(2)}\leqslant\sigma_1\{A_1^{(2)}L_1(s^{(2)})\}^{1-\alpha}(w_1^{(2)})^\alpha,\\ (b_2+\kappa_{22})w_2^{(2)}\leqslant\sigma_2\{A_2^{(2)}l_2\}^{1-\beta}(w_2^{(2)})^\beta,\\ c_1A_1^{(2)}\leqslant g_1(\kappa_{22}w_2^{(2)}), \quad c_2A_2^{(2)}\leqslant g_2(\kappa_{12}w_1^{(2)}).\end{cases}\tag{9}$$

3. Main theorems

By virtue of the theory of ODEs (cf. [5], [1], [2] and [3]), for given parameters $(\{\xi_{0i}\}, \eta_0) \in U_1$ and any $T_1 > 0$, $P_1(0, T_1)$ (hence $P_1(0, \infty)$) has a unique solution $\{s, w, A\}$. We define $t_1(\{\xi_{0i}\}, \eta_0)$ by

$$t_1(\{\xi_{0i}\}, \eta_0) := \min\{t \geqslant 0;\ s(t) \geqslant s^{(1)},\ w(t) \geqslant w^{(1)},\ A(t) \geqslant A^{(1)}\},$$

if $\{t \geqslant 0;\ s(t) \geqslant s^{(1)},\ w(t) \geqslant w^{(1)},\ A(t) \geqslant A^{(1)}\}$ is nonempty, and define $t_1(\{\xi_{0i}\},\ \eta_0) = \infty$ otherwise.

As to the problem in the first period, we have:

Theorem 3.1. *Assume that the initial data w_{k0}, A_{k0} are positive and close to 0 for $k = 1, 2$ so that*

$$(b_1 + \kappa_{11} + \kappa_{12})w_{10} \leqslant \kappa_{21}w_{20},\ (b_2 + \kappa_{21} + \kappa_{22})w_{20} \leqslant \sigma_2(A_{20}l_2)^{1-\beta}w_{20}^{\beta}, \tag{10}$$

$$c_1A_{10} \leqslant g_1(0),\ c_2A_{20} \leqslant g_2(0), \tag{11}$$

and the public fund v satisfies

$$(b_1 + \kappa_{11} + \kappa_{12})w_1^{(1)} + \sum_{i=1}^{N} \max_{s\in[-1,1]^N} F_i(s) + \delta_0 \leqslant v \text{ for some constant } \delta_0 > 0. \tag{12}$$

Then there exist a parameter $(\{\xi_{0i}^*\}, \eta_0^*) \in U_1$ and a finite time $T_1^* > 0$ such that

$$T_1^* = t_1(\{\xi_{0i}^*\}, \eta_0^*) = \inf_{(\{\xi_{0i}\},\eta_0)\in U_1} t_1(\{\xi_{0i}\}, \eta_0). \tag{13}$$

We denote by P_1^* the problem to find $(\{\xi_{0i}^*\}, \eta_0^*) \in U_1$ and a finite time $T_1^* > 0$ satisfying (13) and the solution $\{s^*, w^*, A^*\}$ of (1)–(5) on $[0, T_1^*]$ associated with the parameter $(\{\xi_{0i}\}, \eta_0) = (\{\xi_{0i}^*\}, \eta_0^*)$. The set $\{\{s^*, w^*, A^*\}; \{\xi_{0i}^*\}, \eta_0^*; T_1^*\}$ is called a solution of P_1^*.

Next, we consider the second period problem. Let $\{\{s^*, w^*, A^*\}; \{\xi_{0i}^*\}, \eta_0^*; T_1^*\}$ be a solution of P_1^* as obtained by Theorem 3.1 and let us fix it. Given any parameter $\{\xi_{1i}\} \in U_2$ and any $T_2 > T_1^*$, by the theory of ODEs (cf. [5], [1], [2] and [3]) again we see that (1)–(4) has a unique solution $\{s, w, A\}$ on $[T_1^*, T_2]$ associated with the initial conditions $s(T_1^*) = s^*(T_1^*)$, $w(T_1^*) = w^*(T_1^*)$, $A(T_1^*) = A^*(T_1^*)$. Just as in the first period problem, we define $t_2(\{\xi_{1i}\})$ by

$$t_2(\{\xi_{1i}\}) := \min\{t \geqslant T_1^*;\ s(t) \geqslant s^{(2)},\ w(t) \geqslant w^{(2)},\ A(t) \geqslant A^{(2)}\}, \tag{14}$$

if $\{t \geqslant T_1^*;\ s(t) \geqslant s^{(1)}, w(t) \geqslant w^{(1)}, A(t) \geqslant A^{(1)}\}$ is nonempty, and define time $t_2(\{\xi_{0i}\}) = \infty$ otherwise.

In order to discuss the second period problem we need additional assumptions as follows: Fixing $s \in [-1,1]^N$ as a parameter, we consider two curves in the $w_1 w_2$-plane

$$
\begin{aligned}
C_1(s)\colon\ (b_1+\kappa_{11}+\kappa_{12})w_1 &= \sigma_1\Big\{\frac{L_1(s)}{c_1}g_1(\kappa_{22}w_2)\Big\}^{1-\alpha} w_1^{\alpha} + \kappa_{21}w_2,\\
C_2\colon\ (b_2+\kappa_{21}+\kappa_{22})w_2 &= \sigma_2\Big\{\frac{l_2}{c_2}g_2(\kappa_{12}w_1)\Big\}^{1-\beta} w_2^{\beta}.
\end{aligned}
$$

These curves $C_1(s)$ and C_2 are described in the explicit forms:

$$
C_1(s)\colon\ w_2 = \Gamma_1^s(w_1), \quad C_2\colon\ w_2 = \Gamma_2(w_1) := \Big(\frac{\sigma_2}{b_2+\kappa_{21}+\kappa_{22}}\Big)^{1/(1-\beta)}\frac{l_2}{c_2}g_2(\kappa_{12}w_1).
$$

Theorem 3.2. *Assume that $\Gamma_1^{s^{(2)}}(w_1) < \Gamma_2(w_1)$ for all $w_1 \in [w_1^{(1)}, w_1^{(2)}]$. Let $\{\{s^*, w^*, A^*\}; \{\xi_{0i}^*\}, \eta_0^*; T_1^*\}$ be a solution of P_1^* and let us fix it, and let $t_2(\cdot)$ be the function on U_2 defined by* (14). *Then there exist a parameter $\{\xi_{1i}^*\} \in U_2$ and a finite time $T_2^* > T_1^*$ such that*

$$
T_2^* = t_2(\{\xi_{1i}^*\}) = \inf_{\{\xi_{1i}\}\in U_2} t_2(\{\xi_{1i}\}). \tag{15}
$$

We denote by P_2^* the problem to find $\{\xi_{1i}^*\} \in U_2$ and a finite time $T_2^* > T_1^*$ satisfying (15) and the solution $\{\tilde{s}^*, \tilde{w}^*, \tilde{A}^*\}$ of (1)–(4) on $[T_1^*, T_2^*]$ with the parameter $\{\xi_{1i}\} = \{\xi_{1i}^*\}$ and initial conditions $\tilde{s}^*(T_1^*) = s^*(T_1^*), \tilde{w}^*(T_1^*) = w^*(T_1^*), \tilde{A}^*(T_1^*) = A^*(T_1^*)$. The set $\{\{\tilde{s}^*, \tilde{w}^*, \tilde{A}^*\}; \{\xi_{1i}^*\}; T_1^*, T_2^*\}$ is called a solution of P_2^*.

In the third period problem, fixing a solution $\{\{\tilde{s}^*, \tilde{w}^*, \tilde{A}^*\}; \{\xi_{1i}^*\}; T_1^*, T_2^*\}$ of P_2^*, we define $t_3(\{\hat{\xi}_{1i}\})$ for each $\{\hat{\xi}_{1i}\} \in U_3$ by

$$
t_3(\{\hat{\xi}_{1i}\}) = \min\{t \geqslant T_2^*;\ s(t) = (1,1,\ldots,1)\}, \tag{16}
$$

where $\{s, w, A\}$ is a unique solution of (1)–(4) on $[T_2^*, T_3]$ for any $T_3 > T_2^*$, associated with the parameter $\{\hat{\xi}_{1i}\} \in U_3$ and initial conditions $s(T_2^*) = \tilde{s}^*(T_2^*)$, $w(T_2^*) = \tilde{w}^*(T_2^*)$, $A(T_2^*) = \tilde{A}^*(T_2^*)$.

Theorem 3.3. *Let $\{\{\tilde{s}^*, \tilde{w}^*, \tilde{A}^*\}; \{\xi_{1i}^*\}; T_1^*, T_2^*\}$ be a solution of P_2^*, and let $t_3(\cdot)$ be the function on U_3 defined by* (16). *Then there exist a parameter $\{\hat{\xi}_{1i}^*\} \in U_3$ and $T_3^* \geqslant T_2^*$ such that*

$$
T_3^* = t_3(\{\hat{\xi}_{1i}^*\}) = \inf_{\{\tilde{\xi}_{1i}\}\in U_3} t_3(\{\tilde{\xi}_{1i}\}). \tag{17}
$$

We denote by P_3^* the problem to find $\{\hat{\xi}_{1i}^*\} \in U_2$ and a finite time $T_3^* > T_2^*$ satisfying (17) and the solution $\{\hat{s}^*, \hat{w}^*, \hat{A}^*\}$ of (1)–(4) on $[T_2^*, T_3^*]$ with the parameter $\{\hat{\xi}_{1i}\} = \{\hat{\xi}_{1i}^*\}$ and initial conditions $\hat{s}^*(T_2^*) = \tilde{s}^*(T_2^*)$, $\hat{w}^*(T_2^*) = \tilde{w}^*(T_2^*)$, $\hat{A}^*(T_2^*) = \tilde{A}^*(T_2^*)$. The set $\{\{\hat{s}^*, \hat{w}^*, \hat{A}^*\}; \{\hat{\xi}_{1i}^*\}; T_2^*, T_3^*\}$ is called a solution of P_3^*.

For detailed proofs of Theorems 3.1–3.3, see [2].

After time T_3^*, w and A are governed by the system with technological collaboration in the normal situation:

$$
\begin{gathered}
w_1' + (b_1 + \kappa_{12})w_1 = \sigma_1 (A_1 l_1)^{1-\alpha} w_1^\alpha, \quad t \geqslant T_3^*, \\
w_2' + (b_2 + \kappa_{22})w_2 = \sigma_2 (A_2 l_2)^{1-\beta} w_2^\beta, \quad t \geqslant T_3^*, \\
A_1' + c_1 A_1 = g_1(\kappa_{22} w_2), \quad A_2' + c_2 A_2 = g_2(\kappa_{12} w_1), \quad t \geqslant T_3^*,
\end{gathered}
\tag{18}
$$

since $s(t) = (1, 1, \cdot, 1)$ for $t \geqslant T_3^*$ and it is not necessary to consider (1) any longer. On account of the general result on the asymptotic behaviour (cf. [1], [2]) $w(t)$ and $A(t)$ converge as $t \to \infty$. Also, it is expected from some economical points of view that

$$
\lim_{t\to\infty} w(t) = \Bigl(\Bigl(\frac{\sigma_1}{b_1 + \kappa_{12}}\Bigr)^{1/(1-\alpha)} \frac{g_1^* l_1}{c_1}, \Bigl(\frac{\sigma_2}{b_2 + \kappa_{22}}\Bigr)^{1/(1-\beta)} \frac{g_2^* l_2}{c_2}\Bigr)
\tag{19}
$$

as well as

$$
\lim_{t\to\infty} A(t) = \Bigl(\frac{g_1^*}{c_1}, \frac{g_2^*}{c_2}\Bigr).
\tag{20}
$$

These convergences will be proved under some additional assumptions on functions g_1 and g_2.

4. Numerical experiment

In this section we give a numerical experiment of our model with the following parameters:

$$
\begin{gathered}
b_1 = 1,\ b_2 = 0.9,\ \alpha = 0.5,\ \beta = 0.5,\ \sigma_1 = 0.975,\ \sigma_2 = 0.945,\ \xi_0 = 0.45,\ \eta_0 = 0.55, \\
\kappa_{11} = 0.11,\ \kappa_{12} = 0.4,\ \kappa_{21} = 0.12,\ \kappa_{22} = 0.3, \\
l_1 = 64/9,\ l_2 = 2000/81,\ c_1 = 1/8,\ c_2 = 1/3, \\
v = 5.1,\ w_{10} = 0.01,\ w_{20} = 0.5,\ s_0 = -1,\ A_{10} = 0.1,\ A_{20} = 0.3.
\end{gathered}
$$

As $F(\cdot) := F_1(\cdot)$, $g_1(\cdot)$ and $g_2(\cdot)$ we choose the following functions:

$$g_1(x) = \begin{cases} 0.1, & x \leqslant 0, \\ 2\Big(\dfrac{x}{8}\Big)^2 + 0.1, & 0 < x \leqslant 4.0, \\ -2\Big(\dfrac{x-8}{8}\Big)^2 + 1.1, & 4.0 < x \leqslant 8.0, \\ 1.1, & 8.0 < x, \end{cases}$$

$$g_2(x) = \begin{cases} 0.1, & x \leqslant 0, \\ 2\Big(\dfrac{x}{3}\Big)^2 + 0.1, & 0 < x \leqslant 1.5, \\ -2\Big(\dfrac{x-3}{3}\Big)^2 + 1.1, & 1.5 < x \leqslant 3.0, \\ 1.1, & 3.0 < x, \end{cases}$$

$$F(x) = \begin{cases} -x + 1.0, & -1.0 \leqslant x \leqslant 0, \\ -\dfrac{1}{4}x + 1.0, & 0 < x \leqslant 4.0 \end{cases}$$

and we set up the check points as follows:

$$(w_1^{(1)}, w_2^{(1)}, s^{(1)}, A_1^{(1)}, A_2^{(1)}) = (2, 5, 3.5, 0.5, 0.4),$$
$$(w_1^{(2)}, w_2^{(2)}, s^{(2)}, A_1^{(2)}, A_2^{(2)}) = (7.5, 25, 4, 4, 2).$$

We remark here that in our numerical experiment the upper threshold value of s is 4 in place of 1, namely, the range of s is assumed to be $-1 \leqslant s \leqslant 4$ in place of $-1 \leqslant s \leqslant 1$. This treatment is just for the sake of indicating more effectively the character of our model, and the original one has the same character.

Figure 1 shows the behavior of the economic curve $w(t) := (w_1(t), w_2(t))$ in the case of the above data. The parts of the curve between $(0.01, 0.5)$–A, A–B and B– correspond, respectively, to the first period, the second period and the third period.

P is the first check point $w^{(1)} = (2, 5)$, Q is the second check point $w^{(2)} = (7.5, 25)$.

In the first period, both of $w_1(t)$ and $w_2(t)$ increase in time. The solution curve $\{w, s, A\}$ completely gets over the first check point $(w^{(1)}, s^{(1)}, A^{(1)})$ at time $t = 3.6798$ when A_1 reaches at the check value 0.5 in this numerical experience. By the way, $w(3.6798) = (3.74, 7.21) =: A$, $s(3.6798) = 3.79$ and $A(3.6798) = (0.5, 0.83)$.

The second period starts with initial time $t = 3.6798$ and initial data (3.74, 7.21), 3.79 and (0.5, 0.83) for w, s and A. After time $t = 3.6798$ the capital $w_1(t)$ decreases a little bit, because of the switching of support system. However, as is also seen from the numerical experiment, we see that $w(t)$ never get lower the first check point. After a while, both of $w_1(t)$ and $w_2(t)$ increase in time. This behaviour of $w(t)$ suggests from the economic point of view that the switching time of support system

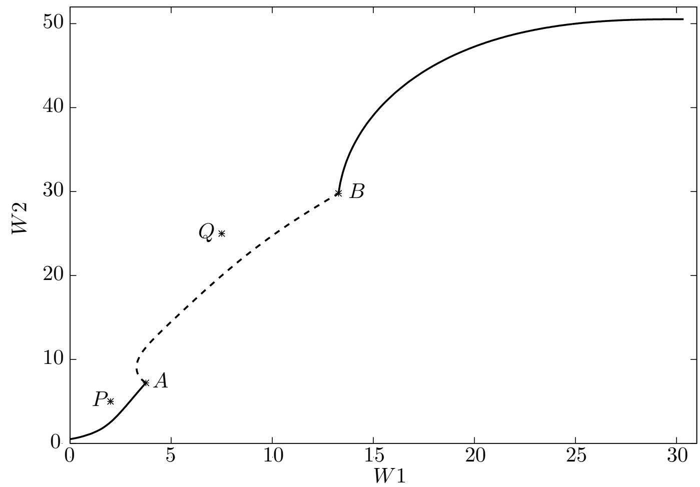

Figure 1.

should be made after $t = 3.6798$, otherwise $w(t)$ might fall down to $(0,0)$ under less support system than the one of the first period; note that in general the support system of the first period is richer than the second period.

At time $t = 10.7178$ the solution curve $\{w, s, A\}$ gets over the second check point $(w^{(2)}, s^{(2)}, A^{(2)})$. By the way, $w(10.7178) = (13.27, 29.77) =: B$, $s(10.7178) = 4.00$ and $A(10.7178) = (4.00, 2.77)$.

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