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A POPULATION BIOLOGICAL MODEL WITH A SINGULAR NONLINEARITY

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Abstract. We consider the existence of positive solutions of the singular nonlinear semipositone problem of the form

$$\begin{cases} -\operatorname{div}(|x|^{-\alpha p}|\nabla u|^{p-2}\nabla u) = |x|^{-(\alpha+1)p+\beta}\Big(au^{p-1} - f(u) - \dfrac{c}{u^\gamma}\Big), & x \in \Omega,\\ u = 0, \quad x \in \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$ with $0 \in \Omega$, $1 < p < N$, $0 \leqslant \alpha < (N-p)/p$, $\gamma \in (0,1)$, and a, β, c and λ are positive parameters. Here $f\colon [0,\infty) \to \mathbb{R}$ is a continuous function. This model arises in the studies of population biology of one species with u representing the concentration of the species. We discuss the existence of a positive solution when f satisfies certain additional conditions. We use the method of sub-supersolutions to establish our results.

Keywords: population biology; infinite semipositone; sub-supersolution

MSC 2010: 35J60, 35J65

1. Introduction

We study the existence of positive solutions to the singular infinite semipositone problem

$$(1) \qquad \begin{cases} -\operatorname{div}(|x|^{-\alpha p}|\nabla u|^{p-2}\nabla u) = |x|^{-(\alpha+1)p+\beta}\Big(au^{p-1} - f(u) - \dfrac{c}{u^\gamma}\Big), & x \in \Omega,\\ u = 0, \quad x \in \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$ with $0 \in \Omega$, $1 < p < N$, $0 \leqslant \alpha < (N-p)/p$, $\gamma \in (0,1)$, a, β, c and λ are positive constants and $f\colon [0,\infty) \to \mathbb{R}$ is a continuous function.

We make the following assumptions:

(A1) There exist $L > 0$ and $b > 0$ such that $f(u) < Lu^b$ for all $u \geqslant 0$.

(A2) There exists a constant $S > 0$ such that $au^{p-1} < f(u) + S$ for all $u \geqslant 0$.

Elliptic problems involving a more general operator, such as the degenerate quasilinear elliptic operator given by $-\operatorname{div}(|x|^{-ap}|\nabla u|^{p-2}\nabla u)$, were motivated by the Caffarelli, Kohn and Nirenberg's inequality (see [3], [17]). The study of this type of problem is motivated by its various applications, for example, in fluid mechanics, in Newtonian fluids, in flow through porous media and in glaciology (see [1], [6]).

More recently, reaction-diffusion models have been used to describe spatiotemporal phenomena in disciplines other than ecology, such as physics, chemistry, and biology (see [5], [13], [15]). In addition, most ecological systems have some form of predation or harvesting of the population, for example, hunting or fishing is often used as an effective means of wildlife management. This model describes the dynamics of the fish population with predation. In such cases, u denotes the population density and the term c/u^γ corresponds to predation. So, the study of positive solutions of (1) has more practical meanings. We refer to [14], [9], [2], [10] for additional results on elliptic problems.

Let $\tilde{f}(u) = au^{p-1} - f(u) - c/u^\gamma$. Then $\lim\limits_{u\to 0} \tilde{f}(u) = -\infty$, and hence we refer to (1) as an infinite semipositone problem. See [11] where the authors discussed the problem (1) when $\alpha = 0$, $\beta = p = 2$. Here we focus on extending the study in [11]. In fact this paper is motivated, in part, by the mathematical difficulty posed by the degenerate quasilinear elliptic operator compared to the Laplacian operator ($\alpha = 0$, $\beta = p = 2$). This extension is nontrivial and requires more careful analysis of the nonlinearity. Our approach is based on the method of sub-supersolutions, see [4], [7].

2. Preliminaries and existence result

In this paper, we denote $W_0^{1,p}(\Omega, |x|^{-ap})$ the completion of $C_0^\infty(\Omega)$, with respect to the norm $\|u\| = \bigl(\int_\Omega |x|^{-ap}|\nabla u|^p \,\mathrm{d}x\bigr)^{1/p}$. To precisely state our existence result we consider the eigenvalue problem

$$(2)\qquad \begin{cases} -\operatorname{div}(|x|^{-\alpha p}|\nabla\varphi|^{p-2}\nabla\varphi) = \lambda|x|^{-(\alpha+1)p+\beta}|\varphi|^{p-2}\varphi, & x \in \Omega,\\ \varphi = 0, \quad x \in \partial\Omega. \end{cases}$$

Let $\varphi_{1,p}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,p}$ of (2) such that $\varphi_{1,p}(x) > 0$ in Ω, and $\|\varphi_{1,p}\|_\infty = 1$ (see [12], [16]). It can be shown that $\partial\varphi_{1,p}/\partial n < 0$ on $\partial\Omega$. Here n is the outward normal. We will also consider the unique solution $\zeta_p(x) \in W_0^{1,p}(\Omega, |x|^{-ap})$ for the problem

$$\begin{cases} -\mathrm{div}(|x|^{-\alpha p}|\nabla u|^{p-2}\nabla u) = |x|^{-(\alpha+1)p+\beta}, & x \in \Omega, \\ u = 0, \quad x \in \partial\Omega, \end{cases}$$

to discuss our existence result. It is known that $\zeta_p(x) > 0$ in Ω and $\partial\zeta_p(x)/\partial n < 0$ on $\partial\Omega$ (see [12]).

Now, we give the definition of weak solution and sub-supersolution of (1). A nonnegative function ψ is called a subsolution of (1) if it satisfies $\psi \leqslant 0$ on $\partial\Omega$ and

$$\int_\Omega |x|^{-\alpha p}|\nabla\psi|^{p-2}\nabla\psi \cdot \nabla w \,\mathrm{d}x \leqslant \int_\Omega |x|^{-(\alpha+1)p+\beta}\Big(a\psi^{p-1} - f(\psi) - \frac{c}{\psi^\gamma}\Big) w \,\mathrm{d}x,$$

and a nonnegative function Ψ is called a supersolution of (1) if it satisfies $\Psi \geqslant 0$ on $\partial\Omega$ and

$$\int_\Omega |x|^{-\alpha p}|\nabla\Psi|^{p-2}\nabla\Psi \cdot \nabla w \,\mathrm{d}x \geqslant \int_\Omega |x|^{-(\alpha+1)p+\beta}\Big(a\Psi^{p-1} - f(\Psi) - \frac{c}{\Psi^\gamma}\Big) w \,\mathrm{d}x,$$

for all $w \in W = \{w \in C_0^\infty(\Omega);\ w \geqslant 0, x \in \Omega\}$. Then the following result holds:

Lemma 2.1 (See [12]). *Suppose there exist sub- and supersolutions ψ and Ψ, respectively, of* (1) *such that $\psi \leqslant \Psi$. Then* (1) *has a solution u such that $\psi \leqslant u \leqslant \Psi$.*

We are now ready to give our existence result.

Theorem 2.2. *Assume* (A1) *and* (A2) *hold. If $a > p\lambda_{1,p}/p - 1 + \gamma$, then there exists $c_0 > 0$ such that if $0 < c < c_0$, then the problem* (1) *admits a positive solution.*

Proof. We start with the construction of a positive subsolution for (1). To get a positive subsolution, we can apply an anti-maximum principle (see [8]), from which we know that there exist a $\delta_1 > 0$ and a solution z_λ of

$$\tag{3} \begin{cases} -\mathrm{div}(|x|^{-\alpha p}|\nabla z|^{p-2}\nabla z) = |x|^{-(\alpha+1)p+\beta}(\lambda z^{p-1} - 1), & x \in \Omega, \\ z = 0, \quad x \in \partial\Omega, \end{cases}$$

for $\lambda \in (\lambda_{1,p}, \lambda_{1,p} + \delta_1)$. Fix $\hat\lambda \in (\lambda_{1,p}, \min\{a(p-1+\gamma)/p, \lambda_{1,p} + \delta_1\})$. Let $\theta = \|z_{\hat\lambda}\|_\infty$, and $z_{\hat\lambda}$ be the solution of (5) when $\lambda = \hat\lambda$. It is well known that $z_{\hat\lambda} > 0$ in Ω and $\partial z_{\hat\lambda}/\partial n < 0$ on $\partial\Omega$, where n is the outer unit normal to Ω. Hence, there exist positive constants $\varepsilon, \delta, \sigma$ such that

$$\tag{4} |x|^{-\alpha p}|\nabla z_{\hat\lambda}|^p \geqslant \varepsilon, \quad x \in \overline{\Omega}_\delta,$$

$$\tag{5} z_{\hat\lambda} \geqslant \sigma, \quad x \in \Omega_0 = \Omega \setminus \overline{\Omega}_\delta,$$

where $\overline{\Omega}_\delta = \{x \in \Omega;\ d(x, \partial\Omega) \leqslant \delta\}$.

Choose $\eta_1, \eta_2 > 0$ such that $\eta_1 \leqslant \min |x|^{-(\alpha+1)p+\beta}$ and $\eta_2 \geqslant \max |x|^{-(\alpha+1)p+\beta}$ in $\overline{\Omega}_\delta$. We construct a subsolution ψ of (1) using $z_{\hat\lambda}$. Define $\psi = M((p-1+\gamma)/p)\, z_{\hat\lambda}^{p/(p-1+\gamma)}$, where

$$M = \min\Big\{\Big(\frac{(p/(p-1+\gamma))^b}{L\theta^{(pb-(1-\gamma)(p-1))/(p-1+\gamma)}}\Big)^{1/(b-p+1)}, \\ \Big(\frac{((p-1)/Lp)[((p-1+\gamma)/p)a-\hat\lambda]}{((p-1+\gamma)/p)^b\theta^{(pb-p(p-1))/(p-1+\gamma)}}\Big)^{1/(b-p+1)}\Big\}.$$

Let $w \in W$. Then a calculation shows that for $\nabla\psi = M z_{\hat\lambda}^{(1-\gamma)/(p-1+\gamma)}\nabla z_{\hat\lambda}$,

$$\begin{aligned}
(6)\qquad &\int_\Omega |x|^{-\alpha p}|\nabla\psi|^{p-2}\nabla\psi\nabla w\,\mathrm{d}x \\
&= M^{p-1}\int_\Omega |x|^{-\alpha p} z_{\hat\lambda}^{((1-\gamma)(p-1))/(p-1+\gamma)}|\nabla z_{\hat\lambda}|^{p-2}\nabla z_{\hat\lambda}\nabla w\,\mathrm{d}x \\
&= M^{p-1}\int_\Omega |x|^{-\alpha p}|\nabla z_{\hat\lambda}|^{p-2}\nabla z_{\hat\lambda}[\nabla(z_{\hat\lambda}^{((1-\gamma)(p-1))/(p-1+\gamma)}w) \\
&\quad - |\nabla z_{\hat\lambda}|^{((1-\gamma)(p-1))/(p-1+\gamma)}w]\,\mathrm{d}x \\
&= M^{p-1}\int_\Omega \Big[|x|^{-(\alpha+1)p+\beta} z_{\hat\lambda}^{((1-\gamma)(p-1))/(p-1+\gamma)}(\hat\lambda z_{\hat\lambda}^{p-1}-1) \\
&\quad - |x|^{-\alpha p}((1-\gamma)(p-1))/(p-1+\gamma)\frac{|\nabla z_{\hat\lambda}|^p}{z_{\hat\lambda}^{\gamma p/(p-1+\gamma)}}\Big] w\,\mathrm{d}x \\
&= \int_\Omega \Big[|x|^{-(\alpha+1)p+\beta}M^{p-1}\hat\lambda z_{\hat\lambda}^{p(p-1)/(p-1+\gamma)} \\
&\quad - |x|^{-(\alpha+1)p+\beta}M^{p-1}z_{\hat\lambda}^{((1-\gamma)(p-1))/(p-1+\gamma)} \\
&\quad - |x|^{-\alpha p}M^{p-1}\frac{(1-\gamma)(p-1)}{p-1+\gamma}\,\frac{|\nabla z_{\hat\lambda}|^p}{z_{\hat\lambda}^{\gamma p/(p-1+\gamma)}}\Big] w\,\mathrm{d}x
\end{aligned}$$

and

$$\begin{aligned}
(7)\qquad &\int_\Omega |x|^{-(\alpha+1)p+\beta}\Big[a\psi^{p-1} - f(\psi) - \frac{c}{\psi^\gamma}\Big] w\,\mathrm{d}x \\
&= \int_\Omega \Big[|x|^{-(\alpha+1)p+\beta} aM^{p-1}\Big(\frac{p-1+\gamma}{p}\Big)^{p-1} z_{\hat\lambda}^{p(p-1)/(p-1+\gamma)} \\
&\quad - |x|^{-(\alpha+1)p+\beta} f\Big(M\Big(\frac{p-1+\gamma}{p}\Big)z_{\hat\lambda}^{p/(p-1+\gamma)}\Big) \\
&\quad - |x|^{-(\alpha+1)p+\beta}\frac{c}{M^\gamma((p-1+\gamma)/p)^\gamma z_{\hat\lambda}^{\gamma p/(p-1+\gamma)}}\Big] w\,\mathrm{d}x.
\end{aligned}$$

Let $c_0 = M^{p-1+\gamma} \min\bigl\{\frac{(1-\gamma)(p-1)}{p-1+\gamma}\bigl(\frac{p-1+\gamma}{p}\bigr)^\gamma \frac{\varepsilon}{\eta_2}, \frac{1}{p}\bigl(\frac{p-1+\gamma}{p}\bigr)^\gamma \sigma^p\bigl(\frac{p-1+\gamma}{p}a - \hat{\lambda}\bigr)\bigr\}$. First, we consider the case when $x \in \overline{\Omega}_\delta$. We have $|x|^{-ap}|\nabla\varphi_{1,p}|^p \geqslant \varepsilon$ on $\overline{\Omega}_\delta$. Since $(p/(p-1+\alpha))^{p-1}\hat{\lambda} \leqslant a$, we have

$$\begin{aligned}
(8)\qquad & |x|^{-(\alpha+1)p+\beta} M^{p-1}\hat{\lambda} z_{\hat{\lambda}}^{p(p-1)/(p-1+\alpha)} \\
&\qquad \leqslant |x|^{-(\alpha+1)p+\beta} a M^{p-1}\Bigl(\frac{p-1+\alpha}{p}\Bigr)^{p-1} z_{\hat{\lambda}}^{p(p-1)/(p-1+\alpha)},
\end{aligned}$$

and from the choice of M, we know that

$$(9)\qquad L M^{b-p+1}\theta^{(pb-(1-\gamma)(p-1))/(p-1+\gamma)} \leqslant \Bigl(\frac{p}{p-1+\gamma}\Bigr)^b.$$

By (9) and (A1) we come to

$$\begin{aligned}
(10)\qquad & -|x|^{-(\alpha+1)p+\beta} M^{p-1} z_{\hat{\lambda}}^{(1-\gamma)(p-1)/(p-1+\gamma)} \\
&\qquad \leqslant -|x|^{-(\alpha+1)p+\beta} L M^b\Bigl(\frac{p-1+\gamma}{p}\Bigr)^b z_{\hat{\lambda}}^{pb/(p-1+\gamma)} \\
&\qquad \leqslant -|x|^{-(\alpha+1)p+\beta} f\Bigl(M\Bigl(\frac{p-1+\gamma}{p}\Bigr) z_{\hat{\lambda}}^{p/(p-1+\gamma)}\Bigr).
\end{aligned}$$

Next, from (4) and the definition of c_0, we arrive at

$$|x|^{-\alpha p} M^{p-1}\frac{(1-\gamma)(p-1)}{p-1+\gamma}|\nabla z_{\hat{\lambda}}|^p \geqslant |x|^{-(\alpha+1)p+\beta}\frac{c}{M^\gamma((p-1+\gamma)/p)^\gamma},$$

and

$$\begin{aligned}
(11)\qquad & -|x|^{-\alpha p} M^{p-1}\frac{(1-\gamma)(p-1)}{p-1+\gamma}\frac{|\nabla z_{\hat{\lambda}}|^p}{z_{\hat{\lambda}}^{\gamma p/(p-1+\gamma)}} \\
&\qquad \leqslant -|x|^{-(\alpha+1)p+\beta}\frac{c}{M^\gamma((p-1+\gamma)/p)^\gamma z_{\hat{\lambda}}^{\gamma p/(p-1+\gamma)}}.
\end{aligned}$$

Hence, by using (8), (10) and (11) for $c \leqslant c_0$, we find that

$$\begin{aligned}
(12)\qquad & \int_{\overline{\Omega}_\delta} |x|^{-\alpha p}|\nabla\psi|^{p-2}\nabla\psi\nabla w\,\mathrm{d}x \\
&\qquad \leqslant \int_{\overline{\Omega}_\delta}\Bigl[|x|^{-(\alpha+1)p+\beta} a M^{p-1}\Bigl(\frac{p-1+\gamma}{p}\Bigr)^{p-1} z_{\hat{\lambda}}^{p(p-1)/(p-1+\gamma)} \\
&\qquad\quad - |x|^{-(\alpha+1)p+\beta} f\Bigl(M\Bigl(\frac{p-1+\gamma}{p}\Bigr) z_{\hat{\lambda}}^{p/(p-1+\gamma)}\Bigr) \\
&\qquad\quad - |x|^{-(\alpha+1)p+\beta}\frac{c}{M^\gamma((p-1+\gamma)/p)^\gamma z_{\hat{\lambda}}^{\gamma p/(p-1+\gamma)}}\Bigr] w\,\mathrm{d}x \\
&\qquad = \int_{\overline{\Omega}_\delta} |x|^{-(\alpha+1)p+\beta}\Bigl[a\psi^{p-1} - f(\psi) - \frac{c}{\psi^\gamma}\Bigr] w\,\mathrm{d}x.
\end{aligned}$$

On the other hand, on $\Omega_0 = \Omega \setminus \overline{\Omega}_\delta$, we have $z_{\hat{\lambda}} \geqslant \sigma$, for some $0 < \sigma < 1$, and from the definition of c_0, for $c \leqslant c_0$ we get

$$
\begin{aligned}
\frac{c}{M^\alpha((p-1+\gamma)/p)^\gamma} &\leqslant \frac{1}{p} M^{p-1} \sigma^p \Big[\frac{p-1+\gamma}{p} a - \hat{\lambda}\Big] \\
&\leqslant \frac{1}{p} M^{p-1} z_{\hat{\lambda}}^p \Big[\frac{p-1+\gamma}{p} a - \hat{\lambda}\Big].
\end{aligned} \tag{13}
$$

Also from the choice of M, we have

$$
LM^{b-p+1}\Big(\frac{p-1+\gamma}{p}\Big)^b z_{\hat{\lambda}}^{(pb-p(p-1))/(p-1+\gamma)} \leqslant \frac{p-1}{p}\Big[\frac{p-1+\gamma}{p} a - \hat{\lambda}\Big]. \tag{14}
$$

Hence, from (12) and (13) we have

$$
\begin{aligned}
&\int_{\Omega_0} |x|^{-\alpha p} |\nabla \psi|^{p-2} \nabla \psi \nabla w \,\mathrm{d}x \\
&= \int_{\Omega_0} \Big[|x|^{-(\alpha+1)p+\beta} M^{p-1} \hat{\lambda} z_{\hat{\lambda}}^{p(p-1)/(p-1+\gamma)} \\
&\quad - |x|^{-(\alpha+1)p+\beta} M^{p-1} z_{\hat{\lambda}}^{((1-\gamma)(p-1))/(p-1+\gamma)} \\
&\quad - |x|^{-\alpha p} M^{p-1} \frac{(1-\gamma)(p-1)}{p-1+\gamma} \frac{|\nabla z_{\hat{\lambda}}|^p}{z_{\hat{\lambda}}^{\gamma p/(p-1+\gamma)}} \Big] w \,\mathrm{d}x \\
&\leqslant \int_{\Omega_0} |x|^{-(\alpha+1)p+\beta} M^{p-1} \hat{\lambda} z_{\hat{\lambda}}^{p(p-1)/(p-1+\gamma)} w \,\mathrm{d}x \\
&= \int_{\Omega_0} |x|^{-(\alpha+1)p+\beta} \frac{1}{z_{\hat{\lambda}}^{\gamma p/(p-1+\gamma)}} \Big[\frac{1}{p} \hat{\lambda} M^{p-1} z_{\hat{\lambda}}^p + \frac{p-1}{p} \hat{\lambda} M^{p-1} z_{\hat{\lambda}}^p\Big] w \,\mathrm{d}x \\
&\leqslant \int_{\Omega_0} |x|^{-(\alpha+1)p+\beta} \frac{1}{z_{\hat{\lambda}}^{\gamma p/(p-1+\gamma)}} \Big[\Big(\frac{1}{p} M^{p-1} \Big(\frac{p-1+\gamma}{p}\Big)^{p-1} a z_{\hat{\lambda}}^p \\
&\quad - \frac{c}{M^\gamma((p-1+\gamma)/p)^\gamma}\Big) + M^{p-1} z_{\hat{\lambda}}^p \Big(\frac{p-1+\gamma}{p}\Big)^{p-1} \\
&\quad \times \Big(\frac{(p-1)a}{p} - LM^{b-p+1}\Big(\frac{p-1+\gamma}{p}\Big)^{b-p+1} z_{\hat{\lambda}}^{(pb-p(p-1))/(p-1+\gamma)}\Big)\Big] w \,\mathrm{d}x \\
&= \int_{\Omega_0} |x|^{-(\alpha+1)p+\beta} \Big[aM^{p-1} \Big(\frac{p-1+\gamma}{p}\Big)^{p-1} z_{\hat{\lambda}}^{p(p-1)/(p-1+\gamma)} \\
&\quad - LM^b \Big(\frac{p-1+\gamma}{p}\Big)^b z_{\hat{\lambda}}^{pb/(p-1+\gamma)} - \frac{c z_{\hat{\lambda}}^{-\gamma p/(p-1+\gamma)}}{M^\gamma((p-1+\gamma)/p)^\gamma} \Big] w \,\mathrm{d}x
\end{aligned} \tag{15}
$$

$$\begin{aligned}
&\leqslant \int_{\Omega_0} |x|^{-(\alpha+1)p+\beta}\Big[aM^{p-1}\Big(\frac{p-1+\gamma}{p}\Big)^{p-1} z_{\hat\lambda}^{p(p-1)/(p-1+\gamma)} \\
&\qquad - f\Big(M\frac{p-1+\gamma}{p} z_{\hat\lambda}^{p/(p-1+\gamma)}\Big) - \frac{c}{M^{\gamma}((p-1+\gamma)/p)^{\gamma} z_{\hat\lambda}^{\gamma p/(p-1+\gamma)}}\Big] w\,\mathrm{d}x \\
&= \int_{\Omega_0} |x|^{-(\alpha+1)p+\beta}\Big[a\psi^{p-1} - f(\psi) - \frac{c}{\psi^{\gamma}}\Big] w\,\mathrm{d}x.
\end{aligned}$$

By using (12) and (15) we see that ψ is a subsolution of (1).

Next, we construct a supersolution Ψ of (1) such that $\Psi \geqslant \psi$. By ($A2$) we can choose a large constant S^* such that $au^{p-1} - f(u) - c/u^{\gamma} \leqslant S^*$ for all $u > 0$. Let $\Psi = (S^*)^{1/(p-1)}\zeta(x)$. We shall verify that Ψ is a supersolution of (1). To this end, let $w \in W$. Then we find that

$$(16)\quad \begin{aligned}
\int_{\Omega} |x|^{-\alpha p}|\nabla\Psi|^{p-2}\nabla\Psi\nabla w\,\mathrm{d}x &= S^*\int_{\Omega} |x|^{-(\alpha+1)p+\beta} w\,\mathrm{d}x \\
&\geqslant \int_{\Omega} |x|^{-(\alpha+1)p+\beta}\Big[a\Psi^{p-1} - f(\Psi) - \frac{c}{\Psi^{\gamma}}\Big] w\,\mathrm{d}x.
\end{aligned}$$

Thus Ψ is a supersolution of (1). Finally, we can choose $S^* \gg 1$ such that $\psi \leqslant \Psi$ in Ω. Hence, for $c \leqslant c_0$ by Lemma 2.1 there exists a positive solution u of (1) such that $\psi \leqslant u \leqslant \Psi$. This completes the proof of Theorem 2.2. $\square$

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