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AM-Compactness of some classes of operators

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Abstract. We characterize Banach lattices on which each regular order weakly compact (resp. b-weakly compact, almost Dunford-Pettis, Dunford-Pettis) operator is AM-compact.

Keywords: AM-compact operator, order weakly compact operator, b-weakly compact operator, almost Dunford-Pettis operator, b-AM-compact operator, order continuous norm, discrete Banach lattice

Classification: 46A40, 46B40, 46B42

1. Introduction and notation

The class of AM-compact operators is introduced and studied by Dodds-Fremlin [14] and its domination problem is characterized in [5]. Recall that a regular operator T from a Banach lattice E into a Banach space F is said to be AM-compact if it carries each order bounded subset of E onto a relatively compact subset of F.

On the other hand, each regular compact operator is AM-compact, but an AM-compact operator is not necessary compact. In fact, the identity operator of the Banach lattice ℓ^1 is AM-compact (because ℓ^1 is discrete with order continuous norm) but it is not compact. However, if E is an AM-space with unit, the class of regular compact operators coincides with that of AM-compact operators. For a more detailed study of this class of operators we refer the reader to the book of Zaanen [21].

In this paper we are interested in three classes of operators. The first one is bigger than that of AM-compact operators. It is the class of order weakly compact operators introduced by Dodds [13]. Recall that an operator T from a Banach lattice E into a Banach space F is said to be order weakly compact if for each $x \in E_+$, the set $T([0,x])$ is relatively weakly compact in F. Note that an order weakly compact operator is not necessarily AM-compact. In fact, the identity operator $Id_{L^1[0,1]} : L^1[0,1] \longrightarrow L^1[0,1]$ is order weakly compact (because the norm of $L^1[0,1]$ is order continuous), but it is not AM-compact (because $L^1[0,1]$ is not discrete).

The second class is that of b-weakly compact operators introduced by Alpay-Altin-Tonyali [3]. An operator T from a Banach lattice E into a Banach space F is said to be b-weakly compact if for each b-order bounded subset A of E (i.e. order bounded in the topological bidual E''), $T(A)$ is relatively weakly compact in F. Note that there is an AM-compact operator which is not b-weakly compact and conversely there is a b-weakly compact operator which is not AM-compact. In fact, the identity operator $Id_{L^1[0,1]} : L^1[0,1] \longrightarrow L^1[0,1]$ is b-weakly compact (because $L^1[0,1]$ is KB-space), but it is not AM-compact (because $L^1[0,1]$ is not discrete), and conversely the identity operator $Id_{c_0} : c_0 \longrightarrow c_0$ is AM-compact (because c_0 is discrete with order continuous norm), but is not b-weakly compact (because c_0 is not KB-space).

The third class is that of almost Dunford-Pettis operators introduced by Sanchez in [18]. Recall from [20] that an operator T from a Banach lattice E into a Banach space F is called almost Dunford-Pettis if the sequence $(\|T(x_n)\|)$ converges to 0 for every weakly null sequence (x_n) consisting of pairwise disjoint elements in E. Note that there is an AM-compact operator which is not almost Dunford-Pettis, and conversely there is an almost Dunford-Pettis operator which is not AM-compact. In fact, the identity operator $Id_{L^1[0,1]} : L^1[0,1] \longrightarrow L^1[0,1]$ is almost Dunford-Pettis (because $L^1[0,1]$ has the positive Schur property) but it is not AM-compact, and conversely the identity operator $Id_{c_0} : c_0 \longrightarrow c_0$ is AM-compact but is not almost Dunford-Pettis (because c_0 does not have the positive Schur property).

In [6], we studied the AM-compactness of positive Dunford-Pettis operators. The aim of this paper is to extend this study to other classes of operators, by characterizing Banach lattices for which each regular order weakly compact (resp. b-weakly compact, almost Dunford-Pettis, Dunford-Pettis) operator is AM-compact. Also, we will give some interesting consequences.

To state our results, we need to fix some notation and recall some definitions. A vector lattice is said to be Dedekind σ-complete if every nonempty countable subset that is bounded from above has a supremum. A nonzero element x of a vector lattice E is discrete if the order ideal generated by x equals the lattice subspace generated by x. The vector lattice E is discrete, if it admits a complete disjoint system of discrete elements. A Banach lattice is a Banach space $(E, \|.\|)$ such that E is a vector lattice and its norm satisfies the following property: for each $x, y \in E$ such that $|x| \leq |y|$, we have $\|x\| \leq \|y\|$. Note that the topological dual E', endowed with the dual norm and the dual order, is also a Banach lattice. A norm $\|.\|$ of a Banach lattice E is order continuous if for each generalized sequence (x_α) such that $x_\alpha \downarrow 0$ in E, the sequence (x_α) converges to 0 for the norm $\|.\|$ where the notation $x_\alpha \downarrow 0$ means that the sequence (x_α) is decreasing, its infimum exists and $\inf(x_\alpha) = 0$. A Banach lattice E is said to be a KB-space whenever every increasing norm bounded sequence of E^+ is norm convergent. As an example, each reflexive Banach lattice is a KB-space. A Banach lattice E is said to be an AM-space if for each $x, y \in E$ such that $\inf(x, y) = 0$, we have $\|x + y\| = \max\{\|x\|, \|y\|\}$. A Banach lattice E is said to have weakly sequentially continuous lattice operations whenever $x_n \longrightarrow 0$ in $\sigma(E, E')$ implies $\|x_n\| \longrightarrow 0$ in $\sigma(E, E')$. Note that every AM-space has this property ([2, Theorem 4.31]). Also, any discrete Banach lattice with an order continuous norm has weakly sequentially continuous lattice operations ([17, Proposition 2.5.23]).

For a bounded linear mapping $T : E \longrightarrow F$ between two Banach lattices, we will use the term operator. It is positive if $T(x) \geq 0$ in F whenever $x \geq 0$ in E. An operator $T : E \longrightarrow F$ is regular if $T = T_1 - T_2$ where T_1 and T_2 are positive operators from E into F. It is well known that each positive linear mapping on a Banach lattice is continuous. If an operator $T : E \longrightarrow F$ between two Banach lattices is positive, then its dual operator $T' : F' \longrightarrow E'$ is likewise positive, where T' is defined by $T'(f)(x) = f(T(x))$ for each $f \in F'$ and for each $x \in E$.

For terminology concerning Banach lattice theory and positive operators, we refer the reader to the excellent book of Aliprantis-Burkinshaw [2].

2. Preliminaries

Recall that an operator T from a Banach space E into another F is said to be Dunford-Pettis if it carries weakly compact subsets of E onto compact subsets of F. A Banach space E has the Dunford-Pettis property if every weakly compact operator defined on E (and taking their values in a Banach space F) is Dunford-Pettis.

Note that if E is a Banach lattice and X, Y are two Banach spaces, and if $T : E \to X$ and $S : X \to Y$ are two operators such that T is order weakly compact and S is Dunford-Pettis, then the composed operator $S \circ T$ is AM-compact.

To give a characterization of AM-compact operators, we need the following lemma.

Lemma 2.1. *Let E be a Banach lattice. Then the following assertions are equivalent.*

(1) *Every positive operator from E into E is AM-compact.*
(2) *The identity operator of the Banach lattice E is AM-compact.*
(3) *E is discrete and its norm is order continuous.*

Proof: (1)$\Longrightarrow$(2) Obvious.

(2)$\Longrightarrow$(3) Since the identity operator of E is AM-compact, then for each $x \in E_+$, the order interval $[0, x]$ is norm relatively compact, and since $[0, x]$ is norm closed, then $[0, x]$ is norm compact. Finally, Corollary 21.13 of [1] implies that E is discrete with order continuous norm.

(3)$\Longrightarrow$(1) Let T be a positive operator from E into E. Since E is discrete and its norm is order continuous, it follows from Corollary 21.13 of [1] that for each $x \in E_+$, the order interval $[0, x]$ is norm compact and hence $T[0, x]$ is norm compact. □

Let E be a Banach lattice. For each $u \in E_+$, we denote E_u the principal ideal generated by u, that we endow with the norm $\|.\|_\infty$ defined by $\|x\|_\infty = \inf\{\lambda > 0 : \|x\| \leq \lambda u\}$. It is an AM-space having u as the unit and $[-u, u]$ as the closed unit ball (see Theorem 4.21 of [2]), and the natural embedding $i_u : (E_u, \|.\|_\infty) \to E$ is continuous.

Moreover, for every $f \in E'$ we have $f \circ i_u \in (E_u)'$ and $\|f \circ i_u\|_{(E_u)'} = \sup\{|(f \circ i_u)(y)| : y \in [-u, u]\} = \sup\{|f(y)| : |y| \leq u\} = |f|(u)$.

Note that an operator $T : E \longrightarrow X$ is AM-compact if and only if for every $u \in E_+$ the composed map $T \circ i_u : E_u \longrightarrow E \longrightarrow X$ is compact. Thus $T : E \to X$ is AM-compact if and only if for every order bounded sequence (x_n) of E, the sequence $(T(x_n))$ has a norm convergent subsequence in X.

Now we are in position to give this characterization.

Proposition 2.2. *Let E be a Banach lattice, X a Banach space and T an operator from E into X. Then T is AM-compact if and only if for every order bounded sequence (x_n) in E such that $(T(x_n))$ converges weakly to x in X, we have* $\lim_n \|T(x_n) - x\| = 0$.

Proof: Let $T : E \longrightarrow X$ be an AM-compact operator and A an order bounded subset of E and let (x_n) be a sequence in A such that the sequence $(T(x_n))$ converges weakly to x in X. Since $T(A)$ is norm relatively compact and $(T(x_n))$ converges weakly to x in X, we obtain $\lim_n \|T(x_n) - x\| = 0$.

Conversely, consider the operator $T : E \longrightarrow X$ and let A be an order bounded subset of E. Choose $x \in E_+$ with $A \subset [-x, x]$. Let E_x be the principal ideal generated by x in E and endowed with the norm $\|\cdot\|_\infty$ and (x_n) be a weakly null sequence in E_x. Since the identity mapping $i_x : (E_x, \|\cdot\|_\infty) \longrightarrow (E, \|\cdot\|)$ is continuous, (x_n) converges weakly to 0 in E. Hence (Tx_n) converges weakly to zero in X, and thus $\|Tx_n\| \to 0$ by the assumption. Thus we have verified that $T \circ i_x : E_x \longrightarrow X$ is a Dunford-Pettis operator. Since $(E_x, \|\cdot\|_\infty)$ is an AM-space with unit, then by Theorem 2.1.3 of [2], $(E_x, \|\cdot\|_\infty)$ can be identified with a suitable $C(K)$-space. It follows from Theorem 4 of [15], that $T \circ i_x$ is weakly compact. Thus $T(A)$ is a relatively weakly compact subset of X.

Now we claim that $T(A)$ is relatively norm compact. Indeed, otherwise there would exist a sequence (Tx_n) in $T(A)$ without a norm convergent subsequence. By the relative weak compactness of $T(A)$ we may assume that (Tx_n) converges weakly to a point $x \in X$. But then we have a contradiction with the assumption. Therefore, $T(A)$ is a norm relatively compact subset of X, and hence $T : E \longrightarrow X$ is AM-compact. □

As a consequence of Proposition 2.2, we obtain the following characterization of a discrete Banach lattice with order continuous norm.

Corollary 2.3. *Let E be a Banach lattice. Then E is discrete and its norm is order continuous if and only if every order bounded weakly convergent sequence (x_n) in E is norm convergent.*

Proof: Let (x_n) be an order bounded and weakly convergent sequence in E. Since E is discrete with order continuous norm, it follows from Lemma 2.1 that its identity operator is AM-compact. And hence Proposition 2.2 implies that (x_n) is norm convergent.

Conversely, let (x_n) be an order bounded and weakly convergent sequence in E. Then (x_n) is norm convergent and it follows from Proposition 2.2 that the identity operator of E is AM-compact. Finally, Lemma 2.1 implies that E is discrete and its norm is order continuous. □

3. Major results

Note that each b-weakly compact operator is order weakly compact, but the converse is false in general. However, if the Banach lattice E has the (b)-property (i.e. each subset $A \subset E$ is order bounded in E whenever it is order bounded in its topological bidual E''), then the class of b-weakly compact operators on E coincides with that of order weakly compact operators on E.

On the other hand, each almost Dunford-Pettis operator is b-weakly compact. (In fact, let (x_n) be a disjoint b-order bounded sequence of E. Then (x_n) is an order bounded disjoint sequence of the topological bidual E''. So, $x_n \to 0$ for the topology $\sigma(E'', E''')$, and hence $x_n \to 0$ for the topology $\sigma(E, E')$. If $T : E \to X$ is almost Dunford-Pettis, then $T(x_n)$ converges in norm to 0 and hence it follows from Proposition 2.8 of [3] that T is b-weakly compact). However, a b-weakly compact operator is not necessarily almost Dunford-Pettis. In fact, the identity operator $Id_{\ell^2} : \ell^2 \to \ell^2$ is b-weakly compact, but it is not almost Dunford-Pettis.

Now, we are in position to give necessary and sufficient conditions under which each regular order weakly compact (resp. b-weakly compact, almost Dunford-Pettis, Dunford-Pettis) operator $T : E \longrightarrow F$ is AM-compact.

Theorem 3.1. *Let E and F be two Banach lattices such that the lattice operations of F are weakly sequentially continuous. Then the following statements are equivalent.*

(1) *Every regular order weakly compact operator $T : E \to F$ is AM-compact.*
(2) *Every regular b-weakly compact operator $T : E \to F$ is AM-compact.*
(3) *Every regular almost Dunford-Pettis operator $T : E \to F$ is AM-compact.*
(4) *One of the following conditions is valid:*
 (i) *E' is discrete,*
 (ii) *F is discrete with order continuous norm.*

Proof: (1)$\Longrightarrow$(2) Since every regular b-weakly compact operator is order weakly compact, it is evident that every regular b-weakly compact operator is AM-compact.

(2)$\Longrightarrow$(3) Since every regular almost Dunford-Pettis operator is b-weakly compact, then every regular almost Dunford-Pettis operator is AM-compact.

(3)$\Longrightarrow$(4) Suppose that E' is not discrete. So, we have to show that F is discrete and its norm is order continuous.

Suppose that F is not discrete or its norm is not order continuous. It follows from Corollary 2.4 the existence of an order bounded sequence $(y_n) \subset F$ which converges weakly to some y and $\lim_n \|y_n - y\| > \varepsilon$. Consider the sequence $(v_n) = (|y_n - y|)$. Since the lattice operations of F are weakly sequentially continuous, then (v_n) converges weakly to 0 and we have $\lim_n \|v_n\| > \varepsilon$. Now, by Corollary 2.3.5 of [17], there exist a subsequence $(k_n) \subset \mathbf{N}$ and a disjoint sequence $(z_n) \subset F_+$ such that $z_n \le v_{k_n}$ and $\|z_n\| \ge \frac{1}{2}$ for all $n \in \mathbf{N}$. Since (v_n) is order bounded then (z_n) is order bounded and hence there exists $z \in F_+$ such that $(z_n) \subset [0, z]$. By Lemma 3.4 of [7] there exists a positive disjoint sequence (g_n) of

F' with $\|g_n\| \leq 1$ such that

$$g_n(z_n) = 1 \text{ for all } n \text{ and } g_n(z_m) = 0 \text{ for } n \neq m.$$

On the other hand, as E' is not discrete, it follows from Theorem 3.1 of Chen-Wickstead [11] the existence of a sequence $(f_n) \subset E'$ such that $f_n \to 0$ in $\sigma(E', E)$ as $n \to \infty$ and $\|f_n\| = f > 0$ for all n and some $f \in E'$.

Now, we consider the operators $S, T : E \to F$ defined by

$$S(x) = \left(\sum_{n=1}^{\infty} f_n(x) \cdot z_n \right) \quad \text{and} \quad T(x) = f(x) \cdot z \quad \text{for all } x \in E.$$

Since $\sum_{n=1}^{\infty} \|f_n(x) \cdot z_n\| \leq \sum_{n=1}^{\infty} f(|x|) \cdot \|z_n\| \leq f(|x|) \cdot \|z\|$, the series defining S converges in norm for each $x \in E$. So, the operator S is well defined and is positive. Note that S and T are the same operators considered in Theorem 2 of [19].

Clearly, $0 \leq S \leq T$ holds. (In fact, for each $x \in E^+$ and each $n \geq 1$, we have $|\sum_{k=1}^{n} f_k(x) \cdot z_k| \leq \sum_{k=1}^{n} f(x) \cdot z_k \leq f(x) \cdot z$. Then $|\sum_{n=1}^{\infty} f_n(x) \cdot z_n| \leq f(x) \cdot z$ for each $x \in E^+$. Hence $0 \leq S(x) \leq T(x)$ for each $x \in E^+$.)

The operator T is compact and hence almost Dunford-Pettis. After that, it follows from the Corollary 2.3 of [9] that the operator S is almost Dunford-Pettis.

It remains to show that S is not AM-compact. Choose $u \in E_+$ such that $f(u) > 0$, and note that $(f_n \circ i_u)_{n=1}^{\infty}$ has no norm convergent subsequence in $(E_u)'$. In fact, for each $y \in E_u$ we have $f_n \circ i_u(y) = f_n(y) \to 0$ as $n \to \infty$. Then $f_n \circ i_u \to 0$ in $\sigma((E_u)', E_u)$. As $\|f_n \circ i_u\|_{(E_u)'} = \|f_n\|(u) = f(u) > 0$ for all n, we conclude that $(f_n \circ i_u)_{n=1}^{\infty}$ has no norm convergent subsequence in $(E_u)'$.

If S is AM-compact, then $S \circ i_u : E_u \to E \to F$ is compact and so is $(S \circ i_u)'$. As we have $(S \circ i_u)'(g) = (\sum_{n=1}^{\infty} g(z_n) \cdot (f_n \circ i_u))$ for all $g \in F'$, then $(S \circ i_u)'(g_k) = (f_k \circ i_u)$ for all k. Hence $((S \circ i_u)'(g_k))$ has a norm convergent subsequence in $(E_u)'$. We conclude that $(f_k \circ i_u)_k$ has a convergent subsequence in $(E_u)'$. This is a contradiction and then S is not AM-compact.

(4)(i)$\Longrightarrow$(1) Follows from Proposition 7 of [4].

(4)(ii)$\Longrightarrow$(1) Since $T : E \to F$ is a regular operator, then the image by T, of each order interval of E, is an order bounded subset of F. Finally, the result follows from Corollary 21.13 of [1]. □

Remark 3.2. The assumption "the lattice operations of F are weakly sequentially continuous" is essential in Theorem 3.1. For instance, every regular operator $T : L^1[0,1] \to L^2[0,1]$ is AM-compact. But neither $(L^1[0,1])'$ is discrete nor $L^2[0,1]$ is discrete with order continuous norm.

As consequences of Theorem 3.1, we obtain the following results:

Corollary 3.3. *Let F be a Banach lattice with weakly sequentially continuous lattice operations. Then the following statements are equivalent.*

(1) *Every regular order weakly compact operator $T : \ell^\infty \to F$ is AM-compact.*

(2) *Every regular b-weakly compact operator* $T : \ell^\infty \to F$ *is AM-compact.*
(3) *Every regular almost Dunford-Pettis operator* $T : \ell^\infty \to F$ *is AM-compact.*
(4) F *is discrete with order continuous norm.*

Corollary 3.4. *Let* E *be a Banach lattice, then the following statements are equivalent.*

(1) *Every regular order weakly compact operator* $T : E \to c$ *is AM-compact.*
(2) *Every regular b-weakly compact operator* $T : E \to c$ *is AM-compact.*
(3) *Every regular almost Dunford-Pettis operator* $T : E \to c$ *is AM-compact.*
(4) E' *is discrete.*

To give another consequence of Theorem 3.1, we need to recall from [8] that an operator T from a Banach lattice E into a Banach space X is said to be b-AM-compact if it carries each b-order bounded subset of E into a relatively compact subset of X.

Note that a regular order weakly compact (resp. b-weakly compact, almost Dunford-Pettis) operator is not necessarily b-AM-compact. In fact, the identity operator $Id_{L^1[0,1]} : L^1[0,1] \to L^1[0,1]$ is order weakly compact (resp. b-weakly compact, almost Dunford-Pettis) but it is not b-AM-compact (because $L^1[0,1]$ is not a discrete KB-space).

Theorem 3.5. *Let* E *and* F *be two Banach lattices such that the norm of* E *is order continuous and the lattice operations of* E *and* F *are weakly sequentially continuous. Then the following statements are equivalent.*

(1) *Every regular operator* $T : E \to F$ *is b-AM-compact.*
(2) *Every regular order weakly compact operator* $T : E \to F$ *is b-AM-compact.*
(3) *Every regular AM-compact operator* $T : E \to F$ *is b-AM-compact.*
(4) *One of the following conditions is valid:*
    (a) E *is a discrete KB-space,*
    (b) F *is a discrete KB-space.*

Proof: (1)$\Longrightarrow$(2) Obvious.

(2)$\Longrightarrow$(3) Since every regular AM-compact is order weakly compact, then every regular AM-compact operator is b-AM-compact.

(3)$\Longrightarrow$(4) Since the norm of E is order continuous and the lattice operations of E are weakly sequentially continuous, it follows from Corollary 2.3 of [12] that E is discrete.

Suppose that E is not a KB-space and that F is not a discrete KB-space. Since the norm of E is order continuous, then it follows from [10] that E contains a complemented copy of c_0. Hence, there exists a positive projection $P : E \to c_0$ and let $i : c_0 \to E$ be the injection of c_0 in E. And as F is not a discrete KB-space, it follows from Corollary 3.9 of [8] that there exists a regular operator $S : c_0 \longrightarrow F$ which is not b-AM-compact.

Consider the operator $T = S \circ P : E \to c_0 \to F$, since S and P are two regular operators and the identity operator Id_{c_0} is AM-compact, then $T = S \circ Id_{c_0} \circ P$

is AM-compact. But T is not b-AM-compact. Otherwise, the operator $T \circ i = S$ would be b-AM-compact, which is a contradiction.

(4)$\Longrightarrow$(1) Follows from [8, Corollary 2.4]. □

Remarks 3.6. (1) The assumption "the norm of E is order continuous" is essential in Theorem 3.5. For instance, every positive operator $T : l^\infty \to c_0$ is b-AM-compact. But neither l^∞ nor c_0 is a discrete KB-space.

(2) The assumption "the lattice operations of E are weakly sequentially continuous" is essential in Theorem 3.5. For instance, from [16, Theorem] it follows that each regular operator $T : L^1[0,1] \to c_0$ is Dunford-Pettis. Since $T = T \circ Id_{L^1[0,1]}$ and $Id_{L^1[0,1]}$ is b-weakly compact, it follows from Proposition 3.4 of [8] that the operator $T : L^1[0,1] \to c_0$ is b-AM-compact. But neither $L^1[0,1]$ nor c_0 is a discrete KB-space.

(3) The assumption "the lattice operations of F are weakly sequentially continuous" is essential in Theorem 3.5. For instance, from Theorem 6.8 of Wnuk [20] every regular operator $T : c_0 \to (l^\infty)'$ is compact. But neither c_0 nor $(l^\infty)'$ is a discrete KB-space.

Let us recall that a Banach space X has the Dunford-Pettis property if $\lim_n x'_n(x_n) = 0$ whenever (x_n) converges weakly to zero in X and (x'_n) converges weakly to zero in X'.

It follows from Theorem 5.82 of [2] that a Banach space X has the Dunford-Pettis property if and only if every weakly compact operator from X to an arbitrary Banach space is Dunford-Pettis.

We end this paper by establishing a result on the AM-compactness of Dunford-Pettis operators.

Theorem 3.7. *Let E and F be two Banach lattices such that E is Dedekind σ-complete and the lattice operations of F are weakly sequentially continuous. Then the following statements are equivalent.*

(1) *Every regular Dunford-Pettis operator $T : E \to F$ is AM-compact.*
(2) *One of the following conditions is valid:*
  (a) *the norm of E is order continuous,*
  (b) *F is discrete with order continuous norm.*

Proof: (2)(a)$\Rightarrow$(1). Let $T : E \to F$ be a regular Dunford-Pettis operator and let A be an order bounded subset of E. Since E has an order continuous norm, then it follows from Theorem 4.9 of [2] that A is weakly relatively compact. On the other hand, since the operator T is Dunford-Pettis, then $T(A)$ is norm relatively compact and hence T is AM-compact.

(2)(b)$\Rightarrow$(1). In this case it follows from Corollary 21.13 of [1] that every regular operator $T : E \to F$ is AM-compact.

(1)$\Rightarrow$(2). Assume that the norm of E is not order continuous and that F is not discrete with order continuous norm. Since E is Dedekind σ-complete, it follows from Corollary 2.4.3 of [17] that E contains a sublattice which is isomorphic to l^∞ and there exists a positive projection P from E onto l^∞. As the lattice operations of F are weakly sequentially continuous and F is not discrete with order continuous norm, it follows from Corollary 3.3 that there exists a regular almost Dunford-Pettis operator $S : l^\infty \to F$ which is not AM-compact. Since $S : l^\infty \to F$ is almost Dunford-Pettis, it is order weakly compact and as l^∞ is an AM-space with unit, $S : l^\infty \to F$ is weakly compact. As l^∞ has the Dunford-Pettis property, then $S : l^\infty \to F$ is Dunford-Pettis. We consider the operator product $T = S \circ P : E \to F$. Note that T is Dunford-Pettis because the operator S is Dunford-Pettis and the class of Dunford-Pettis operators is a two-sided ideal. But it is not AM-compact. If not, the operator $T \circ i = S$ would be AM-compact and this is a contradiction. □

Remark 3.8. The assumption "E is Dedekind σ-complete" is essential in Theorem 3.7. In fact, every regular Dunford-Pettis operator $T : c \to c$ is AM-compact (In fact, since T is Dunford-Pettis then T is almost Dunford-Pettis. As c' is discrete and the lattice operations of c are weakly sequentially continuous, then it follows from Theorem 3.1 that the operator T is AM-compact), but the norm of the Banach lattice c is not order continuous.

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