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A NON-ARCHIMEDEAN DUGUNDJI EXTENSION THEOREM

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Abstract. We prove a non-archimedean Dugundji extension theorem for the spaces $C^*(X,\mathbb{K})$ of continuous bounded functions on an ultranormal space X with values in a non-archimedean non-trivially valued complete field $\mathbb{K}$. Assuming that $\mathbb{K}$ is discretely valued and Y is a closed subspace of X we show that there exists an isometric linear extender $T\colon C^*(Y,\mathbb{K}) \to C^*(X,\mathbb{K})$ if X is collectionwise normal or Y is Lindelöf or $\mathbb{K}$ is separable. We provide also a self contained proof of the known fact that any metrizable compact subspace Y of an ultraregular space X is a retract of X.

Keywords: Dugundji extension theorem, non-archimedean space, space of continuous functions, 0-dimensional space

MSC 2010: 46S10, 54C35

1. Introduction

Let X be a completely regular Hausdorff space and $C^*(X,\mathbb{R})$ the space of all continuous and bounded real-valued functions on X. The classical Tietze-Urysohn theorem asserts that every continuous [and bounded] $\mathbb{R}$-valued map on a closed subspace Y of a normal space X can be extended to a continuous [and bounded] function on X. Then, there exists a linear extender $T\colon C^*(Y,\mathbb{R}) \to C^*(X,\mathbb{R})$, i.e., $T(f)|Y = f$ for each $f \in C^*(Y,\mathbb{R})$. Indeed, if B is any basis of the vector space $C^*(Y,\mathbb{R})$, then for each $f \in B$ there exists an extension $H(f) \in C^*(X,\mathbb{R})$. We can extend the map $H\colon B \to C^*(X,\mathbb{R})$, $f \mapsto H(f)$, linearly over $C^*(Y,\mathbb{R})$ to the map T as desired. It is natural to ask if T can be constructed to be continuous. This line of research has been intensively studied by many specialists, see [12], [16], [17] and the references therein, and we may summarize the early results addressed in the papers [1], [5], [14] by the following result $(*)$:

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If Y is a closed subspace of a metrizable space X, then there exists a continuous linear extender $T\colon C^*(Y,\mathbb{R}) \to C^*(X,\mathbb{R})$, provided both spaces are equipped with the sup-norm topology (in that case T is an isometry), the compact-open topology, or the topology of pointwise convergence.

If X is nonmetrizable, this fails in general; if $X = \beta\mathbb{N}$, where $\mathbb{N}$ is discrete, and $Y = \beta\mathbb{N} \setminus \mathbb{N}$, then no linear extender $T\colon C^*(Y,\mathbb{R}) \to C^*(X,\mathbb{R})$ is continuous, see [10], [12] and [17].

In 1966 Borges, see [3], introduced a class of topological spaces called stratifiable spaces for which $(*)$ is still true. Recall that every metrizable space is stratifiable and stratifiable spaces are included in the class of perfectly paracompact spaces. The only other large class of spaces for which some version of $(*)$ is known is the class of generalized ordered spaces, see [11].

It is known by Ostrovsky's theorem, see [18], that if a complete valued field K is not topologically isomorphic to the field of real numbers $\mathbb{R}$ or to the field of complex numbers $\mathbb{C}$, then on K there exists a non-archimedean valuation generating the original topology of K. In that case we call K non-archimedean.

Let $\mathbb{K}$ be a non-archimedean non-trivially valued complete field. We present a non-archimedean Dugundji extension theorem for spaces $C^*(X,\mathbb{K})$ over ultranormal spaces X. Recall that a topological space X is ultraregular if every point in X has a fundamental system of neigborhoods which are clopen sets; X is ultranormal if any two disjoint closed subsets of X can be separated by clopen sets. Theorems 1 and 2 (see below) motivate us also to (re)prove Theorem 3 stating that every metrizable compact subspace of an ultraregular space X is a retract of X; surely this is known but it is hard to locate. We provide two independent proofs of Theorem 3.

2. Results and proofs

Throughout this chapter X denotes a Hausdorff topological space and $\mathbb{K}$ is a non-archimedean non-trivially valued complete field. If Y is a compact subspace of an ultraregular space X and $\mathbb{K}$ is locally compact, then every $f \in C^*(Y,\mathbb{K})$ admits a continuous extension $g \in C^*(X,\mathbb{K})$, see [6, Theorem]. It is a classical fact that X is ultranormal if and only if for every closed subset $Y \subset X$ any $f \in C^*(Y,\mathbb{K})$ can be extended to some $g \in C^*(X,\mathbb{K})$, see [6, Theorem]. The following stronger result, for the proof see [15, Corollary 2.5.23] or [18, Theorem 5.24], partially motivates our work.

Theorem 1. *Let $\mathbb{K}$ be locally compact and Y be a closed subspace of an ultraregular space X. If Y is compact or X is ultranormal then there exists a linear extender $T\colon C^*(Y,\mathbb{K}) \to C^*(X,\mathbb{K})$ such that $\sup_{x\in X}\|Tf(x)\| = \sup_{y\in Y}\|f(y)\|$ for each $f \in C^*(Y,\mathbb{K})$.*

Theorem 1 provides an essential difference between the real and the non-archimedean Dugundji theorems. If $\mathbb{K}$ is locally compact, $X := \beta\mathbb{N}$ and $Y := \beta\mathbb{N}\setminus\mathbb{N}$, then Theorem 1 applies. The corresponding real case fails as we have mentioned above. The next result extends Theorem 1.

Theorem 2. *Assume that $\mathbb{K}$ is discretely valued and Y is a closed subspace of an ultranormal space X. Then there exists an isometric linear extender $T\colon C^*(Y,\mathbb{K}) \to C^*(X,\mathbb{K})$ if at least one of the following conditions holds:* (i) *X is collectionwise normal;* (ii) *Y is Lindelöf;* (iii) *$\mathbb{K}$ is separable.*

P r o o f. $\mathbb{K}$ is discretely valued, so $C^*(Y,\mathbb{K})$ has an orthonormal basis $(f_i)_{i\in I}$, see [18, Corollary 5.25]. By [15, Theorem 2.5.21], the bounded locally constant functions form a dense subspace of $C^*(Y,\mathbb{K})$, hence applying [18, Theorem 5.16, Excersise 5.C] we can assume that f_i is locally constant for every $i \in I$.

For the cases (i) and (ii) for each f_i let $(U_j)_{j\in I_i}$ be a clopen partition of Y such that $f_i(x) = \lambda_{i,j}$ if $x \in U_j$ ($\lambda_{i,j} \in \mathbb{K}$, $j \in I_i$); note that I_i is countable if Y is a Lindelöf space. Using [16, Lemma 5.1] (since X is strongly zero-dimensional by [8, Theorem 6.2.4]) for the case (i) or applying [7, Theorem 2.1] if (ii) is satisfied, we find a clopen partition $(V_j)_{j\in I_i}$ of X such that $V_j \cap Y = U_j$ for $j \in I_i$. Define

$$F_i(x) = \lambda_{i,j} \quad \text{if } x \in V_j \text{ for } j \in I_i.$$

In this way we obtain an extension F_i of f_i such that $\|F_i\| = \|f_i\|$ for every $i \in I$.

Assume (iii). Using the same proof as in [7, Theorem 3.1], for each $i \in I$ we construct $F_i \in C^*(X,\mathbb{K})$ such that $F_i(x) \in \overline{f_i(Y)}$ for every $x \in X$. Hence, $\|F_i\| \leqslant \|f_i\|$. Since $\|f_i\| \leqslant \|F_i\|$ holds trivially as F_i is an extension of f_i, we get $\|F_i\| = \|f_i\|$.

Then, the map

$$T\colon C^*(Y,\mathbb{K}) \to C^*(X,\mathbb{K}), \quad f = \sum_{i\in I} \lambda_i f_i \longmapsto T(f) = \sum_{i\in I} \lambda_i F_i$$

is an isometric linear extender. Indeed, let $f = \sum_{i\in I} \lambda_i f_i \in C^*(Y,\mathbb{K})$. Then for every $\varepsilon > 0$, the set $\{i \in I\colon \|\lambda_i F_i\| \geqslant \varepsilon\}$ $(= \{i \in I\colon \|\lambda_i f_i\| \geqslant \varepsilon\})$ is finite, so the sum $\sum_{i\in I} \lambda_i F_i$ there exists ([18, Exercise 3.K]). Thus, T is well-defined; clearly it is linear. $T(f)$ is an extension of f, since F_i is an extension of f_i, for every $i \in I$. Clearly, $\|f\| \leqslant \|T(f)\|$. Since $\|F_i\| = \|f_i\|$ for every $i \in I$ and $(f_i)_{i\in I}$ is orthonormal, we have

$$\|T(f)\| \leqslant \max_{i\in I} |\lambda_i| \cdot \|F_i\| = \max_{i\in I} |\lambda_i| \cdot \|f_i\| = \|f\|.$$

Thus $\|Tf\| = \|f\|$, so T is an isometry. □

A result of Arkhangel'skij-Choban, see [2], states that if Y is a metrizable compact subspace of a completely regular space X, then there exists a continuous linear extender $T\colon C_p(Y,\mathbb{R}) \to C_p(X,\mathbb{R})$, although Y need not be a retract of X. This, along with Theorems 1 and 2 motivates us also to (re)prove Theorem 3. We present two proofs of it. The idea of the first one, suggested to the authors by Professor W. Marciszewski, essentially depends on the known fact (due to Sierpinski, see [13, Section 26 II, Corollary 2] or [9, Proposition 17.10]) stating that every nonempty closed subset of the Cantor set is a retract of it. The second proof, totally different from the first one, depends on two lemmas (of their own interest), and although it is longer, it is self-contained.

Theorem 3. *Any metrizable compact subspace Y of an ultraregular space X is a retract of X.*

The first Proof of Theorem 3. By assumption Y is second countable (see [8, Theorems 4.2.8]), hence, we can select (U_n), a sequence of clopen subsets of Y which form a base of the topology of Y. In the same way as in the proof of [15, Corollary 2.5.23] we find (V_n), a sequence of clopen sets of X such that $V_n \cap Y = U_n$ for every $n \in N$. Define a diagonal product $\Delta\colon X \to \{0,1\}^N$ by $\Delta(x) = (\xi_{V_1}(x), \xi_{V_2}(x), \ldots)$, where ξ_{V_i} denotes the characteristic function of V_i. By [8, Theorem 2.3.6], Δ is continuous and $\Delta_{/Y}\colon Y \to \varphi(Y)$ is a homeomorphism (see [8, Theorems 2.3.20 and 6.2.16]). It is a well-known fact that there exists a homeomorphism $h\colon \{0,1\}^N \to C$, where C is the Cantor set. Then, $(h\circ\Delta)(Y) \subset C$ is closed since Y is compact. By [13, Section 26 II, Corollary 2], there exists a continuous retraction $r\colon C \to (h\circ\Delta)(Y)$. Thus, $(\Delta_{/Y})^{-1}\circ h^{-1}\circ r\circ h\circ\Delta$ is the required retraction $X \to Y$. $\square$

To present the promised second proof of Theorem 3, we need some extra work and two lemmas. Let $Y = (Y,d)$ be an ultrametric space. By a closed ball in Y we mean a subset of Y of the form $B(x,r) := \{y \in Y\colon d(x,y) \leqslant r\}$, where $x \in Y$ and $r \in (0,\infty)$; clearly any closed ball in Y is a clopen set. We denote the family of all closed balls in Y by $B(Y)$. Any two balls in Y are either disjoint, or one is contained in the other. Hence for any finite family $\{B_i\colon i \in I\} \subset B(Y)$ there is a subset J of I such that $\{B_i\colon i \in J\}$ is a partition of $\bigcup\limits_{i\in I} B_i$. Thus, if Y is compact, any open subset of Y is the sum of a countable family of pairwise disjoint closed balls in Y.

Lemma 4. *Let $B_i = B(y_i,r_i)$, $i \in \mathbb{N}$, be a sequence of pairwise different closed balls in a compact ultrametric space $Y = (Y,d)$. Then $\lim\limits_i r_i = 0$.*

P r o o f. Suppose, to the contrary, that for some $r > 0$ the set $M_r = \{i \in \mathbb{N}\colon\ r_i > r\}$ is infinite. Put $\mathcal{B}_r = \{B_i\colon\ i \in M_r\}$. Denote by $\mathcal{M}_r$ the family of all maximal totally ordered subsets of $(\mathcal{B}_r, \subseteq)$. Consider two cases:

(1) Any element of $\mathcal{M}_r$ is finite. Denote by $B_{i(M)}$ the minimal element of $M \in \mathcal{M}_r$. Then the balls $B_{i(M)}, M \in \mathcal{M}_r$, are pairwise disjoint. Thus, $d(y_{i(M)}, y_{i(M')}) > r$ for all $M, M' \in \mathcal{M}_r$ with $M \neq M'$. By the compactness of Y we infer that $\mathcal{M}_r$ is finite; so M_r is finite, a contradiction.

(2) Some element M_0 of $\mathcal{M}_r$ is infinite. Let $N_0 = \{i \in \mathbb{N}\colon\ B_i \in M_0\}$; clearly, for $i, j \in N_0$ we have $B_i \subsetneq B_j$ if and only if $r_i < r_j$. The sequence $(r_i)_{i \in N_0}$ has a strictly monotonic subsequence (r_{i_k}), thus (B_{i_k}) is strictly monotonic. Suppose that (B_{i_k}) is strictly decreasing. For every $k \in \mathbb{N}$ select $x_k \in B_{i_k} \setminus B_{i_{k+1}}$, then

$$d(x_k, x_{k+1}) > r_{k+1} \geqslant d(x_{k+1}, x_{k+2}) > r_{k+2} \geqslant \ldots > r,$$

hence, (x_k) has no convergent subsequence. Similarly, assuming that (B_{i_k}) is strictly increasing, we choose a sequence (x_k) with the same property. This contradicts the compactness of Y. So, both cases yield that $\lim\limits_i r_i = 0$. $\square$

Lemma 5. (A) *Let Y be an ultrametric, compact space. Then, there exists (U_n), a sequence of closed balls in Y such that*

(v1) $U_1 = Y$, $U_n \not\subseteq \bigcup\limits_{j=n+1}^{\infty} U_j$ *for all $n \in \mathbb{N}$ and (ξ_{U_n}), where ξ_{U_n} denotes the characteristic function of U_n, is a maximal orthonormal sequence in $C(Y, \mathbb{K})$.*

(B) *Let Y be an ultrametric compact subspace of an ultraregular space X. Then, for every sequence (U_n) of closed balls in Y which satisfies* (v1) *there exists (V_n), a sequence of clopen subsets of X, such that*

(v2) $V_1 = X$ *and* $V_n \cap Y = U_n$;

(v3) $V_n \subset V_m$ *if* $U_n \subset U_m$;

(v4) $V_n \cap V_m = \emptyset$ *if* $U_n \cap U_m = \emptyset$ *for all* $n, m \in \mathbb{N}$.

P r o o f. (A) Denote by $\mathcal{M}$ the family of all $M \subset B(Y)$ with $Y \in M$ such that $\{\xi_B\colon\ B \in M\}$ is linearly independent in $C(Y, \mathbb{K})$. By the Kuratowski-Zorn Lemma, $(\mathcal{M}, \subseteq)$ has a maximal element $M_0 = \{B_i\colon\ i \in I\}$. It is easy to see that I is infinite and countable by Lemma 4; so, we can assume that $I = \mathbb{N}$. Let $B_i = B(y_i, r_i)$ for $i \in \mathbb{N}$. It follows from Lemma 4 that $\lim\limits_i r_i = 0$. Let π be a permutation of $\mathbb{N}$ such that $(r_{\pi(i)})_i$ is decreasing. Put $U_i = B_{\pi(i)}$ for $i \in \mathbb{N}$. Clearly, for $i, j \in \mathbb{N}$ with $i > j$ we have $U_i \subsetneq U_j$ or $U_i \cap U_j = \emptyset$. Moreover, $U_i \not\subseteq \bigcup\limits_{j=i+1}^{\infty} U_j$ for any $i \in \mathbb{N}$. Indeed, in the opposite case there exist $i_0, k \in \mathbb{N}$ and $j(1), \ldots, j(k) \in \{i_0 + 1, i_0 + 2, \ldots\}$

such that $\{U_{j(1)}, \ldots, U_{j(k)}\}$ is a partition of U_{i_0}. Then $\xi_{U_{i_0}} = \sum_{n=1}^{k} \xi_{U_{j(n)}}$; so (ξ_{U_i}) is linearly dependent, a contradiction.

(B) Let $(U_n) \subset Y$ be a sequence of closed balls which satisfies (v1). We form inductively the required sequence (V_n). Set $V_1 = X$. Assume that for some $k \in \mathbb{N}$ we have constructed clopen sets $V_1, \ldots, V_k$ in X that satisfy (v2)–(v4) for all $n, m \in \{1, \ldots, k\}$.

Let $W_1 = \{1 \leqslant n \leqslant k \colon U_{k+1} \subset U_n\}$ and $W_2 = \{1 \leqslant n \leqslant k \colon U_n \cap U_{k+1} = \emptyset\}$. Clearly $\{W_1, W_2\}$ is a partition of $\{1, \ldots, k\}$. The set

$$V'_{k+1} = \bigcap_{n \in W_1} V_n \cap \bigcap_{n \in W_2} (X \setminus V_n)$$

is clopen in X and

$$V'_{k+1} \cap Y = \bigcap_{n \in W_1} U_n \cap \bigcap_{n \in W_2} (Y \setminus U_n) \supset U_{k+1}.$$

By the proof of [15, Corollary 2.5.23], there exists a clopen set V''_{k+1} in X with $V''_{k+1} \cap Y = U_{k+1}$. Thus the set $V_{k+1} = V'_{k+1} \cap V''_{k+1}$ is clopen in X and $V_{k+1} \cap Y = U_{k+1}$. Moreover for all $1 \leqslant n \leqslant k$ we have: $V_{k+1} \subset V_n$ if $U_{k+1} \subset U_n$ and $V_{k+1} \cap V_n = \emptyset$ if $U_{k+1} \cap U_n = \emptyset$. This inductive procedure provides the required sequence (V_n). $\square$

The second Proof of Theorem 3. If Y is finite, then X has a clopen partition $\{U_y \colon y \in Y\}$ such that $y \in U_y$ for $y \in Y$. Define the map $\varphi \colon X \to Y$ by $\varphi(x) = y$ for any $x \in U_y$ and $y \in Y$. Clearly, φ is a retraction. Hence Y is a retract of X.

Assume that Y is infinite. Since every metrizable compact ultraregular space is strongly zero-dimensional, see [8], we apply [4, Theorem II] to deduce that Y is an ultrametric space. Applying Lemma 5, we can select (U_n), a sequence of closed balls in Y, which satisfies (v1) and (V_n), a sequence of clopen subsets of X satisfying (v2)–(v4). Let $x \in X$ and $N_x = \{n \in \mathbb{N} \colon x \in V_n\}$. Consider two cases.

(1) N_x is finite: then, by (v1) of Lemma 5, $W_x := U_{n(x)} \setminus \bigcup \{U_n \colon n > n(x)\}$, where $n(x)$ is the greatest element of N_x, is nonempty. Take $w \in W_x$ and set $\varphi(x) := w$. Fix $n \in N_x$. If $x \in V_n$, then $U_{n(x)} \subset U_n$ by (v4) and (v1), thus $\varphi(x) \in U_n$. On the other hand, $\varphi(x) \in U_n$ implies $U_{n(x)} \subset U_n$ by (v1), hence $x \in V_n$ by (v3). If $x \in Y$, then $W_x = \{x\}$. Indeed, clearly $x \in W_x$. For the converse inclusion assume that there exists $y \in W_x$ with $y \neq x$. If $n < n(x)$ then $x \in U_n$ if and only if $U_{n(x)} \subset U_n$ if and only if $y \in U_n$. If $n = n(x)$ then $x, y \in U_n$. If $n > n(x)$ then $x, y \notin U_n$. Thus, $\xi_{U_n}(x) = \xi_{U_n}(y)$ for any $n \in \mathbb{N}$. Since Y is ultraregular, there exists a closed ball W with $x \in W \subset Y \setminus \{y\}$. Then $\xi_W(x) \neq \xi_W(y)$, so $\xi_W \notin [(\xi_{U_n})_n]$, a contradiction with the maximality of (ξ_{U_n}) given by (v1).

(2) N_x is infinite: By (v_1) of Lemma 5, the sequence $(U_n)_{n\in N_x}$ is strictly decreasing, so its intersection is nonempty since Y is compact. Then we take $\varphi(x) \in \bigcap\{U_n\colon n \in N_x\}$. Note that $\varphi(x) \in U_m$ for some $m \in \mathbb{N}$ implies, by Lemma 4, $U_n \subset U_m$ for some $n \in N_x$; hence, $x \in V_m$ by (v3). If $x \in Y$, by Lemma 4 we get $\bigcap\{U_n\colon n \in N_x\} = \{x\}$.

We see that $x \in V_k$ if and only if $\varphi(x) \in U_k$ for all $x \in X$ and $k \in \mathbb{N}$; hence, $\xi_{U_k} \circ \varphi = \xi_{V_k}$ for all $k \in \mathbb{N}$. Let $U \subset Y$ be a closed ball. By (v1) and [15, Theorem 2.5.22], there exist $n \in \mathbb{N}$ and scalars $\alpha_1, \ldots, \alpha_n \in \mathbb{K}$ such that $\xi_U = \sum\limits_{i=1}^{n} \alpha_i \xi_{U_i}$. Put $f = \sum\limits_{i=1}^{n} \alpha_i \xi_{V_i}$; clearly $f \in C(X, \mathbb{K})$ and $f = \xi_U \circ \varphi$. The set $A = \{\alpha \in \mathbb{K}\colon |\alpha - 1| < 1\}$ is open in $\mathbb{K}$ and $f^{-1}(A) = \varphi^{-1}(\xi_U^{-1}(A)) = \varphi^{-1}(U)$. Thus $\varphi^{-1}(U)$ is open in X for any closed ball $U \subset Y$. It follows that φ is continuous. $\square$

Using Theorem 3 we get the following

Corollary 6. *If X is an ultraregular space and $C_p(X, \mathbb{K})$ is strictly of countable type, then every compact subspace Y of X is a retract of X.*

P r o o f. By [15, Theorem 4.3.4] there exists a continuous injection from X to $\mathbb{K}$. Therefore there exists on X a weaker utrametric topology. Thus any compact subspace Y of X is metrizable and Theorem 3 applies. $\square$

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