Remarks on strongly star-Menger spaces

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Abstract. A space X is *strongly star-Menger* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X, there exists a sequence $(K_n : n \in N)$ of finite subsets of X such that $\{St(K_n,\mathcal{U}_n) : n \in \mathbb{N}\}$ is an open cover of X. In this paper, we investigate the relationship between strongly star-Menger spaces and related spaces, and also study topological properties of strongly star-Menger spaces.

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Classification: 54D20, 54C10

1. Introduction

By a space we mean a topological space. Let us recall that a space X is *countably compact* if every countable open cover of X has a finite subcover. Van Douwen et al. [2] defined a space X to be *strongly starcompact* if for every open cover $\mathcal{U}$ of X, there exists a finite subset F of X such that $St(F,\mathcal{U}) = X$, where $St(F,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap F \neq \emptyset\}$. They proved that every countably compact space is strongly starcompact and every T_2 strongly starcompact space is countably compact, but this does not hold for T_1-spaces (see [10, Example 2.5]).

Van Douwen et al. [2] defined a space X to be *strongly star-Lindelöf* if for every open cover $\mathcal{U}$ of X, there exists countable subset F of X such that $St(F,\mathcal{U}) = X$.

In [5], a strongly starcompact space is called starcompact and in [8], a strongly star-Lindelöf space is called star-Lindelöf.

Kočinac [6], [7] defined a space X to be *strongly star-Menger* if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X, there exists a sequence $(K_n : n \in \mathbb{N})$ of finite subsets of X such that $\{St(K_n,\mathcal{U}_n) : n \in \mathbb{N}\}$ is an open cover of X.

From the above definitions, it is not difficult to see that every strongly starcompact space is strongly star-Menger and every strongly star-Menger space is strongly star-Lindelöf.

The purpose of this paper is to investigate the relationship between strongly star-Menger spaces and related spaces, and study topological properties of strongly star-Menger spaces.

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Throughout this paper, let ω denote the first infinite cardinal, ω_1 the first uncountable cardinal, $\mathfrak{c}$ the cardinality of the set of all real numbers. For a cardinal κ, let κ^+ be the smallest cardinal greater than κ. For each ordinals α, β with $\alpha < \beta$, we write $[\alpha, \beta) = \{\gamma : \alpha \leq \gamma < \beta\}$, $(\alpha, \beta] = \{\gamma : \alpha < \gamma \leq \beta\}$, $(\alpha, \beta) = \{\gamma : \alpha < \gamma < \beta\}$ and $[\alpha, \beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$. As usual, a cardinal is an initial ordinal and an ordinal is the set of smaller ordinals. Every cardinal is often viewed as a space with the usual order topology. Other terms and symbols that we do not define follow [4].

2. Strongly star-Menger spaces

First we give some examples showing relationships between strongly star-Menger spaces and related spaces.

Example 2.1. There exists a Tychonoff strongly star-Menger space X which is not strongly starcompact.

Proof: Let

$$X = ([0, \omega] \times [0, \omega]) \setminus \{\langle \omega, \omega \rangle\}$$

be the subspace of the product space $[0, \omega] \times [0, \omega]$. Then X is not countably compact, since $\{\langle \omega, n \rangle : n \in \omega\}$ is a countable discrete closed subset of X. Hence X is not strongly starcompact.

Next we show that X is strongly star-Menger. To this end, let $\{\mathcal{U}_n : n \in \mathbb{N}\}$ be a sequence of open covers of X. For each $n \in \mathbb{N}$, let $F_n = ([0, \omega] \times \{n-1\}) \cup (\{n-1\} \times [0, \omega])$. Then $X = \bigcup_{n \in \mathbb{N}} F_n$ and F_n is a compact subset of X for each $n \in \mathbb{N}$. We can find a finite subset K_n of F_n such that $F_n \subseteq St(K_n, \mathcal{U}_n)$ for each $n \in \mathbb{N}$. Thus the sequence $(K_n : n \in \mathbb{N})$ witnesses for $(\mathcal{U}_n : n \in \mathbb{N})$ that X is strongly star-Menger. $\square$

Next we give an example of a Tychonoff strongly star-Lindelöf space which is not strongly star-Menger by using the following example from [1]. We make use of two of the cardinals defined in [3]. Define ${}^{\omega}\omega$ as the set of all functions from ω to itself. For all $f, g \in {}^{\omega}\omega$, we say $f \leq^* g$ if and only if $f(n) \leq g(n)$ for all but finitely many n. The unbounding number, denoted by $\mathfrak{b}$, is the smallest cardinality of an unbounded subset of $({}^{\omega}\omega, \leq^*)$. The dominating number, denoted by $\mathfrak{d}$, is the smallest cardinality of a cofinal subset of $({}^{\omega}\omega, \leq^*)$. It is not difficult to show that $\omega_1 \leq \mathfrak{b} \leq \mathfrak{d} \leq \mathfrak{c}$ and it is known that $\omega_1 < \mathfrak{b} = \mathfrak{c}$, $\omega_1 < \mathfrak{d} = \mathfrak{c}$ and $\omega_1 \leq \mathfrak{b} < \mathfrak{d} = \mathfrak{c}$ are all consistent with the axioms of ZFC (see [3] for details).

Example 2.2 ([1])**.** Let $\mathcal{A}$ be an almost disjoint family of infinite subsets of ω (i.e., the intersection of every two distinct elements of $\mathcal{A}$ is finite) and let $X = \omega \cup \mathcal{A}$ be the Isbell-Mrówka space constructed from $\mathcal{A}$ ([2], [4]). Then X is strongly star-Menger if and only if $|\mathcal{A}| < \mathfrak{d}$.

Example 2.3. There exists a Tychonoff strongly star-Lindelöf space X which is not strongly star-Menger.

Proof: Let $X = \omega \cup \mathcal{A}$ be the Isbell-Mrówka space, where $\mathcal{A}$ is the maximal almost disjoint family of infinite subsets of ω with $|\mathcal{A}| = \mathfrak{c}$. Then X is not strongly star-Menger by Example 2.2. Since ω is a countable dense subset of X, X is strongly star-Lindelöf. Thus we complete the proof. $\square$

Since strong starcompactness is equivalent to countable compactness for Hausdorff spaces (see [2]), the extent $e(X)$ of every T_2 strongly starcompact space X is finite. Assuming $\mathfrak{d} = \mathfrak{c}$, let $X = \omega \cup \mathcal{A}$ be the Isbell-Mrówka space with $|\mathcal{A}| = \omega_1$. Then, by Example 2.2, X is a strongly star-Menger space with $e(X) = \omega_1$, since $\mathcal{A}$ is a discrete closed subset of X.

The author does not know if there exists a Tychonoff strongly star-Menger space X such that $e(X) \geq \mathfrak{c}$.

For a T_1-space X, the extent $e(X)$ of a strongly star-Menger space can be arbitrarily large.

Example 2.4. For every infinite cardinal κ, there exists a T_1 strongly star-Menger space X such that $e(X) \geq \kappa$.

Proof: Let κ be an infinite cardinal and let $D = \{d_\alpha : \alpha < \kappa\}$ be a set of cardinality κ. Let $X = [0, \kappa^+) \cup D$. We topologize X as follows: $[0, \kappa^+)$ has the usual order topology and is an open subspace of X; a basic neighborhood of a point $d_\alpha \in D$ takes the form

$$O_\beta(d_\alpha) = \{d_\alpha\} \cup (\beta, \kappa^+) \quad \text{where} \quad \beta < \kappa^+.$$

Then X is a T_1 space and $e(X) = \kappa$, since D is discrete closed in X. To show that X is strongly star-Menger, we only prove that X is strongly starcompact, since every strongly starcompact space is strongly star-Menger. To this end, let $\mathcal{U}$ be an open cover of X. Without loss of generality, we can assume that $\mathcal{U}$ consists of basic open subsets of X. Thus it is sufficient to show that there exists a finite subset F of X such that $St(F, \mathcal{U}) = X$. Since $[0, \kappa^+)$ is countably compact, it is strongly starcompact (see [2, 8]). Then we can find a finite subset F_1 of $[0, \kappa^+)$ such that $[0, \kappa^+) \subseteq St(F_1, \mathcal{U})$. On the other hand, for each $\alpha < \kappa$, there exists $\beta_\alpha < \kappa^+$ such that $O_{\beta_\alpha}(d_\alpha)$ is included in some member of $\mathcal{U}$. If we choose $\beta < \kappa^+$ with $\beta > \sup\{\beta_\alpha : \alpha < \kappa\}$, then $D \subseteq St(\beta, \mathcal{U})$. Let $F = F_1 \cup \{\beta\}$. Then F is finite and $St(F, \mathcal{U}) = X$. Hence X is strongly star-Menger. $\square$

Next we study topological properties of strongly star-Menger spaces. In [11], the author gave an example that assuming $\mathfrak{d} = \mathfrak{c}$, there exists a Tychonoff strongly star-Menger space having a regular-closed subspace which is not strongly star-Menger. But the author does not know if there exists an example in ZFC (that is, without any set-theoretic assumption) showing that a regular-closed subspace (or zero-set) of a strongly star-Menger space is strongly star-Menger.

For a space X, recall that the Alexandroff duplicate $A(X)$ of X is constructed in the following way: The underlying set of $A(X)$ is $X \times \{0,1\}$; each point of $X \times \{1\}$ is isolated and a basic neighborhood of $\langle x, 0\rangle \in X \times \{0\}$ is a set of the form $(U \times \{0\}) \cup ((U \times \{1\}) \setminus \{\langle x, 1\rangle\})$, where U is a neighborhood of x in X. It is well known that a space X is countably compact if and only if so is $A(X)$. In the following, we give two examples to show that the result cannot be generalized to strongly star-Menger spaces.

Example 2.5. Assuming $\mathfrak{d} = \mathfrak{c}$, there exists a Tychonoff strongly star-Menger space X such that $A(X)$ is not strongly star-Menger.

Proof: Assuming $\mathfrak{d} = \mathfrak{c}$, let $X = \omega \cup \mathcal{A}$ be the Isbell-Mrówka space with $|\mathcal{A}| = \omega_1$. Then X is strongly star-Menger by Example 2.2 with $e(X) = \omega_1$, since $\mathcal{A}$ is discrete closed in X. However $A(X)$ is not strongly star-Menger. In fact, the set $\mathcal{A} \times \{1\}$ is an open and closed subset of X with $|\mathcal{A} \times \{1\}| = \omega_1$, and each point $\langle a, 1\rangle$ is isolated for each $a \in \mathcal{A}$. Hence $A(X)$ is not strongly star-Menger, since every open and closed subset of a strongly star-Menger space is strongly star-Menger and $\mathcal{A} \times \{1\}$ is not strongly star-Menger. $\square$

Now we give a positive result. For showing the result, first we give a lemma.

Lemma 2.6. *For T_1-space X, $e(X) = e(A(X))$.*

Proof: Since X is homeomorphic to the closed subset $X \times \{0\}$ of $A(X)$, we have $e(X) \leq e(A(X))$. On the other hand, let F is any closed discrete subset of $A(X)$. Then $F \cap (X \times \{0\})$ is closed in $X \times \{0\}$ by the construction of the topology of $A(X)$. Hence $|F \cap (X \times \{0\})| \leq e(X)$. Moreover it is not difficult to see that $\{\langle x, 0\rangle : \langle x, 1\rangle \in F\}$ is closed discrete in $X \times \{0\}$. This implies that $|F \cap (X \times \{1\})| \leq e(X)$. Thus $e(A(X)) \leq e(X)$. Therefore $e(X) = e(A(X))$. $\square$

Theorem 2.7. *If X is a strongly star-Menger space with $e(X) < \omega_1$, then $A(X)$ is strongly star-Menger.*

Proof: We show that $A(X)$ is strongly star-Menger. To this end, let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of open covers of $A(X)$. For each $n \in \mathbb{N}$ and each $x \in X$, choose an open neighborhood $W_{n_x} = (V_{n_x} \times \{0,1\}) \setminus \{\langle x, 1\rangle\}$ of $\langle x, 0\rangle$ satisfying that there exists some $U \in \mathcal{U}_n$ such that $W_{n_x} \subseteq U$, where V_{n_x} is an open subset of X containing x. For each $n \in \mathbb{N}$, let $\mathcal{V}_n = \{V_{n_x} : x \in X\}$. Then $(\mathcal{V}_n : n \in \mathbb{N})$ is a sequence of open covers of X. There exists a sequence $(K'_n : n \in \mathbb{N})$ of finite subsets of X such that $\bigcup_{n\in\mathbb{N}} St(K'_n, \mathcal{V}_n) = X$, since X is strongly star-Menger. For each $n \in \mathbb{N}$, let $K''_n = K'_n \times \{0,1\}$. Then K''_n is a finite subset of $A(X)$ and $X \times \{0\} \subseteq \bigcup_{n\in\mathbb{N}} St(K''_n, \mathcal{U}_n)$. Let $A = A(X) \setminus \bigcup_{n\in\mathbb{N}} St(K''_n, \mathcal{U}_n)$. Then A is a discrete closed subset of $A(X)$. By Lemma 2.6, the set A is countable and we can enumerate A as $\{a_n : n \in \mathbb{N}\}$. For each $n \in \mathbb{N}$, let $K_n = (K''_n \times \{0,1\}) \cup \{a_n\}$. Then K_n is a finite subset of $A(X)$ and $A(X) = \bigcup_{n\in\mathbb{N}} St(K_n, \mathcal{U}_n)$, which shows that $A(X)$ is strongly star-Menger. $\square$

From the proof of Example 2.5, it is not difficult to show the following result.

Theorem 2.8. *If X is a T_1-space and $A(X)$ is a strongly star-Menger space, then $e(X) < \omega_1$,*

PROOF: Suppose that $e(X) \geq \omega_1$. Then there exists a discrete closed subset B of X such that $|B| \geq \omega_1$. Hence $B \times \{1\}$ is an open and closed subset of $A(X)$ and every point of $B \times \{1\}$ is an isolated point. Thus $A(X)$ is not strongly star-Menger, since every open and closed subset of a strongly star-Menger space is strongly star-Menger and $B \times \{1\}$ is not strongly star-Menger. □

We have the following corollary from Theorems 2.7 and 2.8.

Corollary 2.9. *If X is a strongly star-Menger T_1-space, then $A(X)$ is strongly star-Menger if and only if $e(X) < \omega_1$.*

Remark 2.10. The author does not know if there is a space X such that $A(X)$ is strongly star-Menger, but X is not strongly star-Menger.

It is not difficult to show the following result.

Theorem 2.11. *A continuous image of a strongly star-Menger space is strongly star-Menger.*

Next we turn to consider preimages. We show that the preimage of a strongly star-Menger space under a closed 2-to-1 continuous map need not be strongly star-Menger,

Example 2.12. There exist spaces X and Y, and a closed 2-to-1 continuous map $f : X \to Y$ such that Y is a strongly star-Menger space, but X is not strongly star-Menger.

PROOF: Let Y be the space $\omega \cup \mathcal{A}$ of Example 2.5, and X be the Alexandroff duplicate of Y. Then Y is strongly star-Menger, but X is not. Let $f : X \to Y$ be the projection. Then f is a closed 2-to-1 continuous map, which completes the proof. □

Now, we give a positive result:

Theorem 2.13. *Let f be an open and closed, finite-to-one continuous map from a space X onto a strongly star-Menger space Y. Then X is strongly star-Menger.*

PROOF: Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of open covers of X and let $y \in Y$. Since $f^{-1}(y)$ is finite, for each $n \in \mathbb{N}$ there exists a finite subcollection $\mathcal{U}_{n_y}$ of $\mathcal{U}_n$ such that $f^{-1}(y) \subseteq \bigcup \mathcal{U}_{n_y}$ and $U \cap f^{-1}(y) \neq \emptyset$ for each $U \in \mathcal{U}_{n_y}$. Since f is closed, there exists an open neighborhood V_{n_y} of y in Y such that $f^{-1}(V_{n_y}) \subseteq \bigcup\{U : U \in \mathcal{U}_{n_y}\}$. Since f is open, we can assume that

$$V_{n_y} \subseteq \bigcap\{f(U) : U \in \mathcal{U}_{n_y}\}. \tag{1}$$

For each $n \in \mathbb{N}$, taking such open set V_{n_y} for each $y \in Y$, we have an open cover $\mathcal{V}_n = \{V_{n_y} : y \in Y\}$ of Y. Thus $(\mathcal{V}_n : n \in \mathbb{N})$ is a sequence of open covers of Y, so that there exists a sequence $(K_n : n \in \mathbb{N})$ of finite subsets of Y such that $\{St(K_n, \mathcal{V}_n) : n \in \mathbb{N}\}$ is an open cover of Y, since Y is strongly star-Menger. Since f is finite-to-one, the sequence $(f^{-1}(K_n) : n \in N)$ is the sequence of finite subsets of X. We show that $\{St(f^{-1}(K_n), \mathcal{U}_n) : n \in \mathbb{N}\}$ is an open cover of X. Let $x \in X$. Then there exists $n \in \mathbb{N}$ and $y \in Y$ such that $f(x) \in V_{n_y}$ and $V_{n_y} \cap K_n \neq \emptyset$. Since

$$x \in f^{-1}(V_{n_y}) \subseteq \bigcup\{U : U \in \mathcal{U}_{n_y}\},$$

we can choose $U \in \mathcal{U}_{n_y}$ with $x \in U$. Then $V_{n_y} \subseteq f(U)$ by (1), and hence $U \cap f^{-1}(K_n) \neq \emptyset$. Therefore $x \in St(f^{-1}(K_n), \mathcal{U}_n)$. Consequently, we have $\{St(f^{-1}(K_n), \mathcal{U}_n) : n \in \mathbb{N}\}$ is an open cover of X, which shows that X is strongly star-Menger. □

Example 2.14. Assuming $\mathfrak{d} = \mathfrak{c}$, there exists a strongly star-Menger space X and a compact space Y such that $X \times Y$ is not strongly star-Menger.

Proof: Assuming $\mathfrak{d} = \mathfrak{c}$, let $X = \omega \cup \mathcal{A}$ be the space of Example 2.2 with $|\mathcal{A}| = \omega_1$. Then X is strongly star-Menger by Example 2.2. Let $D = \{d_\alpha : \alpha < \omega_1\}$ be the discrete space of cardinality ω_1 and let $Y = D \cup \{y_\infty\}$ be the one-point compactification of D. We show that $X \times Y$ is not strongly star-Menger. Since $|\mathcal{A}| = \omega_1$, we can enumerate $\mathcal{A}$ as $\{a_\alpha : \alpha < \omega_1\}$. For each $n \in \mathbb{N}$, let

$$\mathcal{U}_n = \{(\{a_\alpha\} \cup a_\alpha) \times (Y \setminus \{d_\alpha\}) : \alpha < \omega_1\} \cup \{X \times \{d_\alpha\} : \alpha < \omega_1\} \cup \{\omega \times Y\}.$$

Then $\mathcal{U}_n$ is an open cover of $X \times Y$. Let us consider the sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of $X \times Y$. It suffices to show that $\bigcup_{n \in \mathbb{N}} St(K_n, \mathcal{U}_n) \neq X \times Y$ for any sequence $(K_n : n \in \mathbb{N})$ of finite subsets of $X \times Y$. Let $(K_n : n \in \mathbb{N})$ be any sequence of finite subsets of $X \times Y$. For each $n \in \mathbb{N}$, since K_n is finite, there exists $\alpha_n < \omega_1$ such that

$$K_n \cap (X \times \{d_\alpha\}) = \emptyset \ \text{ for each } \ \alpha > \alpha_n.$$

Let $\beta = \sup\{\alpha_n : n \in \mathbb{N}\}$. Then $\beta < \omega_1$ and

$$(\bigcup_{n \in \mathbb{N}} K_n) \cap (X \times \{d_\alpha\}) = \emptyset \ \text{ for each } \ \alpha > \beta.$$

If we pick $\alpha > \beta$, then $\langle a_\alpha, d_\alpha \rangle \notin St(K_n, \mathcal{U}_n)$ for each $n \in \mathbb{N}$, since $X \times \{d_\alpha\}$ is the only element of $\mathcal{U}_n$ containing the point $\langle a_\alpha, d_\alpha \rangle$. This shows that $X \times Y$ is not strongly star-Menger. □

Remark 2.15. Example 2.14 also shows that Theorem 2.13 fails to be true if "open and closed, finite-to-one" is replaced by "open perfect".

The following example shows that the product of two strongly star-Menger spaces (even if countably compact) need not be strongly star-Menger. In fact, the following well-known example showing that the product of two countably compact (and hence strongly star-Menger) spaces need not be strongly star-Menger. Here we give the proof roughly for the sake of completeness.

Example 2.16. There exists two countably compact spaces X and Y such that $X \times Y$ is not strongly star-Menger.

PROOF: Let D be a discrete space of cardinality $\mathfrak{c}$. We can define $X = \bigcup_{\alpha<\omega_1} E_\alpha$ and $Y = \bigcup_{\alpha<\omega_1} F_\alpha$, where E_α and F_α are the subsets of βD which are defined inductively so as to satisfy the following conditions (1), (2) and (3):

(1) $E_\alpha \cap F_\beta = D$ if $\alpha \neq \beta$;
(2) $|E_\alpha| \leq \mathfrak{c}$ and $|F_\beta| \leq \mathfrak{c}$;
(3) every infinite subset of E_α (resp., F_α) has an accumulation point in $E_{\alpha+1}$ (resp., $F_{\alpha+1}$).

These sets E_α and F_α are well-defined since every infinite closed set in βD has cardinality $2^{\mathfrak{c}}$ (see [9]). Then $X \times Y$ is not strongly star-Menger. In fact, the diagonal $\{\langle d, d\rangle : d \in D\}$ is an open and closed subset of $X \times Y$ with cardinality $\mathfrak{c}$ and every point of $\{\langle d, d\rangle : d \in D\}$ is isolated. Then $\{\langle d, d\rangle : d \in D\}$ is not strongly star-Menger. Hence $X \times Y$ is not strongly star-Menger, since open and closed subsets of strongly star-Menger spaces are strongly star-Menger. □

In [2, Example 3.3.3], van Douwen et al. gave an example showing that there exists a countably compact (and hence strongly star-Menger) space X and a Lindelöf space Y such that $X \times Y$ is not strongly star-Lindelöf. Therefore, this example shows that the product of a strongly star-Menger space X and a Lindelöf space Y need not be strongly star-Menger.

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