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Robert Bystrický

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DIFFERENT APPROACHES TO WEIGHTED VOTING SYSTEMS BASED ON PREFERENTIAL POSITIONS

Robert Bystrický

Voting systems produce an aggregated result of the individual preferences of the voters. In many cases the aggregated collective preference – the result of the voting procedure – mirrors much more than anything else the characteristics of the voting systems. Preferential voting systems work most of the time with equidistant differences between the adjacent preferences of an individual voter. They produce, as voting systems usually do, some paradoxical results under special circumstances. However, the distances between the preferences can be understood as the function of the position in the sequence of preferences and can be aggregated in different ways fulfilling the basic attributes of the voting system. This approach at least allows us to avoid the worst paradoxical situations or to design a voting system containing some special needs.

Keywords: voting system, preference, position

Classification: 90A28, 90A05, 90A08, 62F07

1. INTRODUCTION

We can understand voting as a method for a group (electorate, meeting, part of society, country) to transfer and combine their will, preferences or motions of individuals represented by the vote. Alternatively, in other words voting and elections are a decision-making process, which aggregates individual preferences to give a social choice or election outcome that reflects the interests or "desires" of the electorate [2]. Elections, as it is largely known, produce many different results according to the voting system used, counting, and tabulations of results. We are aware of many different voting systems and different approaches to them. In many cases the aggregated collective preference – the result of the voting procedure mirrors much more than anything else the characteristics of the voting system. There is a need to have the possibility to influence the selected voting system in a desired direction. This need requires a tool to design and adapt known voting systems and schemes and at the same time to understand the influence of the changes.

In this paper, we will discuss only the situation when the voter is able to provide the ordinal preferential sequence of alternatives or candidates chosen. Preferential voting systems work most of the time with equidistant differences between the adjacent preferences of the individual voter. These voting systems also produce, as voting systems usually do, some paradoxical results under special circumstances. However, the preferential position in the sequence of preferences can be awarded with specific weights for respective positions. These weighted values can be aggregated in different ways fulfilling the basic attributes of the voting system. The described approach could at least allow us to avoid the worst paradoxical situations or to design a voting systems containing some special needs. Another possible application of this approach or reason for using it can be the modification of an existing voting system by just introducing this weighted method into them, in order to secure some specified characteristics of the voting system and it can be a quite simple way how to achieve it. We will discuss the influence of the introduced approach to the classic voting system with a single winner – it means the selection of the "most preferred" alternative from a range of alternatives. The original motivation to start our game with the weights allocated to the preferences is the perception that the differences in the value between e. g. 1st and 2nd preferences doesn't have to be the same as between e. g. the 3rd and 4th positions. It came from the belief that the differences between adjacent preferences could be understood as the distances between them and so they do not have to be the same (equidistant case) as they usually are in common systems.

The reason to study different approaches to the voting systems is, that the choice of the election system can significantly influence the result of the elections. Multicandidate elections allow many paradoxes, so e. g. the election outcome may more accurately reflect the voting method rather than the voters' desires (e. g. Saari [12], Nurmi [11]). We will investigate two groups of classic voting systems with preferential ordering of candidates or alternatives; it means a group of scoring voting systems (such as e. g. Borda count) and Alternative Vote system (AV). Modifications of scoring voting systems with new weights will be compared with Borda count and Alternative Vote with the classic first past the post system (FPTP), where the candidate with most first preferences is the winner of the election.

Any positional voting system with clear ranking designation of preferences can be seen as a weighted (or sometimes – distance) based voting system. The first well-known method that works with this approach is Borda rule [1]. The proposal of J. C. Borda was to rank preferences, mark them according to the ordinal positions provided and finally add up these marks. In this case, we can speak about equidistance between two succesive alternatives in a preferential sequence. His work was extended in a statistical framework by Kendall [10] and by the proposal of Nanson (see e. g. Fishburn [8]). There are other measures taken by studying ordinal preference aggregation problems based on the distances between preferences in individual rankings. Kemeny [9] proposed distance measures in comparison of pairs of alternatives where the relative position of individual preferences is different to the group ordinal preferences. It should help to solve the problem with cycles in majority voting systems. In Cook & Seiford [5, 6] approach l_1 and l_2 metrics were used to find the most representative alternative. A new approach to the weighted (or distance) based method was used also in Saari & Merlin [13], Eckert et al. [7] and Contreras [4]. Several interesting approaches can also be found in Vavríková [14].

In the second section of the paper there is a short description of the mentioned and considered voting systems, in the third part the proposed method. The section after that is dedicated to the examples and the last one to the conclusion of the paper.

2. SHORT DESCRIPTION OF VOTING SYSTEMS

In our paper, we use as comparison or inspiration some of the less or more known voting systems. After the short introduction above, we would like to describe some of the voting systems in this short paragraph. All are based on preferences for respective candidates or alternatives and they are all single-winner methods.

Borda count as one of the scoring voting systems was introduced by J. C. Borda in 1770 in France. It is used in many professional organizations to select some alternatives, also to elect minority candidates for parliament in Slovenia and in modified version in parliamentary elections in Nauru. The basic idea behind this method is to award each ranking (preference) with a certain amount of points. In the case of n candidates the first preference becomes n points, second $n-1$ and so on, the last nth preference is awarded by 1 point. The winner is the candidate with highest cumulative number of points after summation of the results from all voters. It promotes a consensual result, as the winner with the most first preferences does not have to be the winner. There are many modified versions of Borda count such as e. g. ***Nanson method*** or ***Baldwin method***. Both of them use ranked ballots as Borda count does. But after each aggregation the candidates with the fewest points are eliminated (Baldwin), or all candidates with less points than the average are eliminated (Nanson). The next round is recalculated with only the remaining candidates. The procedure lasts until a single winner remains.

Plurality vote elects as the winner the candidate with the most (not necessarily a majority) of first preferences. It is widely used in elections all around the world e. g. India, Canada, United Kingdom. We can also understand it as a scoring voting system since the first preference receives one point and the other preferences zero. It is also often called **First Past The Post (FPTP)**.

Antiplurality vote is the system where voters vote against their least favorite option among the candidates. It can also be understood as the scoring voting system, where the voted option gets 0 points and all the others 1 point. The candidate with the best result – most points in total, is the winner. As we can see all three previously described methods could be taken as scoring voting methods.

Approval voting is an election system, in which each candidate is either approved by the voter, which means he/she receives one point or disapproved, then the candidate receives zero points. The candidate with the most votes in total is the winner. It is used in many professional organizations such as e. g. Mathematical Association of America or American Statistical Association.

Condorcet method is not a scoring voting system, but it is also based on ranking candidates on the ballot according to the voter's preferences. It was introduced by French mathematician Marie Jean Antoine Nicolas Caritat, the Marquis de Condorcet. The scheme was a strong competitor to the Borda scheme in France in the 18th century and we can say that competition continues till this day. The Condorcet winner is the candidate who beats any other candidate in pairwise comparisons. These pairwise comparisons between all the candidates are based on the preferential ordering by voters.

Alternative Vote (AV) is also called Instant-Runoff Voting or Preferential Voting or Ranking Choice Voting. It is also from time to time called Ware's method or Hare-Clark method after the inventors of this type of voting scheme. It is used to elect the Presidents of Ireland and India, members of the Australian House of Representatives, etc. Voters put the candidates into a preferential sequence and counting is ongoing mostly in more than one round until any candidate secures the majority of first preferences, then the winner is found. If nobody has the majority of first preferences, then the last candidate in the respective round is eliminated and its second preference becomes the new first one (positions in the sequence are shifting one up), and all the counting is repeated.

Scoring voting systems assign a special value to each position in a preferential sequence. Aggregation of these values forms the result. Standard Alternative Vote is in the beginning close to the procedure of FPTP, first preferences are counted and the alternative with more than 50 % of votes is the winner; if there is no such alternative, the one with the fewest first preferences is eliminated and its second best is "upgraded" and redistributed with the same value to the remaining first preferences. Again, if one of the alternatives then has more than 50 %, it is the winner; if not the whole procedure is repeated until there is a clear winner with over 50 % of the votes. One of the main differences is that in scoring voting systems, different positions reflect different values in aggregation; Alternative Vote uses the same value when the preferences are counted. This can change with the introduction of weights based on poistions (or distances). Other differences are e. g. that scoring voting systems satisfy criteria of consistency and monotonicity and do not always comply with majority criterion; AV does exactly the opposite (majority criterion only in specific "loose" way). Both methods do not elect a Condorcet winner in all the cases, but they meet the Condorcet loser criterion.

Short description of the above mentioned criteria:

- *Consistency:* If A is the choice of voter group V and B is the choice of group V' disjoint from V and if $A \cap B \neq 0$, then merged groups $V \cup V'$ should choose the winner in $A \cap B$ (as in Young [15]); or in other words the choice which win in the two separated groups should also win overall.

- *Monotonicity:* Ranking an alternative higher should not have negative impact on its overall result.

- *Majority:* The candidate which receives more than half of the votes is the overall winner.

- *Condorcet winner:* The candidate who wins against any other candidates in pairwise comparison should always be the overall winner.

- *Condorcet loser:* The candidate, which loses to any other candidate in pairwise comparison should always be the loser.

3. THE PROPOSED METHOD

3.1. Scoring voting systems

First, we will define the meaning of weights in our way of understanding and computing. Let $\mathbf{A} = \{a_1, a_2, \ldots, a_n\}$ be a set of n alternatives, which can be ordered in a sequence of alternatives by each voter. We assume that each voter will produce a sincere ordering of preferences independently of any possibility of strategic or tactical behavior. For simplicity in this paper, we will use a sequence ordering which guarantees a linear order of alternatives; it means no ties will be taken into account. Lets introduce a discrete function $k : \mathbf{A} \to \{1, 2, \ldots, n\}$, which describes the position of the alternative in the sequence. Lets have a non-decreasing function $F(x)$ as weight value function, where x represents the preferential positions from 1 to n. If a_l is preferred to a_i ($a_l \succ a_i$) we can write that $\exists F : \{1, 2, \ldots, n\} \to \mathbb{R}$, $\forall a_i$, $a_l \in \mathbf{A}$, $a_l \succ a_i \Leftrightarrow F(k(a_l)) < F(k(a_i))$. Weight value function $F(x)$ is a function of position $k(a)$ of the alternative a in the sequence, i. e., it can be understood as a utility function representing the preferences of the discussed voter. E.g. we have a set of alternatives $\{a_1, a_2, a_3, a_4\} = \{A, B, C, D\}$ and the sequence of preferences $ADCB$, $k(A) = 1$ and $k(B) = 4$. Next, we will discuss the utilization of weight value function in scoring voting systems; for Alternative Vote we will use a slightly different approach.

One of the easiest possible ways of introducing a weight value function is the linear function $F(x) = x - 1$, which is actually a slightly modified approach of Borda count. In this case the sequence of weights according to the position would obviously be: $F(1) = 0$, $F(2) = 1$, $F(3) = 2$, $F(4) = 3$, for $n = 4$. Nevertheless, the definition of the function is completely dependent on the designer of the voting system. With careful shaping of this function, we can suppress some negative aspects of the method used, or also sharpen other desired aspects. Let us elaborate on these three examples and see also Figure 1, where ($n = 4$):

a) If we would like to emphasize the role of the winning (first one in the sequence) alternative, the impact will decrease with higher position, so the weight difference between the 2 last alternatives in the sequence should be the smallest one. In this case we can use the square root function e. g.: $F(x) = \sqrt{x-1}$. It shifts the voting method towards the plurality voting method. The sequence of weights is: $F(1) = 0$, $F(2) = 1$, $F(3) = 1.41$, $F(4) = 1.73$, for $n = 4$.

b) If we want to suppress the impact of the least preferred alternative in the preferential sequence of an individual voter – an approach which is closer to antiplurality vote, we can use e. g.: $F(x) = (x-1)^2$. The sequence of weights in this case is: $F(1) = 0$, $F(2) = 1$, $F(3) = 4$, $F(4) = 9$, for $n = 4$.

c) If we would like to emphasize the impact of extremes, enforce the winning alternative and "punish" the last one, some kind of concave function suggests itself like e. g. simple quadratic function. We introduce s as the middle position of our sequence of alternatives, $s = (n-1)/2$ and helping function $h(x) = (x - 1 - s)^2 \cdot \mathrm{sign}(x - 1 - s)$. The weight value function in this case will be:

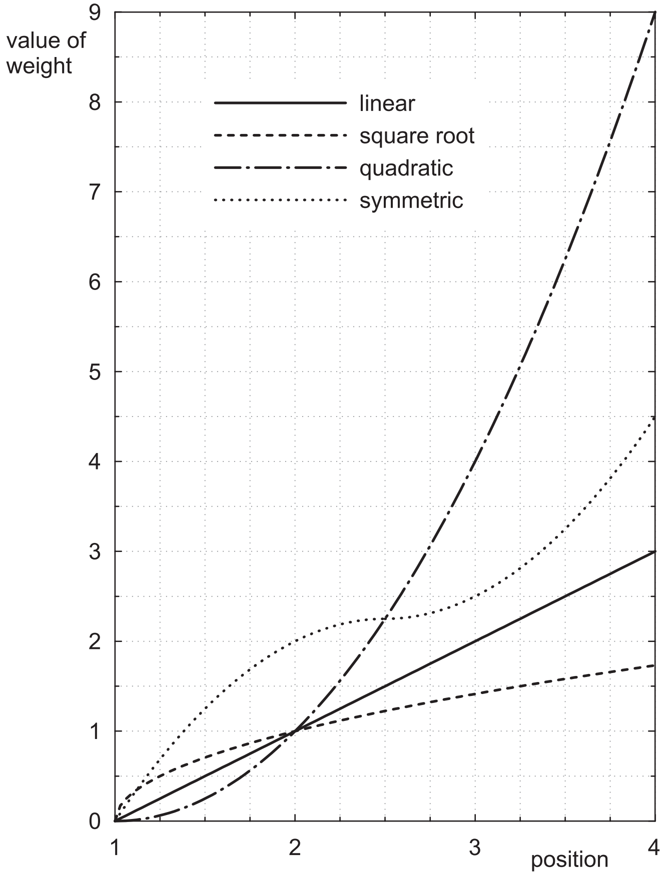

Fig. 1. Description of the weight values for scoring voting method.

$F(x) = h(x) - h(1)$. We will call it a symmetric function. The sequence of weights is: $F(1) = 0$, $F(2) = 2$, $F(3) = 2.5$, $F(4) = 4.5$, for $n = 4$.

If we have a finite set of voters or decision makers $B = \{b_1, b_2, \ldots, b_m\}$, we can introduce $F(k_j(a_i))$ as a weight value function for the jth voter and ith alternative; $j = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$. Finally we can introduce alternative value function $V : \mathbf{A} \to \mathbb{R}$ defined as

$$V(a_i) = \sum_{j=1}^{m} F(k_j(a_i)). \tag{1}$$

The winning alternative is $\mathbf{a} = \arg\min \{V(a_i), i = 1, 2, \ldots, n\}$. We should note that there are other ways of minimizing the value function V. In the case when the winning alternative is not a singleton, we can use different additional tie-breaking procedures. So the decisive rule could be e. g. the most 1st preferences, then the most 2nd preferences, and so on or other invented measures could be used. One additional approach to solve this problem can also be found in the next paragraph.

Alternatively, one can use other aggregation techniques (1) than the classic sum, such as the product,

$$V(a_i) = \prod_{j=1}^{m} (F(k_j(a_i)) + 1),$$

or median,

$$V(a_i) = \mathrm{med}(F(k_1(a_i)), \ldots, F(k_m(a_i))).$$

The relation between sum and product can be seen as the relation between arithmetic and geometric mean. Alternative aggregation methods could be used also as a tie-breaking rule if another rule does not bring a clear winner.

3.2. Alternative Vote

Motivation to adopt a weight based system into Alternative Vote comes from discussions between supporters of AV and FPTP before the referendum about possible change to the voting system in the UK in May 2011. In the end after the referendum FPTP remained. There are doubts whether and how correct it is to count upgraded second, third or even fourth preference of some alternatives in respective rounds of voting, with the same value as first preferences of other alternatives. The introduction of weights to the system can solve this problem and at the same time maintain the main characteristics of AV, the possibility to vote freely for marginal candidates or alternatives without the risk of in reality wasting the vote completely.

We will slightly modify the weight value function $F(x)$, which will be non-increasing in general in this case with default value 1 for first preference i.e. $F(1) = 1$. It allows us to preserve aggregation in the same way as standard Alternative Vote without any difference. In fact the weight value function for standard AV is $F(x) = 1$, for all the possible preferential positions. The weight value actually means in this case the vote value aggregated in subsequent rounds of counting (so it is also described in Figure 2, where used functions are depicted).

One typical weight value function for alternative vote can be $F(x) = 1 - \frac{x-1}{x}$. We will call it the hyperbolic function, because it is the same situation as we would get from allocating weight $1/p$ for each pth preference in the sequence, where $p \in \{1, 2, \ldots, n\}$. This function enforces the better-positioned candidates. The sequence of weight (or vote) values in this case is: $F(1) = 1$, $F(2) = 1/2$, $F(3) = 1/3$, $F(4) = 1/4$, for $n = 4$. As we already pointed out in the previous section, we can influence the outcome of an election by designing this function and we show some examples:

a) Concave quadratic function $F(x) = \sqrt{(n - x + 1)/n}$ strengthens the importance of higher preferences, decrease of the value of each position is slower, at least for the higher ranks, it is closer to the standard AV than the next two functions. The sequence of weight (or vote) values in this case is: $F(1) = 1$, $F(2) = 0.87$, $F(3) = 0.71$, $F(4) = 0.5$, for $n = 4$.

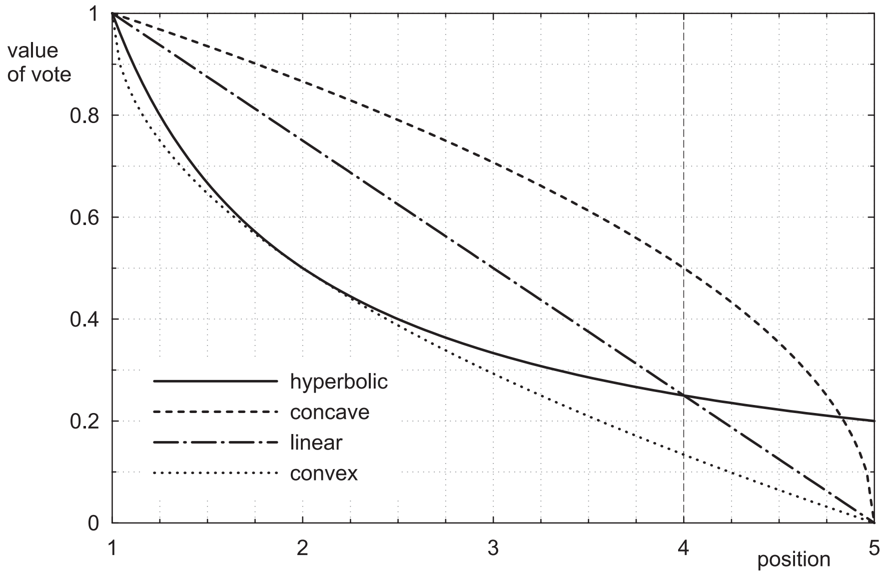

Fig. 2. Description of the vote (weight) values for Alternative Vote method.

b) Linear function $F(x) = 1 - (x-1)/n$ secures an equivalent drop in the value of position. The sequence of weight (or vote) values is: $F(1) = 1$, $F(2) = 0.75$, $F(3) = 0.5$, $F(4) = 0.25$, for $n = 4$.

c) Convex quadratic function $F(x) = 1 - \sqrt{(x-1)/n}$ strengthens the importance solely of the first preference, because the drop in value of other ranks is more significant. As a result, it is closer to FPTP than the previous two examples. It is very close to previously described hyperbolic function, but the decrease of values in lower positions is even more significant. The sequence of weight (or vote) values in this case is: $F(1) = 1$, $F(2) = 0.5$, $F(3) = 0.29$, $F(4) = 0.13$, for $n = 4$.

We can construct many other similar functions, which are non-increasing and $F(1) = 1$. We can also designate the minimal value v for last nth rank as $F(n) = v$.

The aggregation is a bit more complicated. In specific consequent rounds for each alternative or candidate, we will aggregate the results only if the respective alternative is available for counting (it is the first preference originally or by "upgrade" after redistribution of votes from previous round). It means, for the jth voter and ith alternative; $j = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$ we can write the alternative value function $V(a_i)$:

$$V(a_i) = \sum_{j=1}^{m} \left\{ F(k_j(a_i)) \mid \text{only for actually } 1^{\text{st}} \text{ placed } a_i \right\}. \tag{2}$$

The procedure is finished and the winning alternative a_i is found if in the respective round this alternative fulfills the following condition:

$$V(a_i) > \sum_{j=1, j \neq i}^{n} V(a_j).$$

There is a need to mention, that the condition for the winning alternative to secure more than 50% of original votes is valid only for the first round of counting, the total amount of votes for each next round will decrease, because some of the aggregated values of votes will be less than 1.

4. EXAMPLES

4.1. Scoring voting system

According to Table 1 we can investigate the situation in the classic voting system with 72 voters and 3 different voting schemes chosen by voters. This example and the next one can also be found in Bystrický [3].

Scheme	**Number of voters**	**Preferences**
1	24	ABC
2	23	BCA
3	25	CAB

Tab. 1.

$F(x)$	$V(A)$	$V(B)$	$V(C)$
$x-1$	**71**	74	**71**
$\sqrt{x-1}$	57.5	59.4	**56.9**
$(x-1)^2$	**117**	124	119

Tab. 2.

$F(x)$	$V(A)$	$V(B)$	$V(C)$	
$x-1$	3.16	14.22	**2.37**	$E+18$
$\sqrt{x-1}$	5.16	6.22	**1.29**	$E+16$
$(x-1)^2$	**4**	50	5	$E+23$

Tab. 3.

Results based on the application of three weight value functions are summarized in Table 2, which shows the different outcomes (winner is always in bold) for each function according to the anticipation. There is no clear winner in the case of the linear function; the winner for the other two functions is according to the designer's wish. Equation 1 was used for aggregation.

Just for comparison, we can compute the same case with another aggregation method – product, as it is proposed above. The results are shown in Table 3. They are very similar to Table 2, only there is no tie in the case of linear identity function. Accordingly this approach can be used e. g. as a tie-breaking rule in the case of the value function calculated as a sum.

Another example is investigating the situation with one hundred voters, which distribute their votes in four possible profiles according to the Table 4.

Scheme	**Number of voters**	**Preferences**
1	42	$ABCD$
2	26	$BCDA$
3	15	$CDAB$
4	17	$DABC$

Tab. 4.

$F(x)$	$V(A)$	$V(B)$	$V(C)$	$V(D)$
$x-1$	125	**121**	163	193
$\sqrt{x-1}$	**83.25**	92.02	114.84	124.52
$(x-1)^2$	311	**245**	347	497
sym	**188.5**	194	233.5	284

Tab. 5. SUM.

$F(x)$	$V(A)$	$V(B)$	$V(C)$	$V(D)$
$x-1$	**8.47 E+27**	6.10 E+29	1.26 E+38	1.16 E+42
$\sqrt{x-1}$	**1.61 E+22**	4.99 E+25	2.11 E+31	6.31 E+32
$(x-1)^2$	4.00 E+41	**3.36 E+39**	1.53 E+54	4.88 E+64
sym	**3.32 E+35**	2.48 E+40	6.95 E+47	2.50 E+52

Tab. 6. PRODUCT.

The results for the situation described above can be found in Tables 5 and 6. The outcomes are very similar for both aggregation operators, with only one difference for square root function where alternative B is the winner in the case of value function calculated as a product. Otherwise, the results are as expected according to the chosen value added functions.

4.2. Alternative Vote

As the first example in the Section of examples for Alternative Vote, we will analyze the same scheme as described in Table 4. In Tables 7 – 10 we show four different results for standard alternative vote, followed by versions with introduced weight value functions (see Section 3.2) – convex quadratic, linear and concave quadratic. The outcome for hyperbolic function is homothetic with the result for convex quadratic function with the same winner. As we can see, we obtained similar results with the same winner D in the case of standard AV as well as AV with concave quadratic weight value function. AV with linear function shows very tight competition with a different winner, A for this scheme and finally AV with convex quadratic function again brings clear winner A, which is all in line with the choice and design of the weight value functions. In standard AV, even if the winner D secures only 17 first preferences (in comparison to 42 first preferences for A – winner in FPTP); it is a clear victory in the third and final round. Application of a weight based function can influence these outcomes according to the value assigned for each position and change the results.

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	42	26	15	17
2	42	26		32
3	42			**58**

Tab. 7. Standard AV

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	42	26	15	17
2	42	26		24.5
3	**54.9**	26		

Tab. 8. Convex AV

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	42	26	15	17
2	42	26		28.25
3	**42**			41.25

Tab. 9. Linear AV

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	42	26	15	17
2	42	26		30
3	42			**48.4**

Tab. 10. Concave AV

We can describe how we obtained the results e. g. in Table 8 for convex function. The first row is simple as we calculated only the first preferences for respective candidates. After the 1st round C is eliminated and second preference D becomes the first one and it is added to $V(D)$. Then $V(D) = 17 * F(1) + 15 * F(2) = 17 + 7.5 = 24.5$, which is the smallest value in the 2nd round of calculation and D is eliminated. All the votes from D are redistributed accordingly to A and after 3 rounds A becomes the winner, as $V(A) = 42 * F(1) + 17 * F(2) + 15 * F(3) = 42 * 1 + 17 * 0.5 + 15 * 0.29 = 54.85$, where $F(1)$, $F(2)$ and $F(3)$ are naturally taken from the weight sequence for the convex function, 15 from scheme 4 from Table 4 and 17 from the scheme 3 from the same Table. All the other calculations are made in a similar way.

Scheme	**Number of voters**	**Preferences**
1	48	$BCDA$
2	10	$CDAB$
3	12	$DABC$
3	30	$ABCD$

Tab. 11.

The second example works with a slightly modified scheme as shown in Table 11. The leading preferences, first B and second A are clearly ahead of the two other alternatives. Counting in the first two rounds leads to the transfer of second and third ranks for alternative A adding to the 30 votes originally allocated as first preference. Therefore, A finally wins after three rounds with 52 total votes compared to 48 votes for alternative B, which could not secure any other votes from second and/or third places thanks to the distribution of preferences (see Table 12).

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	30	48	10	12
2	30	48		22
3	**52**	48		

Tab. 12. Standard AV.

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	30	48	10	12
2	30	48		17
3	38.9	**48**		

Tab. 13. Convex AV.

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	30	48	10	12
2	30	48		19.5
3	43	**48**		

Tab. 14. Linear AV.

Round	$V(A)$	$V(B)$	$V(C)$	$V(D)$
1	30	48	10	12
2	30	48		20.7
3	47.5	**48**		

Tab. 15. Concave AV.

Also in other results for AV with introduced weight based function (see Tables 13 – 15), we can see that the initial advantage for alternative B (48 preferences) was good enough to stay as winner under all circumstances. We can see also that, as expected, the introduction of concave quadratic function brings the best result regarding alternative A for competing with alternative B, as this is the closest function by nature to standard AV. In addition, the other results contain anticipated characteristics according to the attributes of the selected functions. Imagine that if we would distribute the 22 transferred votes to alternative A otherwise, alternative A could still be the winner with the introduction of the concave quadratic function, despite the huge advantage of 48 votes for alternative B. The desired distribution would be at least 16 votes (instead of 12) for sub-profile $DABC$, and the remaining 6 remain for sub-profile $CDAB$. This result would also be similar if we would use the hyperbolic function, but we cannot secure the win with the transfer of 22 votes as above just by applying any other of the proposed functions. Simply the advantage of 18 votes is too big and the result will tend to the result of FPTP.

5. CONCLUSION

The change of method certainly brings new possible paradoxes and shortages in other cases, as it is impossible to construct a general always-functioning model how to deal with voting and achieve "justified" outcomes. In our recalculations, the winner is e. g. not always Condorcet winner, but the outputs are promising.

In the scoring voting schemes it allows us to design a voting system according to the wishes of the designers, and the scheme is always monotonic. Different aggregation operators can produce different results in particular cases, it is necessary to be aware of it and use it according to expectations, e. g. also as tie-breaking rule.

The introduction of weights into Alternative Vote system allows us to secure resuls similar to typical plurality voting systems, like FPTP, if this is desired. With the appropriate choice of weight based function AV can be calibrated as "continuous" transformation from standard AV to FPTP system or stay in between, according to the wishes. Of course it can bring some additional problems, but this approach contains the demanded characteristics from both systems.

Weight based functions can be designed in many other ways, they should only keep the basic characteristics. It is necessary to study this approach more precisely in order to find shortcomings. In addition, it is necessary to investigate the mathematical characteristics of this approach more broadly. The way which method and approach is used should depend on our expectation from the voting. Stakeholders should be able to determine the expected characteristics of voting systems together with identifying which negative outcomes should be excluded. With appropriate weight based function it could be possible to address at least some of their wishes. However, the whole approach should be studied further, especially focusing on generalizing formulas replacing the classic sum or product with other aggregation functions.

Further research should also include the analysis of properties also according to changes in used weight based functions in order to have better knowledge about the whole system. It will allow more possibilities and options for designers of voting systems.

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Robert Bystrický, Department of Mathematics, Faculty of Civil Engineering Slovak University of Technology, Radlinského 11, 813 68 Bratislava. Slovak Republic.

e-mail: rb@math.sk