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Problems Concerning Subclasses of Analytic Functions *

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Abstract

In the present paper, we establish some interesting results concerning the quasi-Hadamard product for certain subclasses of analytic functions.

Key words: analytic, Starlike, Convex, Quasi-Hadamard product

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1 Introduction and preliminaries

Throughout this paper, let the of functions of the form

$$f(z) = a_1z + \sum_{n=2}^{\infty} a_n z^n, \quad a_1 > 0, a_n \geq 0, \quad (1)$$

$$f_i(z) = a_{1,i}z + \sum_{n=2}^{\infty} a_{n,i} z^n, \quad a_{1,i} > 0, a_{n,i} \geq 0, \quad (2)$$

$$g(z) = b_1z + \sum_{n=2}^{\infty} b_n z^n, \quad b_1 > 0, b_n \geq 0, \quad (3)$$

$$g_j(z) = b_{1,j}z + \sum_{n=2}^{\infty} b_{n,j} z^n, \quad b_{1,i} > 0, b_{n,j} \geq 0, \quad (4)$$

be regular and univalent in the unit disc $\mathbb{U} = \{z: |z| < 1\}$.

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For a function $f(z)$ defined by (1) (with $a_1 = 1$) we define

$$\begin{aligned} D^0 f(z) &= f(z), \\ D_\lambda^1(\alpha, \beta, \mu) f(z) &= \left(\frac{\alpha - \mu + \beta - \lambda}{\alpha + \beta}\right) f(z) + \left(\frac{\mu + \lambda}{\alpha + \beta}\right) z f'(z), \\ D_\lambda^2(\alpha, \beta, \mu) f(z) &= D(D_\lambda^1(\alpha, \beta, \mu) f(z)), \\ &\vdots \\ D_\lambda^k(\alpha, \beta, \mu) f(z) &= D(D_\lambda^{k-1}(\alpha, \beta, \mu) f(z)). \end{aligned} \quad (5)$$

If f is given by (1) then from (5) we have

$$D_\lambda^k(\alpha, \beta, \mu) f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta}\right)^k a_n z^n, \quad (6)$$

where $f \in A$, $\alpha, \beta, \mu, \lambda \geq 0$, $\alpha + \beta \neq 0$, $n \in N_o$. The operator $D_\lambda^k(\alpha, \beta, \mu) f(z)$ was introduced by authors, Darus and Faisal in [7].

By specializing the parameters of $D_\lambda^k(\alpha, \beta, \mu) f(z)$ we get the following differential operators. If we substitute

- $\beta = 0$, we get $D_\lambda^k(\alpha, 0, \mu) f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1)}{\alpha}\right)^k a_n z^n$ of differential operator given by Darus and Faisal [8].
- $\beta = 1$, $\mu = 0$, we get $D_\lambda^k(\alpha, 1, 0) f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\alpha + \lambda(n-1) + 1}{\alpha + 1}\right)^k a_n z^n$ of differential operator given by Aouf et. al [3].
- $\alpha = 1$, $\beta = 0$, and $\mu = 0$, we get $D_\lambda^k(1, 0, 0) f(z) = z + \sum_{n=2}^{\infty} (1 + \lambda(n-1))^k a_n z^n$ of differential operator given by Al-Oboudi [1].
- $\alpha = 1$, $\beta = 0$, $\mu = 0$ and $\lambda = 1$, we get $D_1^k(1, 0, 0) f(z) = z + \sum_{n=2}^{\infty} (n)^k a_n z^n$ of differential operator given by Sălăgean's [12].
- $\alpha = 1$, $\beta = 1$, $\lambda = 1$ and $\mu = 0$, we get $D_1^k(1, 1, 0) f(z) = z + \sum_{n=2}^{\infty} \left(\frac{n+1}{2}\right)^k a_n z^n$ of differential operator given by Uralegaddi and Somanatha [16].
- $\beta = 1$, $\lambda = 1$ and $\mu = 0$, we get $D_1^k(\alpha, 1, 0) f(z) = z + \sum_{n=2}^{\infty} \left(\frac{n+\alpha}{\alpha+1}\right)^k a_n z^n$ of differential operator given by Cho and Srivastava [5, 6].

With the help of the differential operator $D_\lambda^k(\alpha, \beta, \mu) f(z)$, we define the classes $\Psi(k, \rho, \delta, \eta)$, and $R_\rho(n, \rho, \delta, \eta)$ as follows:

For, $0 \leq \rho < 1$, $0 \leq \delta < 1$ and $\eta \geq 0$, we let $\Psi(k, \rho, \delta, \eta)$ denote the class of functions $f(z)$ defined by (1) and satisfying the analytic criterion

$$\begin{aligned} &\Re \left\{ \frac{z(D_\lambda^k(\alpha, \beta, \mu) f(z))'}{(1-\rho)(D_\lambda^k(\alpha, \beta, \mu) f(z)) + \rho z(D_\lambda^k(\alpha, \beta, \mu) f(z))'} - \delta \right\} \\ &> \eta \left| \frac{z(D_\lambda^k(\alpha, \beta, \mu) f(z))'}{(1-\rho)(D_\lambda^k(\alpha, \beta, \mu) f(z)) + \rho z(D_\lambda^k(\alpha, \beta, \mu) f(z))'} - 1 \right|, \quad z \in \mathbb{U}. \quad (7) \end{aligned}$$

Suitably specializing the parameters of the class $\Psi(k, \rho, \delta, \eta)$ generalize the classes defined by some well known authors see [13]–[15].

Using similar arguments as given in [11], we can easily get the following results for functions in the classes $\Psi(n, \rho, \delta, \eta)$ and $\psi(k + 1, \rho, \delta, \eta)$ respectively.

A function $f(z) \in \Psi(k, \rho, \delta, \eta)$ ($0 \leq \rho < 1, 0 \leq \delta < 1, \eta \geq 0$) if and only if

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^k [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_n| \leq (1 - \delta), \tag{8}$$

Now we introduce the following class of analytic functions which plays an important role in our discussion as follows.

A function $f(z)$ which is analytic in U belongs to the class $R_p(n, \rho, \delta, \eta)$ if and only if

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^p [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_n| \leq (1 - \delta), \tag{9}$$

where $0 \leq \rho < 1, 0 \leq \delta < 1, \eta \geq 0$ and p is any fixed nonnegative real number.

For $p = k$, it is identical to $\Psi(k, \rho, \delta, \eta)$. Also for $p = \beta = \mu = 0$, it is identical to $S_p(\rho, \delta, \eta)$ defined in [11]. Further, for any positive integer $p > h > h - 1 > \dots > k + 1 > k$, we have the inclusion relation

$$R_p(n, \rho, \delta, \eta) \subseteq R_h(n, \rho, \delta, \eta) \subseteq \dots \subseteq \psi(k + 1, \rho, \delta, \eta) \subseteq \Psi(k, \rho, \delta, \eta).$$

The class $R_p(n, \rho, \delta, \eta)$ is nonempty for any nonnegative real number p as the functions of the form

$$f(z) = a_1 z + \sum_{n=2}^{\infty} \frac{(\alpha + \beta)^p (1 - \delta)}{\left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^p [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)]} \lambda_n z^n, \tag{10}$$

where $a_1 > 0, \lambda_n \geq 0$ and $\sum_2^{\infty} \lambda_n \leq 1$; satisfy the inequality (10).

Let us define the quasi-Hadamard product of the functions $f(z)$ and $g(z)$ by

$$f * g(z) = a_1 b_1 z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad a_1 > 0, b_1 > 0, a_n \geq 0. \tag{11}$$

In this work we establish certain results concerning the quasi-Hadamard product of functions for the classes $\Psi(k, \rho, \delta, \eta)$ and $R_p(n, \rho, \delta, \eta)$ analogous to the results due to V. Kumar [9] and [10] as well.

2 The main results

Theorem 1 Let the functions $f_i(z)$ defined by (2) be in the class $\psi(k+1, \rho, \delta, \eta)$ for every $i = 1, 2, \dots, r$ and let the functions $g_j(z)$ defined by (4) be in the class $\Psi(n, \rho, \delta, \eta)$ for every $j = 1, 2, \dots, q$. Then the quasi-Hadamard product $f_1 * f_2 * \dots * f_r * g_1 * g_2 * \dots * g_q(z)$ belongs to the class $R_{r(k+2)+q(k+1)-1}(n, \rho, \delta, \eta)$.

Proof We denote the quasi-Hadamard product $f_1 * f_2 * \dots * f_r * g_1 * g_2 * \dots * g_q(z)$ by the function $G(z)$ for the sake of convenience. Clearly,

$$G(z) = \left[\prod_{i=1}^r |a_{1,i}| \right] \left[\prod_{j=1}^q |b_{1,j}| \right] z + \sum_{n=2}^{\infty} \left[\prod_{i=1}^r |a_{n,i}| \right] \left[\prod_{j=1}^q |b_{n,j}| \right] z^n.$$

Since $f_i(z) \in \psi(k+1, \rho, \delta, \eta)$ implies

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{k+1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,i}| \leq (1 - \delta) |a_{1,i}|, \quad (12)$$

for every $i = 1, 2, \dots, r$. Implies

$$|a_{n,i}| \leq \frac{(\alpha + \beta)^{k+1}(1 - \delta)}{\left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{k+1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)]} |a_{1,i}|,$$

for every $i = 1, 2, \dots, r$. Implies

$$|a_{n,i}| \leq \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{-k-2} |a_{1,i}|, \quad (13)$$

for every $i = 1, 2, \dots, r$. Similarly $g_j(z) \in \Psi(k, \rho, \delta, \eta)$, implies

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^k [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] |b_{n,j}| \leq (1 - \delta) |b_{1,j}|, \quad (14)$$

for every $i = 1, 2, \dots, q$. Implies

$$|b_{n,j}| \leq \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{-k-1} |b_{1,j}|, \quad (15)$$

for every $j = 1, 2, \dots, q$. To prove the theorem, we need to show that

$$\sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^t [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] \times \left[\prod_{i=1}^r |a_{n,i}| \right] \left[\prod_{j=1}^q |b_{n,j}| \right] \leq (1 - \delta) \left[\prod_{i=1}^r |a_{1,i}| \right] \left[\prod_{j=1}^q |b_{1,j}| \right],$$

where $t = r(k+2) + q(k+1) - 1$.

Using (13), (14) and (15) for $i = 1, 2, \dots, r$, $j = q$ and $j = 1, 2, \dots, q - 1$ respectively. we have (consider $t = r(k + 2) + q(k + 1) - 1$)

$$\begin{aligned} & \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^t [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] \\ & \quad \times \left[\prod_{i=1}^r |a_{n,i}| \right] \left[\prod_{j=1}^q |b_{n,j}| \right] \\ \leq & \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^{r(k+2)+q(k+1)-1} [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] \\ & \times \left[\left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^{-r(k+2)} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^{-(q-1)(k+1)} \right. \\ & \quad \times |b_{n,q}| \prod_{i=1}^r |a_{1,i}| \left. \right] \left[\prod_{j=1}^{q-1} |b_{1,j}| \right] = \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^k \\ & \quad \times [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] |b_{n,q}| \left[\prod_{i=1}^r |a_{1,i}| \right] \left[\prod_{j=1}^{q-1} |b_{1,j}| \right] \\ & \leq (1 - \delta) \left[\prod_{i=1}^r |a_{1,i}| \right] \left[\prod_{j=1}^q |b_{1,j}| \right]. \end{aligned}$$

Implies

$$\begin{aligned} & \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n - 1) + \beta}{\alpha + \beta} \right)^t [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] \\ & \quad \times \left[\prod_{i=1}^r |a_{n,i}| \right] \left[\prod_{j=1}^q |b_{n,j}| \right] \\ & \leq (1 - \delta) \left[\prod_{i=1}^r |a_{1,i}| \right] \left[\prod_{j=1}^q |b_{1,j}| \right], \quad t = r(k + 2) + q(k + 1) - 1. \end{aligned}$$

Using (9) we conclude that

$$f_1 * f_2 * \dots * f_r * g_1 * g_2 * \dots * g_q(z) \in R_{r(k+2)+q(k+1)-1}(n, \rho, \delta, \eta). \quad \square$$

Theorem 2 Let the functions $f_i(z)$ defined by (2) be in the class $\psi(k + 1, \rho, \delta, \eta)$ for every $i = 1, 2, \dots, r$. Then the Hadamard product $f_1(z) * f_2(z) * \dots * f_r(z)$ belongs to the class $R_{r(k+2)-1}(n, \rho, \delta, \eta)$.

Proof To prove the theorem, we need to show that

$$\begin{aligned} \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{r(k+2)-1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] \prod_{i=1}^r |a_{n,i}| \\ \leq (1 - \delta) \prod_{i=1}^r |a_{1,i}|. \end{aligned}$$

Since $f_i(z) \in \psi(k+1, \rho, \delta, \eta)$, implies

$$\begin{aligned} \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{k+1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,i}| \\ \leq (1 - \delta) |a_{1,i}|, \end{aligned} \quad (16)$$

for every $i = 1, 2, \dots, r$. Implies

$$|a_{n,i}| \leq \frac{(\alpha + \beta)^{k+1}(1 - \delta)}{(\alpha + (\mu + \lambda)(n-1) + \beta)^{k+1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)]} |a_{1,i}|,$$

for every $i = 1, 2, \dots, r$. Implies

$$|a_{n,i}| \leq \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{-k-2} |a_{1,i}|, \quad (17)$$

for every $i = 1, 2, \dots, r$.

Simultaneously applying (16) as well as (17) for $i = r$ and $i = 1, 2, \dots, r-1$ respectively, we get

$$\begin{aligned} \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{r(k+2)-1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] \prod_{i=1}^r |a_{n,i}| \\ \leq \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{r(k+2)-1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,r}| \\ \times \left[\left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{-(r-1)(k+2)} \prod_{i=1}^{r-1} |a_{1,i}| \right] \\ = \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{k+1} [n(1+\eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,r}| \left[\prod_{i=1}^{r-1} |a_{1,i}| \right] \\ = (1 - \delta) \prod_{i=1}^r |a_{1,i}|. \end{aligned}$$

Hence $f_1(z) * f_2(z) * \dots * f_r(z) \in R_{r(k+2)-1}(k, \rho, \delta, \eta)$. \square

Theorem 3 Let the functions $f_i(z)$ defined by (2) be in the class $\Psi(k, \rho, \delta, \eta)$ for every $i = 1, 2, \dots, r$. Then the Hadamard product $f_1(z) * f_2(z) * \dots * f_r(z)$ belongs to the class $R_{r(k+1)-1}(n, \rho, \delta, \eta)$.

Proof Since $f_i(z) \in \Psi(k, \rho, \delta, \eta)$, implies

$$\begin{aligned} \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^k [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,i}| \\ \leq (1 - \delta) |a_{1,i}|, \end{aligned} \quad (18)$$

for every $i = 1, 2, \dots, r$. Implies

$$\left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^k [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,i}| \leq (1 - \delta) |a_{1,i}|,$$

or

$$|a_{n,i}| \leq \frac{(1 - \delta)}{\left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^k [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)]} |a_{1,i}|,$$

for every $i = 1, 2, \dots, r$. Implies

$$|a_{n,i}| \leq \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{-k-1} |a_{1,i}|, \quad (19)$$

for every $i = 1, 2, \dots, r$.

To prove that $f_1(z) * f_2(z) * \dots * f_r(z) \in R_{r(k+1)-1}(n, \rho, \delta, \eta)$, it is enough to show that

$$\begin{aligned} \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{r(k+1)-1} [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] \prod_{i=1}^r |a_{n,i}| \\ \leq (1 - \delta) \prod_{i=1}^r |a_{1,i}|. \end{aligned} \quad (20)$$

Using (18) and (19) for $i = r$ and $i = 1, 2, \dots, r-1$ respectively, then we have

$$\begin{aligned} \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{r(k+1)-1} [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] \prod_{i=1}^r |a_{n,i}| \\ \leq \sum_{n=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{r(k+1)-1} [n(1 + \eta) - (\delta + \eta)(1 + n\rho - \rho)] |a_{n,r}| \\ \times \left[\left(\frac{\alpha + (\mu + \lambda)(n-1) + \beta}{\alpha + \beta} \right)^{-(r-1)(k+1)} \prod_{i=1}^{r-1} |a_{1,i}| \right] = (1 - \delta) \prod_{i=1}^r |a_{1,i}|. \end{aligned}$$

Hence $f_1(z) * f_2(z) * \dots * f_r(z) \in R_{r(k+1)-1}(n, \rho, \delta, \eta)$. \square

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