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FINITE-TIME OUTPUT FEEDBACK STABILIZATION AND CONTROL FOR A QUADROTOR MINI-AIRCRAFT

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This paper focuses on the finite-time output feedback control problem for a quad-rotor mini-aircraft system. First, a finite-time state feedback controller is designed by utilizing the finite-time control theory. Then, considering the case that the velocity states are not measurable, a finite-time stable observer is developed to estimate the unmeasurable states. Thus a finite-time output feedback controller is obtained and the stability analysis is provided to ensure the finite-time stability of the closed loop system. The proposed control method improves the convergence and disturbance rejection properties with respect to the asymptotically control results. Simulation results show the effectiveness of the proposed method.

Keywords: quadrotor mini-aircraft, finite-time stability, finite-time observer, output feedback

Classification: 93E12, 62A10

1. INTRODUCTION

The autonomous mini-aerial vehicles are drawing an increasing interest in recent years due to the growing numbers of civil and military applications of UAVs (unmanned aerial vehicles). Helicopter is one of the most important machines of the UAVs for its versatility and manoeuver ability to perform many types of tasks. One trend of such aircraft is the development of small flying machines capable of performing hover as well as forward flight. A mini-aircraft which has four rotors is an interesting alternative to the classical helicopter because this novel rotorcraft is mechanically simpler than a regular helicopter since it does not require a swashplate. The autonomous quadrotor aircrafts have been investigated for a variety of applications both as individual vehicles and in multiple vehicle teams, including surveillance, search, rescue and mobile sensor networks.

The quad-rotor mini-aircraft is an under-actuated dynamic rotorcraft with four input forces and six output coordinates. Various effective control methods have been developed to meet the increasing demands on the UAV performance for the quadrotors, such as feedback linearization method [6, 25] backstepping control technique [8, 21], sliding mode control [3, 9] and backstepping integrated with sliding mode control [7]. From the optimization control aspect, optimal control is drawing an increasing attention because it is often required for the quadrotor UAVs to carry out a hovering motion maneuver in minimum-time or in minimum-energy [1, 2, 18].

Different from classical time-optimal control approaches, Continuous and nonsmooth control via state or dynamic output feedback can provide a finite-time convergence rate for the equilibrium of the system, but these closed-loop systems are of interest not only because of faster convergence rates around the equilibria, but also because they can perform better disturbance rejection abilities [4, 5]. Therefore, there are more and more attentions focusing on the continuous (nonsmooth) control and many important results have been achieved.

For the second-order integrator systems, [4] proposes a homogeneous state feedback control method which is very inspiring for the following works by the nonlinear community, then [15] studies the problem of output feedback finite-time stabilization and gives a class of nonsmooth finite-time observers. For the high order systems, [28] presents an design procedure which is called adding a power integrator technique to solve a class of nth-order lower-triangular systems. [14] proposes a finite-time feedback control law for a class of STLC systems. For the nth-order finite-time observer problem, [24, 26, 29] handle the problem of the finite-time homogeneous observer and give significant results for finite-time output feedback control. And recently for the nth-order upper-triangular systems, the global asymptotical and finite-time stabilization problems have been addressed by integrating the adding a power integrator design technique with a series of nested saturation techniques in [10, 11] respectively.

When revisiting the actual systems such as the spacecraft attitude, [20] proposes a global set stabilization method using finite-time control technique. And for the rigid robot manipulators, [12, 16] use different control strategy to handle the finite-time control problem. [13] proposes a global finite-time output feedback control law to stabilize a PVTOL aircraft. When we consider the finite-time control problem for the quad-rotor UAVs, compared to the numerous interesting results achieved for the system that aim to achieve the asymptotical convergence rate research in the past several decades, there are fewer results on the finite-time stabilization for these intensively used systems. [3, 9] deal with the finite-time control problem by applying a super twisting algorithm [19, 27] and the authors have demonstrated the effectiveness of proposed methods with respect to the stabilization, tracking and disturbance rejection. Hence the research into the finite-time control for these highly used aircraft systems will lead to some theoretical and practical significance. In this paper, we use another nonsmooth control approach newly developed in [11] to propose an alternative strategy to finite-time stabilize and control the quadrotor mini-aircraft system.

Considering the mini-rotor aircraft which has four rotors, the authors first design a homogeneous finite-time controller to track and control the altitude and yaw angular movements, then extend the roll and pitch state feedback control result to finite-time stable case by employing the approach proposed in [11]. Considering the case when the translational velocities are not available from measurement due to weight consideration, cost limit or other design requirement, etc., a finite-time observer is constructed to estimate the translational velocities. Finally a finite-time output feedback controller is obtained by replacing the unmeasured velocity states with the observed ones. A rigorous stability analysis is provided to ensure the effectiveness of the proposed method.

This paper is organized as follows. Some preliminaries are presented in Section 2. Section 3 describes the system model and gives the control objective. In Section 4, the finite-time state feedback control law is proposed. Section 5 solves the finite-time output feedback control problem. Numerical simulations in Section 6 show the performance of closed loop system under the proposed controller. A conclusion and reference list end the paper.

2. PRELIMINARIES

Definition 2.1. (Bhat and Bernstein [5]) Consider a system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \tag{1}$$

where $f : U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. The zero solution of system (1) is finite-time convergent if there are an open neighborhood U of the origin and a function $T : U\backslash\{0\} \to (0, \infty)$, such that $\forall x_0 \in U$ every solution $x(t, 0, x_0)$ of system (1) with x_0 as the initial condition is defined and $x(t, 0, x_0) \in U\backslash\{0\}$ for $t \in [0, T(x_0))$, and $\lim_{t \to T(x_0)} x(t, 0, x_0) = 0$. The zero solution of this system is finite-time stable if it is Lyapunov stable and finite-time convergent. When $U = R^n$, then the zero solution is said to be globally finite-time stable.

Lemma 2.2. (Bhat and Bernstein [4]) Consider the following double integrator system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u. \end{aligned} \tag{2}$$

There exists a homogeneous controller

$$u = -k_1 sig^{\alpha_1}(x_1) - k_2 sig^{\alpha_2}(x_2), \tag{3}$$

where $sig^{\alpha}(x) = sign(x)|x|^{\alpha}$, $k_1, \ k_2 > 0$ are proper constants, $0 < \alpha_2 < 1$ and $\alpha_1 = \frac{\alpha_2}{2 - \alpha_2}$. Then the equilibrium of this system will be globally finite-time stable.

Lemma 2.3. (Hong et al. [15]) For the double integrator system (2), there exists a finite-time observer

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + k_1 sig^{\sigma_1}(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= u + k_2 sig^{\sigma_2}(x_1 - \hat{x}_1), \end{aligned} \tag{4}$$

where $k_1, \ k_2 > 0$, $\sigma_2 \in (0, 1)$, $\sigma_2 = 2\sigma_1 - 1$. Then the output feedback control law described as follows

$$u = -k_1 sig^{\alpha_1}(\hat{x}_1) - k_2 sig^{\alpha_2}(\hat{x}_2), \tag{5}$$

will globally finite-time stabilize system (2).

Next, consider the following system

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, 2, \ldots, n-1 \\ \dot{x}_n &= u. \end{aligned} \tag{6}$$

Lemma 2.4. (Ding et al. [11]) For system (6), if we construct the controller as the following form

$$u = u_n(X_n(t)) = -\beta_n \sigma_\epsilon^{r_{n+1}}(x_n^{1/r_n} - u_{n-1}^{1/r_n}(X_{n-1})), \tag{7}$$

where

$$\begin{array}{l} u_0 = 0, X_i = (x_1, x_2, \dots, x_i), \\ u_i(X_i(t)) = -\beta_i \sigma_\epsilon^{r_{i+1}}(x_i^{1/r_i} - u_{i-1}^{1/r_i}(X_{i-1})), i = 1, 2, \dots, n-1, \\ \sigma_\epsilon(x) = \begin{cases} \epsilon sign(x), & for\ |x| > \epsilon \\ x, & for\ |x| \le \epsilon \end{cases} \end{array} \tag{8}$$

with $r_i := (i-1)\tau + 1,\ i = 1, 2, \dots, n+1$ are fractions of two odd numbers with a constant $-\frac{1}{n} < \tau < 0$. Then there exist appropriate gains $\beta_i's$ such that controller (7) globally finite-time stabilizes system (6) for any small constant $\epsilon > 0$.

3. SYSTEM MODEL DESCRIPTION AND PROBLEM FORMULATION

A dynamic model of a quad-rotor mini-aircraft is expressed as follows [8]

$$\begin{array}{l} m\ddot{\xi} = u \begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \\ J\ddot{\eta} = -C(\eta, \dot{\eta})\dot{\eta} + \tau, \end{array} \tag{9}$$

where $(\xi^T, \dot{\xi}^T, \eta^T, \dot{\eta}^T) = (x, y, z, \dot{x}, \dot{y}, \dot{z}, \psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi})$ the states of system, x and y the coordinates in the horizontal plane, z the vertical position, ψ the yaw angle, θ the pitch angle, ϕ the roll angle, u the throttle control input, τ the generalized moments, and $C(\eta, \dot{\eta})$ the Coriolis term. Here a change of the input variables is used as follows

$$\tau = J\tilde{\tau} + C(\eta, \dot{\eta})\dot{\eta}, \tag{10}$$

where $\tilde{\tau} = [\tilde{\tau}_\psi\ \tilde{\tau}_\theta\ \tilde{\tau}_\phi]^T$, then (9) can be rewritten as:

$$m\ddot{x} = -u\sin\theta \tag{11}$$

$$m\ddot{y} = u\cos\theta\sin\phi \tag{12}$$

$$m\ddot{z} = u\cos\theta\cos\phi - mg \tag{13}$$

$$\ddot{\psi} = \tilde{\tau}_\psi \tag{14}$$

$$\ddot{\theta} = \tilde{\tau}_\theta \tag{15}$$

$$\ddot{\phi} = \tilde{\tau}_\phi, \tag{16}$$

where the control input u is the throttle input, and $\tilde{\tau}_\psi$, $\tilde{\tau}_\theta$, $\tilde{\tau}_\phi$ are the new angular moment inputs (yaw, pitch and roll moments).

The control objective in this paper is to design the throttle input u and the angular moment inputs $\tilde{\tau}_\psi$, $\tilde{\tau}_\theta$, $\tilde{\tau}_\phi$ to control the states to a desired value.

4. FINITE-TIME STATE FEEDBACK CONTROLLER DESIGN

In this section, a local finite-time state feedback control scheme is designed for the quad-rotor mini-aircraft system.

Assumption 4.1. Assume the pitch and roll angles θ and ϕ are both bounded in $(-\pi/2, \pi/2)$.

Theorem 4.2. Suppose the quad-rotor mini-aircraft system (11) – (16) satisfy Assumption 4.1, it will be finite-time stabilized by the following controller

$$\begin{aligned} u &= \tfrac{1}{\cos\phi\cos\theta}(-k_{z1}sig^{\alpha_1}(z-z_d) - k_{z2}sig^{\alpha_2}(\dot{z}) + mg) \\ \tilde{\tau}_\psi &= -k_{\psi 1}sig^{\tilde{\alpha}_1}(\psi) - k_{\psi 2}sig^{\tilde{\alpha}_2}(\dot{\psi}) \\ \tilde{\tau}_\theta &= \beta_4 \tfrac{\cos^2\theta}{g}\sigma_\epsilon^{1+4\gamma}((\tfrac{-g\dot{\theta}}{\cos^2\theta})^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((-g\tan\theta)^{\frac{1}{1+2\gamma}} \\ &\quad + \beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\dot{x}^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(x)))) - 2\tan\theta\dot{\theta}^2 \\ \tilde{\tau}_\phi &= -\beta_4 \tfrac{\cos^2\phi}{g}\sigma_\epsilon^{1+4\gamma}((\tfrac{g\dot{\phi}}{\cos^2\phi})^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((g\tan\phi)^{\frac{1}{1+2\gamma}} \\ &\quad + \beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\dot{y}^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(y)))) - 2\tan\phi\dot{\phi}^2, \end{aligned} \tag{17}$$

where $k_{z1},\ k_{\psi 1},\ k_{z2},\ k_{\psi 2} > 0$ are positive constants, $0 < \alpha_2 < 1,\ 0 < \tilde{\alpha}_2 < 1$ and $\alpha_1 = \frac{\alpha_2}{2-\alpha_2},\ \tilde{\alpha}_1 = \frac{\tilde{\alpha}_2}{2-\tilde{\alpha}_2},\ z_d$ is the desired altitude, $\beta_i,\ i = 1,2,3,4$ and γ, ϵ are proper gains defined as in Lemma 2.4.

Proof. For the subsystem (13), with Assumption 4.1 in mind, we notice $\cos\theta\cos\phi \neq 0$, then let

$$u = \frac{1}{\cos\theta\cos\phi}(u^* + mg), \tag{18}$$

then the subsystem (13) will reduce to

$$\ddot{z} = \frac{1}{m}u^*. \tag{19}$$

By using Lemma 2.2, the homogeneous controller

$$u^* = -k_{z1}sig^{\alpha_1}(z-z_d) - k_{z2}sig^{\alpha_2}(\dot{z}), \tag{20}$$

where $k_{z1}, k_{z2}, \alpha_1, \alpha_2$ are defined in Theorem 4.2, will drive z to a desired altitude z_d in a finite-time T_1. Similarly for the subsystem (14), the controller

$$\tilde{\tau}_\psi = -k_{\psi 1}sig^{\tilde{\alpha}_1}(\psi) - k_{\psi 2}sig^{\tilde{\alpha}_2}(\dot{\psi}), \tag{21}$$

where the parameters are defined similar with that in (20), will stabilize the state ψ to zero in a finite time T_2.

Substituting the control law (20), (21) into system (11) – (16), the system will reduce to

$$\begin{aligned} m\ddot{x} &= -(u^* + mg)\tfrac{\tan\theta}{\cos\phi} \\ m\ddot{y} &= (u^* + mg)\tan\phi \\ m\ddot{z} &= -k_{z1}sig^{\alpha_1}(z-z_d) - k_{z2}sig^{\alpha_2}(\dot{z}) \\ \ddot{\psi} &= -k_{\psi 1}sig^{\tilde{\alpha}_1}(\psi) - k_{\psi 2}sig^{\tilde{\alpha}_2}(\dot{\psi}) \\ \ddot{\theta} &= \tilde{\tau}_\theta \\ \ddot{\phi} &= \tilde{\tau}_\phi. \end{aligned} \tag{22}$$

With Lemma 2.2 in mind, after the time $t = \max\{T_1, T_2\}$, one can have $\psi = 0,\ z = z_d$. So the control inputs u^*, $\tilde{\tau}_\psi$ are equal to 0. Then the subsystem (11), (12), (15), (16) will reduce to

$$\begin{array}{l} \ddot{x} = -g\frac{\tan\theta}{\cos\phi} \\ \ddot{y} = g\tan\phi \\ \ddot{\theta} = \tilde{\tau}_\theta \\ \ddot{\phi} = \tilde{\tau}_\phi. \end{array} \tag{23}$$

In order to handle the right nonlinear items for (23), here feedback linearization method [17] is applied, first we consider $(y-\phi)$ subsystem

$$\begin{array}{l} \ddot{y} = g\tan\phi \\ \ddot{\phi} = \tilde{\tau}_\phi. \end{array} \tag{24}$$

The fourth derivative of y with respect to time can be easily obtained as follows

$$y^{(4)} = 2g\frac{\sin\phi}{\cos^3\phi}\dot{\phi}^2 + \frac{g}{\cos^2\phi}\tilde{\tau}_\phi. \tag{25}$$

Let $\tilde{\tau}_\phi = \frac{\cos^2\phi}{g}(v - 2g\frac{\sin\phi}{\cos^3\phi}\dot{\phi}^2) = \frac{\cos^2\phi}{g}v - 2\tan\phi\dot{\phi}^2$, then we have

$$y^{(4)} = v. \tag{26}$$

Applying Lemma 2.4, it is easily to obtain the controller for (26) as follows

$$v = -\beta_4\sigma_\epsilon^{1+4\gamma}((y^(3))^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((y^(2))^{\frac{1}{1+2\gamma}} + \beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\dot{y}^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(y)))). \tag{27}$$

Then for $y-\phi$ subsystem (24), we have the following controller

$$\begin{array}{rl} \tilde{\tau}_\phi & = -\beta_4\frac{\cos^2\phi}{g}\sigma_\epsilon^{1+4\gamma}((\frac{g\dot{\phi}}{\cos^2\phi})^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((g\tan\phi)^{\frac{1}{1+2\gamma}} \\ & +\beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\dot{y}^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(y)))) - 2\tan\phi\dot{\phi}^2 \end{array} \tag{28}$$

with proper gains β_i, γ and ϵ which are defined in Lemma 2.4. The controller (28) guarantees the convergence of the equilibrium of the closed loop subsystem (24) $--$ (28) within a finite time interval T_3, that is for any $t > T_3$, we have $\phi = 0$ and $y = 0$. Then $(x-\theta)$ subsystem reduces to

$$\begin{array}{l} \ddot{x} = -g\tan\theta \\ \ddot{\theta} = \tilde{\tau}_\theta. \end{array} \tag{29}$$

With a similar procedure, the fourth derivative of x is as follows

$$x^{(4)} = -2g\frac{\sin\theta}{\cos^3\theta}\dot{\theta}^2 - \frac{g}{\cos^2\theta}\tilde{\tau}_\theta, \tag{30}$$

with $\tilde{\tau}_\theta = -\frac{\cos^2\theta}{g}(\tilde{v} + 2g\frac{\sin\theta}{\cos^3\theta}\dot{\theta}^2) = -\frac{\cos^2\theta}{g}\tilde{v} - 2\tan\theta\dot{\theta}^2$ in mind, we have $x^{(4)} = \tilde{v}$. Similarly by Lemma 2.4, we have the following control law for subsystem (29)

$$\begin{array}{rl} \tilde{\tau}_\phi & = \beta_4\frac{\cos^2\theta}{g}\sigma_\epsilon^{1+4\gamma}((\frac{-g\dot{\theta}}{\cos^2\theta})^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((-g\tan\theta)^{\frac{1}{1+2\gamma}} \\ & +\beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\dot{x}^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(x)))) - 2\tan\theta\dot{\theta}^2. \end{array} \tag{31}$$

The above controller (31) guarantees the convergence for the states (θ, x) to zero within a finite time instant. This completes the proof. □

Remark 4.3. Letting $\alpha_1 = \tilde{\alpha}_1 = \alpha_2 = \tilde{\alpha}_2 = 1$, the controller (20) and (21) reduce to the PD controller proposed in [8].

Remark 4.4. In the normal case, gains are usually chosen sufficiently large to guarantee the system a better disturbance rejection ability. However the gains cannot be selected arbitrarily large when considering the control energy and the practical system's stability aspect. But in the finite-time control strategy, the fractional power items such as γ in (28), (31) can be regulated to achieve a better disturbance rejection ability while the control energy is bounded in a design level. So the finite-time control strategy leads to a better disturbance rejection ability with conventional asymptotic stability control method.

5. FINITE-TIME OUTPUT FEEDBACK CONTROLLER DESIGN

Considering the case when the translational velocities are not available from measurement, next a finite-time observer will be developed to estimate the translational velocities. Then naturally an output feedback finite-time controller based on the estimated states from observer can be obtained. However, it should be pointed out that the famous separation principle for the linear control system is usually not available for the nonlinear control system, which means that a rigorous proof should be given to guarantee the stability of the whole system including the state feedback control subsystem and the observation subsystem. The key point is to prove that during the procedure when the states of observer finite-time converge to the translational velocities, the states of closed loop system under the output feedback finite-time controller will not escape to infinity.

Define the whole states

$$[x, \dot{x}, y, \dot{y}, z, \dot{z}, \psi, \dot{\psi}, \theta, \dot{\theta}, \phi, \dot{\phi}] = [x_1, x_2, \ldots, x_{12}] = \mathbf{x}^T, \tag{32}$$

then rewrite the mini-rotorcraft system as

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= -\tfrac{u}{m}\sin x_9\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= \tfrac{u}{m}\cos x_9 \sin x_{11}\\
\dot{x}_5 &= x_6\\
\dot{x}_6 &= \tfrac{u}{m}\cos x_9 \cos x_{11} - mg\\
\dot{x}_7 &= x_8\\
\dot{x}_8 &= \tilde{\tau}_\psi\\
\dot{x}_9 &= x_{10}\\
\dot{x}_{10} &= \tilde{\tau}_\theta\\
\dot{x}_{11} &= x_{12}\\
\dot{x}_{12} &= \tilde{\tau}_\phi.
\end{aligned} \tag{33}$$

Provided that ψ, θ, ϕ are measurable and the angular velocities $(\dot{\psi}, \dot{\theta}, \dot{\phi})$ can be obtained from three gyros. For the translational velocities, a finite-time stable observer will be constructed to estimate the velocities $(\dot{x}, \dot{y}, \dot{z})$. The finite-time observer developed here is based on homogeneous method, as inspired by [15].

Theorem 5.1. Under the following observer

$$\begin{array}{l}\dot{\hat{x}}_1 = \hat{x}_2 + l_1 sig^{\delta_1}(x_1 - \hat{x}_1)\\ \dot{\hat{x}}_2 = -\frac{u}{m}\sin x_9 + l_2 sig^{\delta_2}(x_1 - \hat{x}_1)\\ \dot{\hat{x}}_3 = \hat{x}_4 + l_3 sig^{\delta_1}(x_3 - \hat{x}_3)\\ \dot{\hat{x}}_4 = \frac{u}{m}\cos x_9 \sin x_{11} + l_4 sig^{\delta_2}(x_3 - \hat{x}_3)\\ \dot{\hat{x}}_5 = \hat{x}_6 + l_5 sig^{\delta_1}(x_5 - \hat{x}_5)\\ \dot{\hat{x}}_6 = \frac{u}{m}\cos x_9 \cos x_{11} - mg + l_6 sig^{\delta_2}(x_5 - \hat{x}_5),\end{array} \tag{34}$$

where $l_i,\ i = 1, 2, \ldots, 6$ are positive gains to be determined later, and $\delta_1 \in (1/2, 1), \delta_2 = 2\delta_1 - 1$, the estimated states $\hat{x}_2, \hat{x}_4, \hat{x}_6$ can converge to the states x_2, x_4, x_6 of system (33) in a finite time.

P r o o f. Define the error $e_i = x_i - \hat{x}_i$, then the dynamics of observation error is given by

$$\begin{array}{l}\dot{e}_1 = e_2 - l_1 sig^{\delta_1}(e_1)\\ \dot{e}_2 = -l_2 sig^{\delta_2}(e_1)\\ \dot{e}_3 = e_4 - l_3 sig^{\delta_1}(e_3)\\ \dot{e}_4 = -l_4 sig^{\delta_2}(e_3)\\ \dot{e}_5 = e_6 - l_5 sig^{\delta_1}(e_5)\\ \dot{e}_6 = -l_6 sig^{\delta_2}(e_5).\end{array} \tag{35}$$

The above equations can be considered as three independent subsystems and each one is in the form

$$\begin{array}{l}\dot{e}_1 = e_2 - l_1 sig^{\delta_1}(e_1)\\ \dot{e}_2 = -l_2 sig^{\delta_2}(e_1).\end{array} \tag{36}$$

So using Lemma 2.3, one obtain that error system (35) is finite-time stable, i. e., there exists a time instant T_4, for $t > T_4$, such that

$$\hat{x}_2 = x_2, \hat{x}_4 = x_4, \hat{x}_6 = x_6. \tag{37}$$

The theorem is thus proved. □

In what follows, by replacing the unmeasurable states x_2, x_4, x_6 in (33) with the estimated states $\hat{x}_2,\ \hat{x}_4,\ \hat{x}_6$ generated from the finite-time observer (34), the main result of this section is obtained.

Theorem 5.2. The quad-rotor min-aircraft system (11) – (16) can be finite-time stabilized by the following output feedback controller

$$\begin{array}{ll} u & = \frac{1}{\cos x_9 \cos x_{11}}(-k_{z1} sig^{\alpha_1}(x_5 - z_d) - k_{z2} sig^{\alpha_2}(\hat{x}_6) + mg)\\ \tilde{\tau}_\psi & = -k_{\psi 1} sig^{\tilde{\alpha}_1}(x_7) - k_{\psi 2} sig^{\tilde{\alpha}_2}(x_8)\\ \tilde{\tau}_\theta & = \beta_4 \frac{\cos^2 x_9}{g} \sigma_\epsilon^{1+4\gamma}((\frac{-g x_{10}}{\cos^2 x_9})^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}} \sigma_\epsilon((-g \tan x_9)^{\frac{1}{1+2\gamma}}\\ & \quad + \beta_2^{\frac{1}{1+2\gamma}} \sigma_\epsilon(\hat{x}_2^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}} \sigma_\epsilon(x_1)))) - 2\tan x_9 x_{10}^2\\ \tilde{\tau}_\phi & = -\beta_4 \frac{\cos^2 x_{11}}{g} \sigma_\epsilon^{1+4\gamma}((\frac{g x_{12}}{\cos^2 x_{11}})^{\frac{1}{1+3\gamma}} + \beta_3^{\frac{1}{1+3\gamma}} \sigma_\epsilon((g \tan x_{11})^{\frac{1}{1+2\gamma}}\\ & \quad + \beta_2^{\frac{1}{1+2\gamma}} \sigma_\epsilon(\hat{x}_4^{\frac{1}{1+\gamma}} + \beta_1^{\frac{1}{1+\gamma}} \sigma_\epsilon(x_3)))) - 2\tan x_{11} x_{12}^2.\end{array} \tag{38}$$

Proof. Note that for $t > T_4$, with (37) in mind, then the finite-time output feedback control law (38) reduces to the state feedback control law (17). What follows is to prove the states x_i will be bounded at any finite-time interval T.

Substituting (38) into (33), yields

$$\begin{array}{ll}
\dot{x}_1 & = x_2 \\
\dot{x}_2 & = -\frac{-k_{z1}sig^{\alpha_1}(x_5-z_d)-k_{z2}sig^{\alpha_2}(\hat{x}_6)+mg}{m}\frac{\tan x_9}{\cos x_{11}} \\
\dot{x}_3 & = x_4 \\
\dot{x}_4 & = \frac{-k_{z1}sig^{\alpha_1}(x_5-z_d)-k_{z2}sig^{\alpha_2}(\hat{x}_6)+mg}{m}\tan x_{11} \\
\dot{x}_5 & = x_6 \\
\dot{x}_6 & = -\frac{k_{z1}}{m}sig^{\alpha_1}(x_5-z_d)-\frac{k_{z2}}{m}sig^{\alpha_2}(\hat{x}_6) \\
\dot{x}_7 & = x_8 \\
\dot{x}_8 & = -k_{\psi 1}sig^{\tilde{\alpha}_1}(x_7)-k_{\psi 2}sig^{\tilde{\alpha}_2}(x_8) \\
\dot{x}_9 & = x_{10} \\
\dot{x}_{10} & = \beta_4\frac{\cos^2 x_9}{g}\sigma_\epsilon^{1+4\gamma}((\frac{-gx_{10}}{\cos^2 x_9})^{\frac{1}{1+3\gamma}}+\beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((-g\tan x_9)^{\frac{1}{1+2\gamma}} \\
 & \quad +\beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\hat{x}_2^{\frac{1}{1+\gamma}}+\beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(x_1))))-2\tan x_9 x_{10}^2 \\
\dot{x}_{11} & = x_{12} \\
\dot{x}_{12} & = -\beta_4\frac{\cos^2 x_{11}}{g}\sigma_\epsilon^{1+4\gamma}((\frac{gx_{12}}{\cos^2 x_{11}})^{\frac{1}{1+3\gamma}}+\beta_3^{\frac{1}{1+3\gamma}}\sigma_\epsilon((g\tan x_{11})^{\frac{1}{1+2\gamma}} \\
 & \quad +\beta_2^{\frac{1}{1+2\gamma}}\sigma_\epsilon(\hat{x}_4^{\frac{1}{1+\gamma}}+\beta_1^{\frac{1}{1+\gamma}}\sigma_\epsilon(x_3))))-2\tan x_{11}x_{12}^2.
\end{array} \tag{39}$$

Denoting the right hand side of (39) as $\mathbf{F}(x_1, x_2, \hat{x}_2, \dots, x_{12})$, so system (39) can be written as

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{F}(\cdot) = [F_1(\cdot), F_2(\cdot), \dots, F_{12}(\cdot)]^T. \tag{40}$$

First, since x_2, x_4 converge to $\hat{x}_2, \hat{x}_4$ within a finite-time T_4, from the saturation property of (28), (31), it is easy to conclude that $x_9, x_{10}, x_{11}, x_{12}$ is bounded at any finite-time interval $[0, T]$. Associate with Assumption 4.1, then there exist two positive constants c_1, c_2 such that

$$\begin{array}{l}
\frac{\tan x_9}{\cos x_{11}} \le c_1 \\
\tan x_9 \tan x_{11} \le c_2.
\end{array} \tag{41}$$

Note that for $0 < \alpha_1, \tilde{\alpha}_1, \alpha_2, \tilde{\alpha}_2 < 1$, one has

$$\begin{array}{l}
sig^{\alpha_1}(x_5-z_d) \le 1+|x_5-z_d| \le 1+|z_d|+|x_5| \\
sig^{\alpha_2}(\hat{x}_6) \le 1+|\hat{x}_6| \\
sig^{\tilde{\alpha}_1}(x_7) \le 1+|x_7| \\
sig^{\tilde{\alpha}_2}(x_8) \le 1+|x_8|.
\end{array} \tag{42}$$

With (41) and (42) in mind, there exists proper positive constants c_i, $i = 3, 4, \dots, 11$ such that

$$\begin{array}{l}
||F_2(\cdot)|| \le c_3(|x_5|+|\hat{x}_6|)+c_4 \\
||F_4(\cdot)|| \le c_5(|x_5|+|\hat{x}_6|)+c_6 \\
||F_6(\cdot)|| \le c_7(|x_5|+|\hat{x}_6|)+c_8 \\
||F_8(\cdot)|| \le c_9(|x_7|+|x_8|)+c_{10} \\
||F_{10}(\cdot)|| \le c_{11} \\
||F_{12}(\cdot)|| \le c_{11}.
\end{array} \tag{43}$$

Construct a Lyapunov function for system (39) as the following form

$$V(X) = \frac{1}{2}\sum_{i=1}^{12} x_i^2, \tag{44}$$

then the derivative of V is

$$\begin{aligned}\dot{V}(X) &= \sum_{i=1}^{n} x_i F_i(\cdot) \\ &\leq \sum_{i=1}^{6} x_{2i-1}x_{2i} + (|x_5| + |\hat{x}_6|)(c_3x_2 + c_5x_4 + c_7x_6) \\ &\quad +(c_4x_2 + c_6x_4 + c_8x_6) + c_{11}(x_{10} + x_{12}).\end{aligned} \tag{45}$$

Applying the inequality $xy \leq \frac{1}{2}(x^2 + y^2),\ x, y \in R$ to the terms in (45), one can have

$$\sum_{i=1}^{6} x_{2i-1}x_{2i} \leq \frac{1}{2}\sum_{i=1}^{12} x_i^2 \tag{46}$$

$$(c_4x_2+c_6x_4+c_8x_6)+c_{11}(x_{10}+x_{12}) \leq \frac{1}{2}(c_4^2+c_6^2+c_8^2+2c_{11}^2)+\frac{1}{2}(x_2^2+x_4^2+x_6^2+x_{10}^2+x_{12}^2) \tag{47}$$

$$\begin{aligned}(|x_5| + |\hat{x}_6|)(c_3x_2 + c_5x_4 + c_7x_6) &\leq \tfrac{1}{2}c_3(2x_2^2 + x_5^2 + \hat{x}_6^2) + \tfrac{1}{2}c_5(2x_4^2 + \hat{x}_6^2 + x_5^2) \\ &\quad +\tfrac{1}{2}c_7(x_5^2 + 2x_6^2 + \hat{x}_6^2) \\ &\leq C_1(x_2^2 + x_4^2 + x_5^2 + x_6^2 + \hat{x}_6^2),\end{aligned} \tag{48}$$

where $C_1 = \max\{c_3, c_5, c_7\}$. Note that in the finite-time observer (34), $\hat{x}_6$ is always bounded for any finite time t. Then following (46), (47) and (48), one can have

$$\begin{aligned}\dot{V}(X) &\leq \tfrac{1}{2}\sum_{i=1}^{12} x_i^2 + C_1(x_2^2 + x_4^2 + x_5^2 + x_6^2 + \hat{x}_6^2) + \tfrac{1}{2}(c_4^2 + c_6^2 + c_8^2 + 2c_{11}^2) \\ &\quad +\tfrac{1}{2}(x_2^2 + x_4^2 + x_6^2 + x_{10}^2 + x_{12}^2) \\ &\leq C_2(\tfrac{1}{2}\sum_{i=1}^{12} x_i^2) + C_3 \\ &= C_2V(X) + C_3,\end{aligned} \tag{49}$$

where $C_2 = \max\{2, 2C_1 + 2\},\ \ C_3 = \frac{1}{2}(c_4^2 + c_6^2 + c_8^2 + 2c_{11}^2) + C_1\hat{x}_6^2$. Define the initial value of $V(X)$ as V_0, from (49), one can obtain

$$V(X) \leq \left(V_0 + \frac{C_3}{C_2}\right)e^{C_2t} - \frac{C_3}{C_2}. \tag{50}$$

It can be easily concluded from (50) that $V(X)$ will not escape to infinity for any finite time t, which implies that the states $x_1, \ldots, x_{12}$ will not escape to infinity in the time interval $[0, T_4]$. Then when $t > T_4$, the output feedback controller (38) reduces to the state feedback controller (17), with Theorem 4.2 in mind, the closed-loop system (33), (34) and (38) is finite-time stable. $\square$

6. NUMERICAL SIMULATION RESULTS

In what follows, Matlab is used to simulate the performance of the proposed finite-time controller. The control objective is to drive the mini-rotor craft to an altitude of 20(m), i. e., the state of the system model is expected to be driven to the position $(x, y, z) = (0, 0, 20)(m)$ while $(\psi, \theta, \phi) = (0, 0, 0)(rad)$. Here the parameters for the control law (17) are selected as $m = 2(kg), \gamma = -2/13, k_{z1} = 1, k_{\psi 1} = 1/2, k_{z2} = 1, k_{\psi 2} = 1/2, \alpha_1 = \tilde{\alpha}_1 = 1/5, \alpha_2 = \tilde{\alpha}_2 = 1/3,\ \beta_1 = 1, \beta_2 = 3, \beta_3 = 10, \beta_4 = 30, \epsilon = 0.2$. The initial states for conducting the simulation are

$$\begin{aligned}
&(x(0), y(0), z(0)) = (10, -10, 0)(m),\\
&(\dot{x}(0), \dot{y}(0), \dot{z}(0)) = (0.5, 0.5, 0.5)(m/s),\\
&(\psi(0), \theta(0), \phi(0)) = (0.5, 0.5, 0.5)(rad),\\
&(\dot{\psi}(0), \dot{\theta}(0), \dot{\phi}(0)) = (0.5, 0.5, 0.5)(rad/s).
\end{aligned} \tag{1}$$

The initial values for observer (34) are selected as $(\dot{\hat{x}}(0), \dot{\hat{y}}(0), \dot{\hat{z}}(0)) = (0, 0, 0)(rad/s)$ and the parameters are $\delta_1 = 3/5, \delta_2 = 1/5, l_i = 0.5, i = 1, 2, \ldots, 6$.

Figure 1 shows the response curves under the finite-time state feedback controller (17). Figure 2 shows the time response of the control input signals. Figure 3 shows the performance of finite-time observer (34) which performs a fast convergence rate. Figure 4 shows the response curves under the finite-time output feedback controller (38).

In what follows, The results in this paper will be compared with the previous controller in [8] for controlling the four-rotor mini-aircraft. That controller is given as follows

$$\begin{aligned}
&u = \tfrac{1}{\cos\theta\cos\phi}(-a_{z1}\dot{z} - a_{z2}(z - z_d) + mg),\\
&\tilde{\tau}_\psi = -a_{\psi 1}\dot{\psi} - a_{\psi 2}(\psi - \psi_d),\\
&\tilde{\tau}_\phi = -b_1\sigma_{\phi_1}(\dot{\phi} + \sigma_{\phi_2}(\dot{\phi} + \phi + \sigma_{\phi_3}(\dot{\phi} + 2\phi + \tfrac{\dot{y}}{g} + \sigma_{\phi_4}(\dot{\phi} + 3\phi + 3\tfrac{\dot{y}}{g} + \tfrac{y}{g})))),\\
&\tilde{\tau}_\theta = -b_2\sigma_{\theta_1}(\dot{\theta} + \sigma_{\theta_2}(\dot{\theta} + \theta + \sigma_{\theta_3}(\dot{\theta} + 2\theta - \tfrac{\dot{x}}{g} + \sigma_{\theta_4}(\dot{\theta} + 3\theta - 3\tfrac{\dot{x}}{g} - \tfrac{x}{g})))).
\end{aligned} \tag{2}$$

To have a fair comparison, proper gains are carefully chosen for the controller (2) so that the control signal amplitudes of both controllers are in the same level.

Here two disturbances $d_1(t) = 0.3\sin(t), d_2(t) = 16\sin(2t)$ are added to subsystems (14), (15), (16). The corresponding model now is

$$\begin{aligned}
\ddot{\psi} &= \tilde{\tau}_\psi + d_1(t)\\
\ddot{\theta} &= \tilde{\tau}_\theta + d_2(t)\\
\ddot{\phi} &= \tilde{\tau}_\phi + d_2(t).
\end{aligned} \tag{3}$$

By spending time on regulating the control parameters for these two controllers to make the performances of their closed loop systems as good as possible. Here the parameters in (2) are selected as $a_{z1} = 0.1, a_{z2} = 0.2, a_{\psi 1} = 1, a_{\psi 2} = 1, b_1 = b_2 = 20, \phi_i = \theta_i = 1, i = 1, 2, 3, 4$ and the initial values are selected the same as (1). Figure 5 shows the state responses under these two different controllers. It can be observed that the finite-time controller can produce a faster convergence rate and a better disturbance rejection ability. From Figure 6, one can see the control input amplitudes of both controller are almost in the same level.

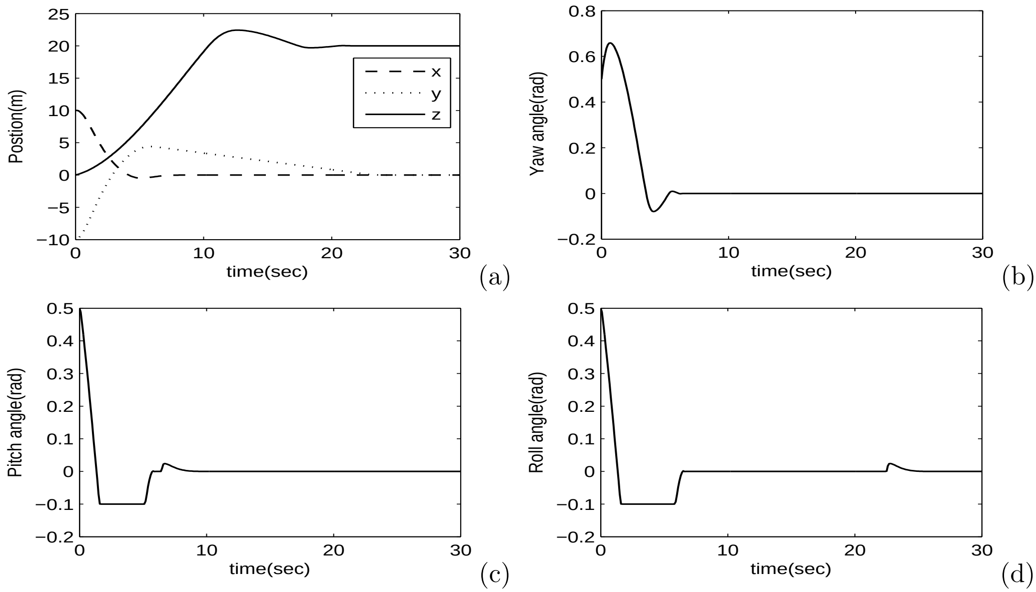

Fig. 1. Responses of states under finite-time state controller (17).
(a) position. (b) yaw angle. (c) pitch angle. (d) roll angle.

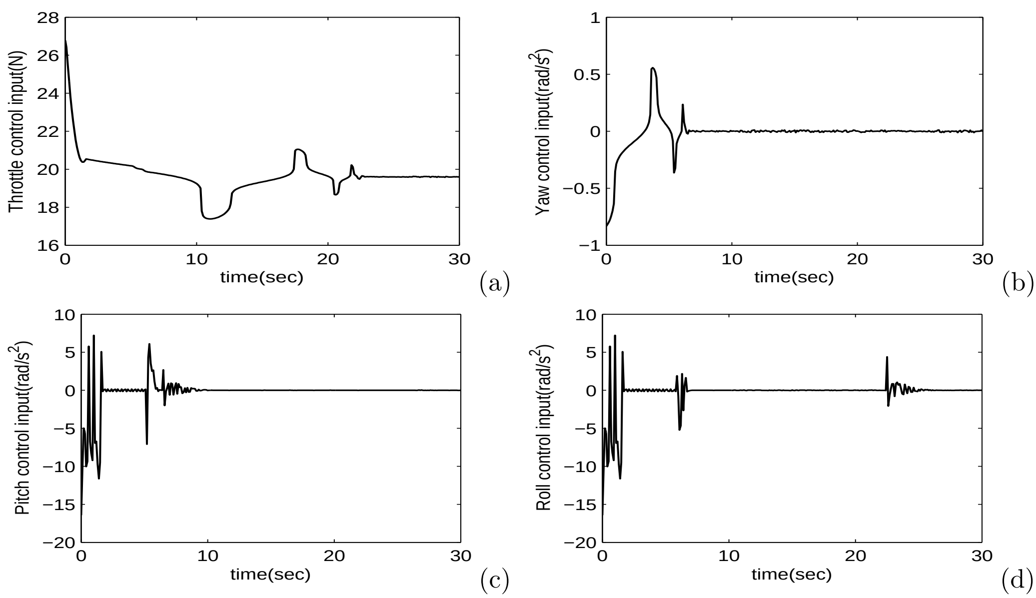

Fig. 2. Responses of control signals under finite-time state controller (17).
(a) throttle input. (b) yaw moment. (c) pitch moment. (d) roll moment.

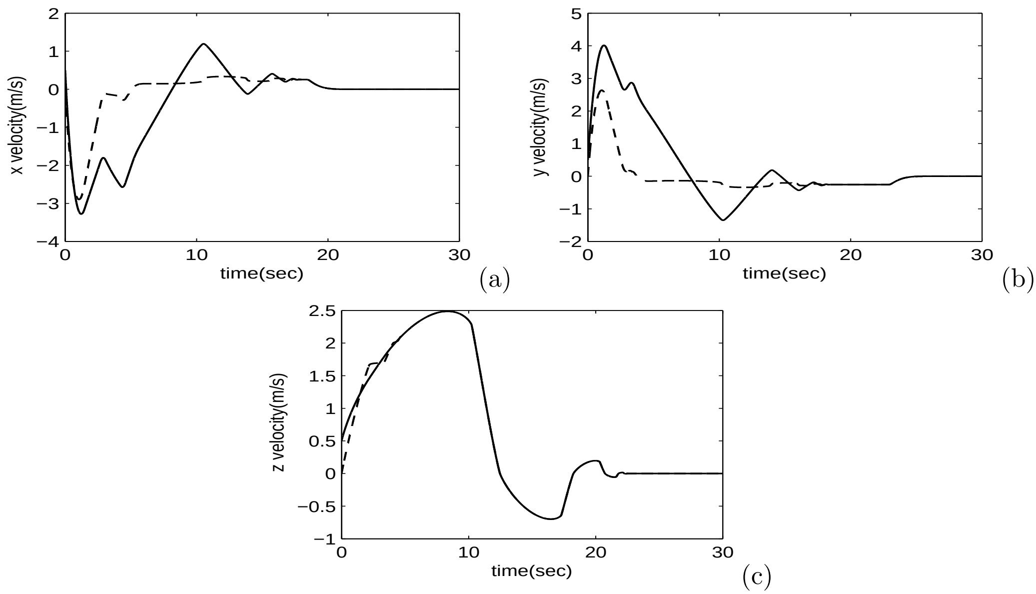

Fig. 3. State responses of finite-time observer (34). (a) $\dot{x}$. (b) $\dot{y}$. (c) $\dot{z}$.
(Solid lines denote the velocities and dotted lines denote their estimates.)

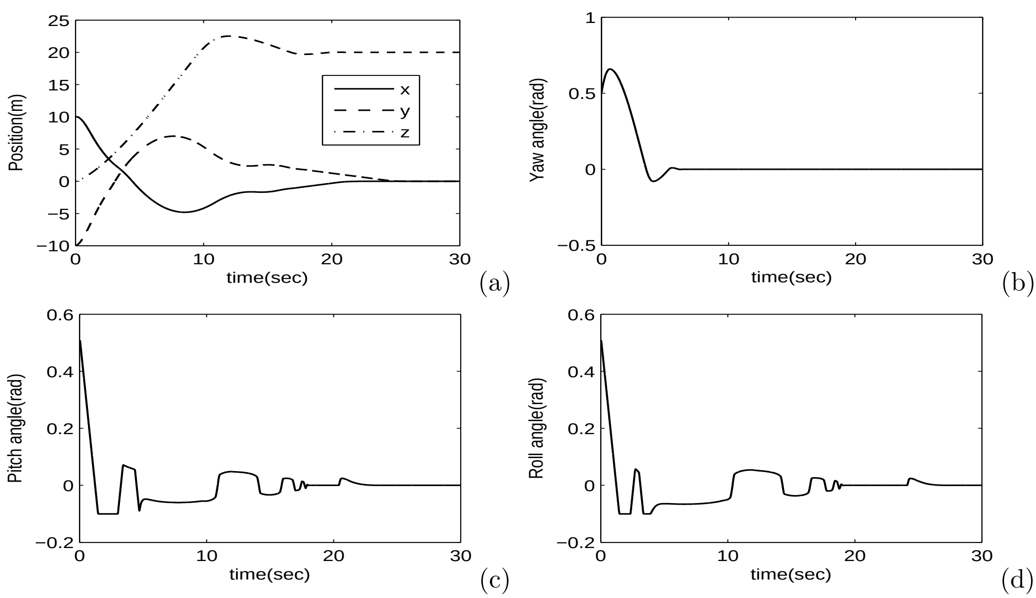

Fig. 4. State responses under the finite-time output feedback controller (38).
(a) position. (b) yaw angle. (c) pitch angle. (d) roll angle.

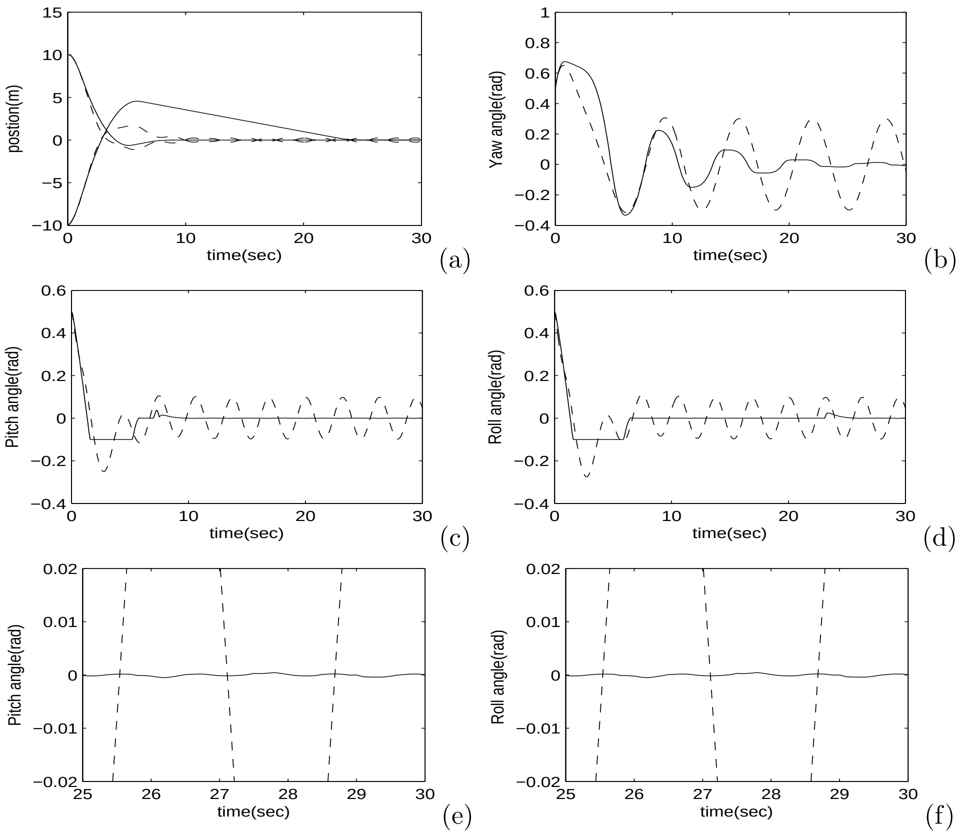

Fig. 5. State responses in the presence of disturbances under finite-time controller (17) (solid line) and the controller (2) (dotted line).
(a) position. (b) yaw angle. (c) pitch angle. (d) roll angle. (e) an enlarged diagram of (c). (f) an enlarged diagram of (d).

7. CONCLUSION

In this paper, considering the case that the velocity states are not measurable, the finite-time output feedback control problem for the quad-rotor mini-aircraft system has been addressed. First, under some assumptions, a finite-time state feedback controller has been designed by employing the finite-time control techniques. Then, a finite-time stable observer has been developed to estimate the unmeasurable states. Finally a finite-time output feedback controller has been obtained. A stability analysis has been provided to ensure the finite-time stability of the closed loop system. This control method has a better robustness against disturbances and performs a faster convergence rate than the conventional control method. Simulation results have shown the effectiveness of the proposed control method.

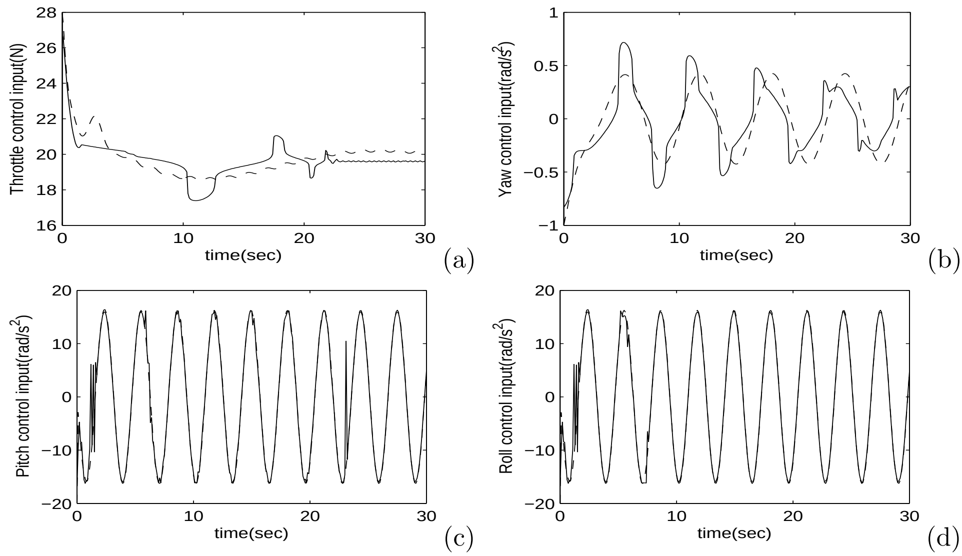

Fig. 6. Responses of control signals in the presence of disturbances under finite-time controller (17) (solid line), the controller (2) (dotted line).
(a) throttle input. (b) yaw moment. (c) pitch moment. (d) roll moment.

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REFERENCES

[1] K. Alexis, G. Nikolakopoulos, and A. Tzes: Constrained-control of a quadrotor helicopter for trajectory tracking under wind-gust disturbances. In: Proc. 15th IEEE Mediterranean Electrotechnical Conference 2010, pp. 1411–1416.

[2] K. Alexis, G. Nikolakopoulos, and A. Tzes: Constrained optimal attitude control of a quadrotor helicopter subject to wind-gusts: Experimental studies. In: Proc. Amer. Control Conference 2010, pp. 4451–4455.

[3] A. Benallegue, A. Mokhtari, and L. Fridman: High-order sliding-mode observer for a quadrotor UAV. Internat. J. Robust and Nonlinear Control *18* (2008), 427–440.

[4] S. P. Bhat and D. S. Bernstein: Finite-time stability of homogeneous systems. In: Proc. Amer. Control Conference 1997, pp. 2513-2514.

[5] S. P. Bhat and D. S. Bernstein: Continuous finite-time stabilization of the translational and rotational double integrators. IEEE Trans. Automat. Control *43* (1998), 678–682.

[6] S. Bouabdallah, E. Murrieri, and R. Siegwart: Design and control of an indoor micro quadrotor. In: Proc. IEEE International Conference on Robotics and Automation 2004, pp. 4393–4398.

[7] S. Bouabdallah and R. Siegwart: Backstepping and sliding-mode techniques applied to an indoor micro quadrotors. In: Proc. IEEE International Conference on Robotics and Automation 2005, pp. 2247–2252.

[8] P. Castillo, A. Dzul, and R. Lozano: Real-time stabilization and tracking of a four-rotor mini-aircraft. IEEE Trans. Control Systems Technol. *12* (2004), 510–516.

[9] L. Derafa, L. Fridman, A. Benallegue, and A. Ouldali: Super twisting control algorithm for the four rotors helicopter attitude tracking problem. In: Proc. 11th International Workshop on Variable Structure Systems 2010, pp. 62–67.

[10] S. H. Ding, C. J. Qian, and S. H. Li: Global stabilization of a class of feedforward systems with lower-order nonlinearities. IEEE Trans. Automat. Control *55* (2010), 691–696.

[11] S. H. Ding, C. J. Qian, and S. H. Li: Global stabilization of a class of upper-triangular systems with unbounded or uncontrollable linearizations. Internat. J. Robust and Nonlinear Control *21* (2010), 271–294.

[12] Y. Feng, X. H. Yu, and Z. H. Man: Non-singular terminal sliding mode control of rigid manipulators. Automatica *38* (2002), 2159–2167.

[13] M. T. Frye, S. H. Ding, C. J. Qian, and S. H. Li: Global Finite-time stabilization of a PVTOL aircraft by output feedback. In: Proc. 48th IEEE Conference on Decision and Control 2009, pp. 2831–2836.

[14] Y. G. Hong: Finite-time stablization and stabilizability of a class of controllable systems. Systems Control Lett. *46* (2002), 231–236.

[15] Y. G. Hong, J. Huang, and Y. S. Xu: On an output feedback finite-time stabilization problem. IEEE Trans. Automat. Control *46* (2001), 305–309.

[16] Y. G. Hong, Y. S. Xu, and J. Huang: Finite time control for robot manipulators. Systems Control Lett. *46* (2002), 243–253.

[17] A. Isidori: Nonlinear Control systems: An Introduction, Lecture Notes in Control and Information Sciences. Springer, Berlin 1985.

[18] L. C. Lai, C. C. Yang, and C. J. Wu: Time-optimal control of a hovering quad-rotor helicopter. J. Intell. and Robotic Systems *45* (2006), 115–135.

[19] A. Levant: Sliding order and sliding accuracy in sliding mode control. Internat. J. of Control *58* (1993), 1247–1263.

[20] S. H. Li, S. H. Ding, and Q. Li: Global set stabilization of the spacecraft attitude using finite-time control technique. Internat. J. Control *82* (2009), 822–836.

[21] T. Madani and A. Benallegue: Control of a quadrotor mini-helicopter via full state backstepping technique. In: Proc. 45th IEEE Conference on Decision and Control 2006.

[22] T. Madani and A. Benallegue: Backstepping control with exact 2-sliding mode estimation for a quadrotor unmanned aerial vehicle. In: Proc. International Conference on Intelligent Robots and Systems (2007), 141–146.

[23] T. Madani and A. Benallegue: Sliding mode observer and backstepping control for a quadrotor unmanned aerial vehicles. In: Proc. Amer. Control Confer. 2007, pp. 5887–5892.

[24] T. Ménard, E. Moulay, and W. Perruquetti: A global High-gain Finite-time Observer. IEEE Trans. Automat. Control *55* (2010), 1500–1506.

[25] V. Mistler, A. Benallegue, and N. K. M. Sirdi: Exact Linearization and Noninteracting Control of a 4 Rotors Helicopter via Dynamic Feedback. In: Proc. 2001 IEEE International Workshop on Robot and Human Interactive Communication 2001.

[26] W. Perruquetti, T. Floquet, and E. Moulay: Finite time observers and secure communication. IEEE Trans. Automat. Control *53* (2008), 356–360.

[27] A. Polyakov and A. Poznyak: Reaching time estimation for "super-twisting" second order sliding mode controller via Lyapunov function designing. IEEE Tran. Automat. Control *54* (2009), 1951–1955.

[28] C. J. Qian and W. Lin: A continuous feedback approach to global strong stabilization of nonlinear systems. IEEE Trans. Automat. Control *46* (2001), 1061–1079.

[29] Y. J. Shen and X. H. Xia: Semi-global finite-time observers for nonlinear systems. Automatica *44* (2008), 3152–3156.

[30] A. R. Teel: A nonlinear small gain theorem for the analysis of control systems with saturation. IEEE Trans. Automat. Control *41* (1996), 1256–1270.

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